AP Calculus Limits Definition of Derivative Critical Homework (Answers)

1b) $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^3 - 12(x + \Delta x)\right] - \left[x^3 - 12x\right]}{\Delta x}$$

1c) $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x + 4} - \sqrt{x + 4}}{\Delta x}$$

1d) $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\frac{4}{\sqrt{x+\Delta x}} - \frac{4}{\sqrt{x}}}{\Delta x}$$

1e) $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\tan(x+\Delta x) - \tan x}{\Delta x}$$

2b) $$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{\left[(x + \Delta x)^2 + 3\right] - \left[x^2 + 3\right]}{\Delta x}$$
$$= \lim_{\Delta x \to 0} \frac{2x\,\Delta x + (\Delta x)^2}{\Delta x}$$
$$= \lim_{\Delta x \to 0} (2x + \Delta x) = 2x$$

At $(1, 4)$, the slope of the tangent line is $m = 2(1) = 2$.

The equation of the tangent line is $y - 4 = 2(x - 1)$
$$y - 4 = 2x - 2$$
$$y = 2x + 2.$$

2c) $$f'(x) = \lim_{\Delta x \to 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \cdot \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$
$$= \lim_{\Delta x \to 0} \frac{(x + \Delta x) - x}{\Delta x\left(\sqrt{x + \Delta x} + \sqrt{x}\right)}$$
$$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

At $(1, 1)$, the slope of the tangent line is $m = \frac{1}{2\sqrt{1}} = \frac{1}{2}$.

The equation of the tangent line is
$$y - 1 = \frac{1}{2}(x - 1)$$
$$y = \frac{1}{2}x + \frac{1}{2}.$$