

Find the derivative of each function.

1a) $y = \ln(x\sqrt{x^2 - 1})$

$$y = \ln[x\sqrt{x^2 - 1}] = \ln x + \frac{1}{2} \ln(x^2 - 1)$$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2} \left(\frac{2x}{x^2 - 1} \right) = \frac{2x^2 - 1}{x(x^2 - 1)}$$

1b) $y = \ln(t + 1)^2$

1c) $f(x) = \ln\left(\frac{\sqrt{4 + x^2}}{x}\right)$

1d) $g(t) = \frac{\ln t}{t^2}$

1e) $y = \ln(\ln x^2)$

Find the equation of the tangent line at the given point of each function.

2a) $f(x) = x^3 \ln x, (1, 0)$

$$f'(x) = 3x^2 \ln x + x^2$$

$$f'(1) = 1$$

Tangent line: $y - 0 = 1(x - 1)$

$$y = x - 1$$

2b) $f(x) = \ln\sqrt{1 + \sin^2 x}, \left(\frac{\pi}{4}, \ln\sqrt{\frac{3}{2}}\right)$

Use implicit differentiation to find $\frac{dy}{dx}$.

3a) $x^2 - 3 \ln y + y^2 = 10$

$$2x - \frac{3}{y} \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$2x = \frac{dy}{dx} \left(\frac{3}{y} - 2y \right)$$

$$\frac{dy}{dx} = \frac{2x}{(3/y) - 2y} = \frac{2xy}{3 - 2y^2}$$

3b) $4x^3 + \ln y^2 + 2y = 2x$

4a) Find the equation of the tangent line at the given point of the function below.

$$x + y - 1 = \ln(x^2 + y^2), \quad (1, 0)$$

Use logarithms to help find $\frac{dy}{dx}$.

5a) $y = x\sqrt{x^2 + 1}$

$$\ln y = \ln x + \frac{1}{2} \ln(x^2 + 1)$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \frac{1}{x} + \frac{x}{x^2 + 1}$$

$$\frac{dy}{dx} = y \left[\frac{2x^2 + 1}{x(x^2 + 1)} \right] = \frac{2x^2 + 1}{\sqrt{x^2 + 1}}$$

5b) $y = \frac{x^2 \sqrt{3x - 2}}{(x + 1)^2}$