

## Limits Containing Trig Functions

### Limits of Expressions Containing Trigonometric Functions

Note: Since it is common to mix algebraic expressions with trig. functions **ALWAYS** use radians when graphing!

$$\lim_{x \rightarrow \pi/3} \csc x$$

As always, try to evaluate the limit using direct substitution. If undefined and you know there is an asymptote, the limit DNE. Sine and Cosine will of course always have a defined limit since they are continuous functions.

$$\lim_{x \rightarrow \pi} \cot x$$

### Limits of Expressions Containing Trigonometric Functions Along With Algebraic Expressions

Use the graphing calculator to find each limit:

$$\text{A) } \lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$\text{B) } \lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$$

Although direct substitution yields an undefined answer, there is a HOLE IN THE GRAPH instead of an ASYMPTOTE even though the expression CANNOT be algebraically simplified to remove the expression making it undefined.

Three KEY limits where this happens and are very important to know!

$$1) \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$2) \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$3) \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

Also remember that because limits are directly correlated their graphs, any translation of these are also true.

Lastly, the reciprocals of both limits that equal 1 will also equal 1 (#1 and #3).

Use *algebra* and the *properties of limits* to evaluate limits containing trig. functions and algebraic expressions.

$$\text{EX) } \lim_{x \rightarrow 0} \frac{\sin 2x}{x}$$

$$\text{EX) } \lim_{x \rightarrow 0} \frac{\cos x \tan x}{x}$$

$$\text{EX) } \lim_{x \rightarrow 0} \frac{2 \csc x - 2 \cot x}{3x \csc x}$$