Limits of Functions

W-up: Sketch $y=\dfrac{1}{x-2}$ without a calculator

Graph $y=\dfrac{x^2-4}{x-2}$ using the graphing calculator and sketch in your notebook. Where are the asymptotes?

We say that the equation $y=\dfrac{x^2-4}{x-2}$ and $y=x+2$ agree at ALL points except one.

Limit: the "y-value" that a function APPROACHES from the left and the right(the two MUST be in agreement)

Notation: $\lim\limits_{x\to 2}$ means the "limit as *x* approaches two"

Find the $\lim\limits_{x\to 2} f(x)$ for each function:

A) $f(x)=x^2+2$

B) $f(x)=-x^3-2$

So, for continuous functions such as polynomials, direct substitution will yield the limit $\lim\limits_{x\to c} f(x)=f(c)$

C) $f(x)=\dfrac{x^2-4}{x-2}$

D) $f(x)=\dfrac{1}{x-2}$

E) $f(x)=3$

Methods for finding limits when a hole in the graph exists(the x-value being approached makes the function UNDEFINED but can be rewritten so it is not)

FACTORING

EX) $\lim_{x \to -3} \frac{x^2 - 9}{x+3}$ Factor expression and try direct substitution again.

EX) $\lim_{x \to -2} \frac{x^3 + 8}{x+2}$

RATIONALIZATION

EX) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$ Rationalize the numerator by multiplying numerator and denominator by the conjugate. Try direct substitution again.

NOTE: DO NOT MULTIPLY OUT THE DENOMINATOR!!!!

ALGEBRA SIMPLIFICATION

$\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$ Simplify any algebraic expression and try direct substitution again.

When Limits DO NOT EXIST(DNE)

Asymptotes

EX) $\lim_{x \to 4} \frac{1}{x-4}$

Discontinuity(where limit from the left does NOT equal limit from the right)

$$\lim_{x \to 0} \begin{cases} x+2, & x \leq 0 \\ x^2-1, & x > 0 \end{cases}$$

Note: No limit can exist if the value approached is NOT in the domain of the function.

$$\lim_{x \to 5} \sqrt{x-8}$$

Note: Since it is common to mix algebraic expressions with trig. functions ALWAYS use radians when graphing!

$\lim_{x \to \pi/3} \csc x$

As always, try to evaluate the limit using direct substitution. If undefined and you know there is an asymptote, the limit DNE. Sine and Cosine will of course always have a defined limit since they are continuous functions.

$\lim_{x \to \pi} \cot x$

Issues with Limits

Use the graphing calculator and the table to find the limit:

$\lim_{x \to 0} \frac{\sin x}{x}$

Although direct substitution yields an undefined answer, there is a HOLE IN THE GRAPH instead of an ASYMPTOTE **even though the expression CANNOT be algebraically simplified to remove the expression that makes it undefined.** So…even if an expression is undefined at an x-value and cannot be changed or simplified to make it defined there DOES NOT NECESSARILY MAKE IT AN ASYMPTOTE! Note: One case of this happening is when trig. functions and variable expressions are divided.

Three KEY limits where this happens!

1) $\lim_{x \to 0} \frac{\sin x}{x} = 1$ 2) $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$ 3) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

Note: We will have a way to find these algebraically(by hand) at a later time!

Properties of Limits

Properties of limits can be used to help us find limits of complex expressions where we do not necessarily know the graph as a frame of reference

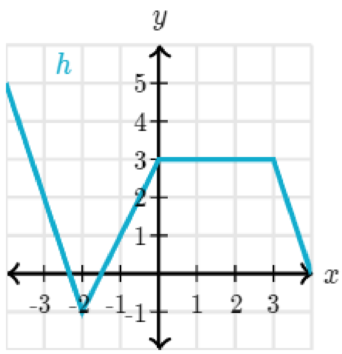
Scalar: $$\lim_{x \to c} 3 \bullet f(x) = 3 \lim_{x \to c} f(x)$$

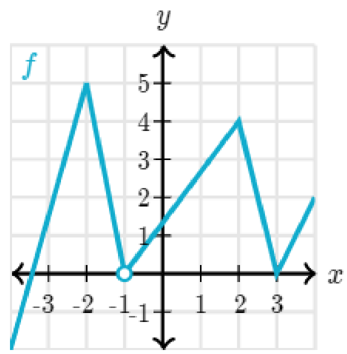
Mult./Div.: $$\lim_{x \to c} f(x) \bullet g(x) = \lim_{x \to c} f(x) \bullet \lim_{x \to c} g(x)$$

AND

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$$

EX)

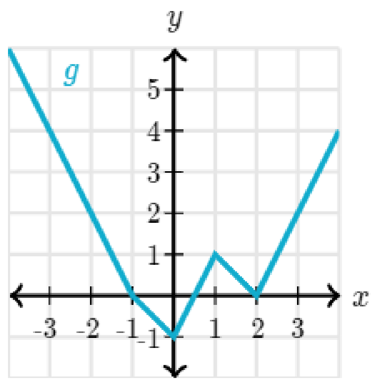


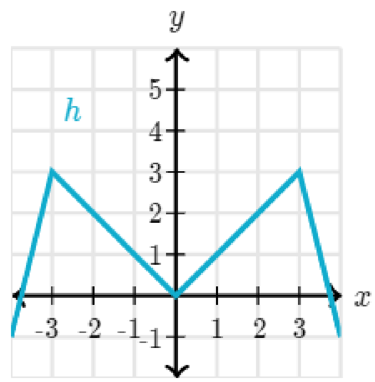


Find $\lim_{x \to -1} \frac{h(x)}{f(x)}$.

Add/Subt.: $$\lim_{x \to c} f(x) \pm g(x) = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x)$$

EX)



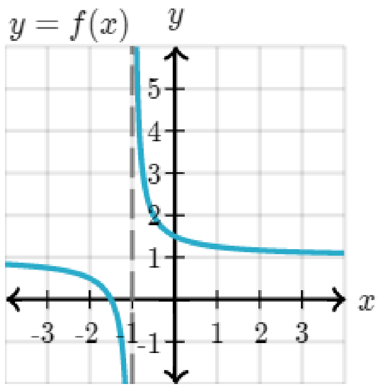


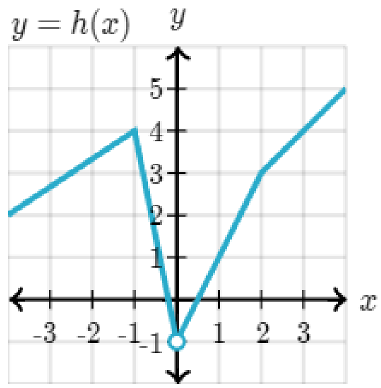
Find $\lim_{x \to 0} \left(g(x) - h(x)\right)$.

Composite Functions

$$\lim_{x\to c} f\left[g(x)\right] = f\left(\lim_{x\to c} g(x)\right)$$

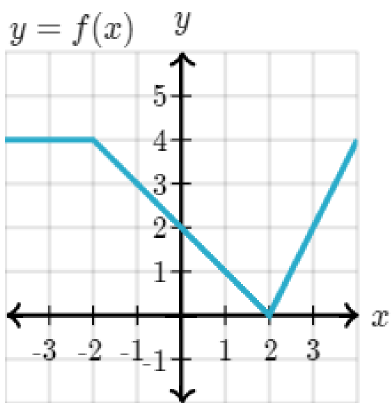
EX)

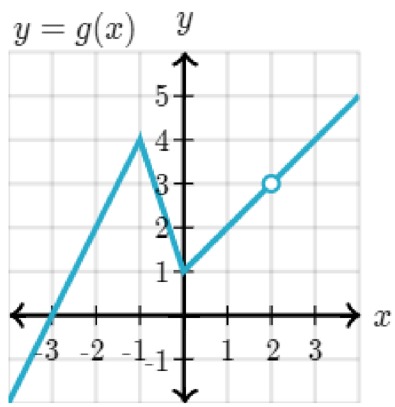




Find $\lim_{x\to 0} f\Big(h(x)\Big)$.

EX)





Find $\lim_{x\to 2} f\Big(g(x)\Big)$.

How the AP Test uses this property

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$f(x)$	-0.1054	-0.0101	-0.001	0.001	0.0099	0.0953

The function f is continuous and increasing for $x > -1$. The table above gives values of f at selected values of x. Of the following, which is the best approximation for $\lim_{x\to 0} e^{-2f(x)}$?

(A) -2

(B) 0

(C) 1

(D) The limit does not exist.