Limits of Functions

W-up: Graph $y=\frac{x^2-4}{x-2}$ using the graphing calculator and sketch in your notebook. Where are the asymptotes?

We say that the equation $y=\frac{x^2-4}{x-2}$ and $y=x+2$ agree at ALL points except one.

Limit: the "y-value" that a function APPROACHES from the left and the right(the two MUST be in agreement)

Notation: $\lim_{x\to 2}$ means the "limit as *x* approaches two"

Find the $\lim_{x\to 2} f(x)$ for each function:

A) $f(x)=x^2+2$

B) $f(x)=-x^3-2$

So, for continuous functions such as polynomials, direct substitution will yield the limit $\lim_{x\to c} f(x)=f(c)$

C) $f(x)=\frac{x^2-4}{x-2}$

D) $f(x)=\frac{1}{x-2}$

E) $f(x)=3$

Methods for finding limits when a hole in the graph exists(the x-value being approached makes the function UNDEFINED but can be rewritten so it is not)

FACTORING

EX) $\lim_{x\to -3} \frac{x^2-9}{x+3}$ Factor expression and try direct substitution again.

EX) $\lim_{x\to -2} \frac{x^3+8}{x+2}$

RATIONALIZATION

EX) $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$ Rationalize the numerator by multiplying numerator and denominator by the conjugate. Try direct substitution again.

NOTE: DO NOT MULTIPLY OUT THE DENOMINATOR!!!!

ALGEBRA SIMPLIFICATION

$\lim_{x\to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$ Simplify any algebraic expression and try direct substitution again.

When Limits DO NOT EXIST(DNE)

Asymptotes

EX) $\lim_{x\to 4}\frac{1}{x-4}$

Discontinuity(where limit from the left does NOT equal limit from the right)

$$\lim_{x\to 0}\begin{cases} x+2, & x\le 0 \\ x^2-1, & x>0 \end{cases}$$

Oscillation

$\lim_{x\to 0}\sin\left(\frac{1}{x}\right)$ see graph on the graphing calculator(note: must use radians)

Note: No limit can exist if the value approached is NOT in the domain of the function.

$\lim_{x\to 5}\sqrt{x-8}$

Properties of Limits

Scalar: $\lim_{x\to c} 3 \bullet f(x) = 3 \lim_{x\to c} f(x)$

Mult./Div.: $\lim_{x\to c} f(x) \bullet g(x) = \lim_{x\to c} f(x) \bullet \lim_{x\to c} g(x)$

Note: true for division also!

Add/Subt.: $\lim_{x\to c} f(x) \pm g(x) = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$

Power: $\lim_{x\to c} \left[f(x) \right]^p = \left[\lim_{x\to c} f(x) \right]^p$