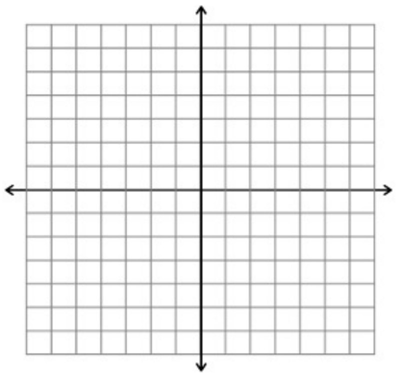
Review of Logarithmic Functions

Find the inverse of $y = 3^x$ and graph below.



An alternate way to write an exponential function is LOGARITHMIC FORM:

From above, $x = 3^y$ is the same as $y = \log_3 x$

So, a logarithm is an EXPONENT of some exponential

In general, $a = b^c$ can be used interchangeably with $c = \log_b a$

Logarithmic Function : Function with general equation $y = \log_b x$,

$b > 0$ and $b \neq 1$ where (1, 0) is the x-intercept and $x = 0$ (y-axis) is the vertical asymptote.

Special Logarithms

$\log x$ implies $\log_{10} x$ called the *common logarithm*

$\ln x$ implies $\log_e x$ called the *natural logarithm*

EX) Graph and state the domain & range for each function:

Hint: Rewrite in exponential form and pick values for "y"

A) $y=\log_2 x$ B) $y=\log_2(x-3)+4$

Note: the base of a logarithm affects the "steepness of the graph"

EX) Evaluate each limit using its graph if necessary.

A) $\lim_{x\to 8}\log_2 x+2$ B) $\lim_{x\to\infty}\log_2 x+2$ C) $\lim_{x\to -2^+}\ln(x+2)-5$

Evaluate and simplify logarithmic expressions using the equivalent exponential form

EX) $\log_2 32$ = EX) $\log_3 \frac{1}{27}$ = EX) $\log_2 1$=

EX) Solve each equation for x

A) $\log_4 x=-2$ B) $\log_x 81=4$ C) $\log_2 x=1$

EX) $\ln e^3$ =

Because Exponential and Logarithmic functions are inverses the following statements must be true:

$$\log_b b^x = x \quad \text{AND} \quad b^{\log_b x} = x$$

Properties of Logarithms

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$\log_b x^a = a \log_b x$$

Properties are useful when solving equations, simplifying expressions, and *finding derivatives.*

Solving Equations Containing Exponential Functions

If an equation with common bases CANNOT be found, isolate the exponential and take the logarithm(common or natural) of both sides. Use the third property of logarithms to move the exponent in front of logarithm and solve remaining equation. Note: final answer can be exact(containing logarithms) or approximate if a calculator is used.

EX) Solve $(4)^{2x-1} = 10$

This same idea is used to evaluate logarithms with **irrational** solutions!

Rewrite and solve using above strategy for the following example

EX) $\log_3 10 =$

So, since the $\log_3 10 = \frac{\log 10}{\log 3}$, all logarithms can be evaluated using this pattern. This is called the change of base formula!

$$\log_b a = \frac{\log a}{\log b}$$

Strategies for solving equations containing logarithms with the same bases

1) Use the properties to write an equation with only one logarithm isolated on one side of the equation. Rewrite exponentially and solve.
   CHECK FOR EXTRANEOUS SOLUTIONS!

   EX) $\log_6 x + \log_6 (x+1) = 1$

2) If there is NO CONSTANT in the equation, use the properties to combine logarithms on both sides. Drop the log symbol from both sides and solve the remaining equation. **CHECK FOR EXTRANEOUS SOLUTIONS!**
   Note: this is allowed since bases are equal and if rewritten, exponents must also be equal

   EX) $\log_2 5 + \log_2 (2x-4) = \log_2 (x+70)$