AP Calculus Limits of Functions Critical Homework (Answers)

1b) $\lim\limits_{x\to -4}(x+3)^2 = (-4+3)^2 = 1$

1c) $\lim\limits_{x\to 7}\dfrac{3x}{\sqrt{x+2}} = \dfrac{3(7)}{\sqrt{7+2}} = \dfrac{21}{3} = 7$

2b) $$\lim_{x\to -3}\frac{x^2+x-6}{x^2-9} = \lim_{x\to -3}\frac{(x+3)(x-2)}{(x+3)(x-3)}$$
$$= \lim_{x\to -3}\frac{x-2}{x-3} = \frac{-5}{-6} = \frac{5}{6}$$

2c) $$\lim_{x\to \pi/4}\frac{1-\tan x}{\sin x-\cos x} = \lim_{x\to \pi 4}\frac{1-\frac{\sin x}{\cos x}}{\sin x-\cos x}\bullet\frac{\cos x}{\cos x} = \lim_{x\to \pi 4}\frac{\cos x-\sin x}{\cos x(\sin x-\cos x)}$$
$$= \lim_{x\to \pi/4}\frac{-1}{\cos x} = \lim_{x\to \pi/4}(-\sec x) = -\sqrt{2}$$

3b) $$\lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4}$$
$$= \lim_{x\to 4}\frac{\sqrt{x+5}-3}{x-4}\cdot\frac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$$
$$= \lim_{x\to 4}\frac{(x+5)-9}{(x-4)\left(\sqrt{x+5}+3\right)}$$
$$= \lim_{x\to 4}\frac{1}{\sqrt{x+5}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

4b) $$\lim_{x\to 0}\frac{\frac{1}{2+x}-\frac{1}{2}}{x}$$
$$= \lim_{x\to 0}\frac{2-(2+x)}{2(2+x)}\cdot\frac{1}{x}$$
$$= \lim_{x\to 0}\frac{-x}{2(2+x)}\cdot\frac{1}{x}$$
$$= \lim_{x\to 0}\frac{-1}{2(2+x)} = -\frac{1}{4}$$

5b) $\lim\limits_{\Delta x \to 0} \dfrac{(x+\Delta x)^2 - 2(x+\Delta x) + 1 - (x^2 - 2x + 1)}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} \dfrac{x^2 + 2x\Delta x + (\Delta x)^2 - 2x - 2\Delta x + 1 - x^2 + 2x - 1}{\Delta x}$

$= \lim\limits_{\Delta x \to 0} (2x + \Delta x - 2) = 2x - 2$

6b) $\lim\limits_{x \to 0} \sec 2x$

$= \sec 0 = 1$

6c) $\lim\limits_{x \to 3} \tan\left(\dfrac{\pi x}{4}\right)$

$= \tan \dfrac{3\pi}{4} = -1$

6d) $\lim\limits_{x \to 2} \cot\left(\dfrac{3\pi}{2}x\right)$

$= \cot(3\pi) = DNE$

7c) $\lim\limits_{x \to 0} h(f(x))$

$= h\left(\lim\limits_{x \to 0}(f(x))\right)$

$= h(-1)$

$= 1$

7d) $\lim\limits_{x \to -1} f(x) \bullet 3h(x)$

$= \lim\limits_{x \to -1} f(x) \bullet \lim\limits_{x \to -1} 3h(x)$

$= \lim\limits_{x \to -1} f(x) \bullet 3 \lim\limits_{x \to -1} h(x)$

$= 4 \bullet 3(1)$

$= 12$

7e) $\lim\limits_{x \to 2} \left[f(2h(x)) + h(3f(x)) \right]$

$= \lim\limits_{x \to 2} f(2h(x)) + \lim\limits_{x \to 2} h(3f(x))$

$= f(\lim\limits_{x \to 2} 2h(x)) + h(\lim\limits_{x \to 2} 3f(x))$

$= f(2 \lim\limits_{x \to 2} h(x)) + h(3 \lim\limits_{x \to 2} f(x))$

$= f(2 \bullet 2) + h(3 \bullet 1)$

$= f(4) + h(3)$

$= 3 + 3$

$= 6$

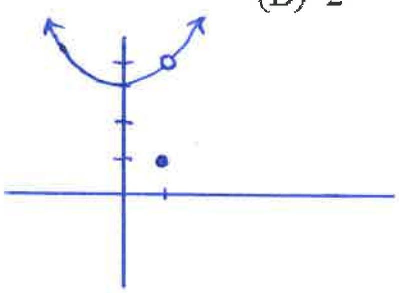
8) AP MULTIPLE CHOICE EXAMPLES

1) $\lim\limits_{x \to \frac{\pi}{6}} \sec^2 x =$ $\quad (\sec \frac{\pi}{6})^2 = \left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$

(A) $\frac{3}{4}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{4}{3}$ (D) $\frac{2\sqrt{3}}{3}$

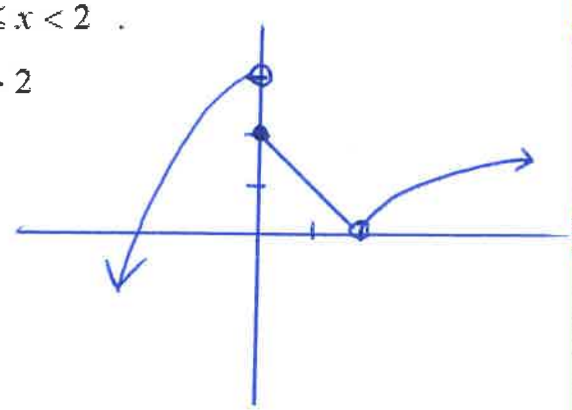
2) If $f(x) = \begin{cases} x^2 + 3, & x \neq 1 \\ 1, & x = 1 \end{cases}$, then $\lim\limits_{x \to 1} f(x) =$

(A) 1 (B) 2 (C) 3 (D) 4



limit is y-value that gets APPROACHED
not necessarily what y-value it equals

3) Let f be a function given by $f(x) = \begin{cases} 3 - x^2, & \text{if } x < 0 \\ 2 - x, & \text{if } 0 \leq x < 2 \\ \sqrt{x - 2}, & \text{if } x > 2 \end{cases}$.



Which of the following statements are true about f ?

I. $\lim\limits_{x \to 0} f(x) = 2$ FALSE, same y-value not approached from left & right

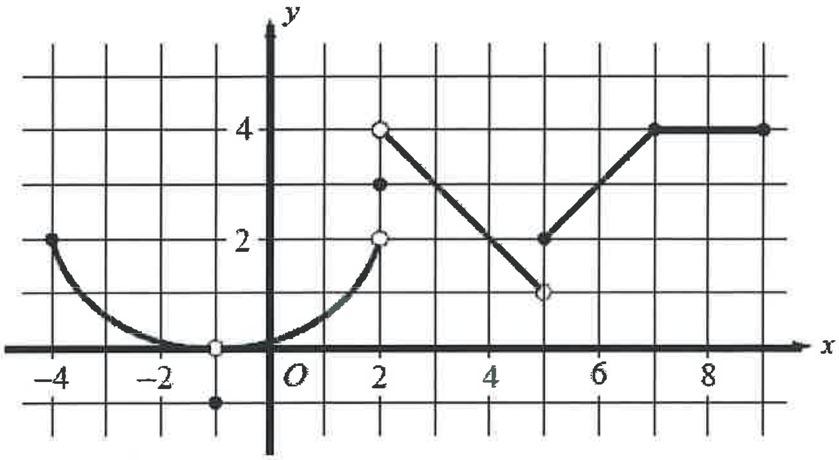
II. $\lim\limits_{x \to 2} f(x) = 0$ TRUE

III. $\lim\limits_{x \to 1} f(x) = \lim\limits_{x \to 6} f(x)$ FALSE $1 \neq 2$

(1) (2)

(A) I only (B) II only (C) II and III only (D) I, II, and III

4)



The figure above shows the graph of $y = f(x)$ on the closed interval $[-4, 9]$.

Find $\lim_{x \to -1} \cos(f(x))$.

$\cos\left[\lim_{x \to -1} f(x)\right] = \cos(0) = \boxed{1}$

(A) DNE

$\boxed{\text{(B) } 1}$

(C) 0

(D) -1

5) $\lim_{x \to 0} \frac{-x^2+4}{x^2-1} = \frac{4}{-1} = -4$

A) 1

B) 0

C) −4

D) −1

NO NEED TO FACTOR, AS "DIRECT SUBSTITUTION" WORKS!

6) $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{x}=$

$\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{x}\cdot\dfrac{(\sqrt{4+x}+2)}{(\sqrt{4+x}+2)}=\dfrac{\cancel{4}+x+\cancel{2\sqrt{4+x}}-\cancel{2\sqrt{4+x}}-\cancel{4}}{x(\sqrt{4+x}+2)}$

(A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) nonexistent

$=\lim\limits_{x\to 0}\dfrac{\cancel{x}^{\,1}}{\cancel{x}_{\,1}(\sqrt{4+x}+2)}=\dfrac{1}{\sqrt{4+0}+2}$

$=\dfrac{1}{2+2}$

$=\dfrac{1}{4}$

7) $\lim\limits_{x\to 3}\dfrac{\frac{1}{x}-\frac{1}{3}}{x-3}=$

$\lim\limits_{x\to 3}\dfrac{(\frac{1}{x}-\frac{1}{3})}{(\frac{x}{1}-\frac{3}{1})}\cdot\dfrac{3x}{3x}=\lim\limits_{x\to 3}\dfrac{3-x}{(x-3)\cdot 3x}$

(A) $-\frac{1}{9}$ (B) $\frac{1}{9}$ (C) -9 (D) 9

$=\lim\limits_{x\to 3}\dfrac{-\cancel{(x-3)}}{\cancel{(x-3)}\cdot 3x}=\dfrac{-1}{3(3)}$

$=\dfrac{-1}{9}$