

Find the derivative of each function.

1a) $y = \ln(x\sqrt{x^2 - 1})$

$$y = \ln[x\sqrt{x^2 - 1}] = \ln x + \frac{1}{2} \ln(x^2 - 1)$$

$$\frac{dy}{dx} = \frac{1}{x} + \frac{1}{2} \left(\frac{2x}{x^2 - 1} \right) = \frac{2x^2 - 1}{x(x^2 - 1)}$$

1b) $y = \ln(t + 1)^2$

1c) $f(x) = \ln\left(\frac{\sqrt{4 + x^2}}{x}\right)$

1d) $g(t) = \frac{\ln t}{t^2}$

1e) $y = \ln(\ln x^2)$

Find the equation of the tangent line at the given point of each function.

2a) $f(x) = x^3 \ln x, (1, 0)$

$$f'(x) = 3x^2 \ln x + x^2$$

$$f'(1) = 1$$

Tangent line: $y - 0 = 1(x - 1)$

$$y = x - 1$$

2b) $f(x) = \ln\sqrt{1 + \sin^2 x}, \left(\frac{\pi}{4}, \ln\sqrt{\frac{3}{2}}\right)$

Use implicit differentiation to find $\frac{dy}{dx}$.

3a) $x^2 - 3 \ln y + y^2 = 10$

3b) $4x^3 + \ln y^2 + 2y = 2x$

$$2x - \frac{3}{y} \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$2x = \frac{dy}{dx} \left(\frac{3}{y} - 2y \right)$$

$$\frac{dy}{dx} = \frac{2x}{(3/y) - 2y} = \frac{2xy}{3 - 2y^2}$$

4a) Find the equation of the tangent line at the given point of the function below.

$$x + y - 1 = \ln(x^2 + y^2), \quad (1, 0)$$

Use logarithms to help find $\frac{dy}{dx}$.

5a) $y = x\sqrt{x^2 + 1}$

5b) $y = \frac{x^2\sqrt{3x-2}}{(x+1)^2}$

5c) $y = x^{2/x}$

$$\ln y = \ln x + \frac{1}{2} \ln(x^2 + 1)$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \frac{1}{x} + \frac{x}{x^2 + 1}$$

$$\frac{dy}{dx} = y \left[\frac{2x^2 + 1}{x(x^2 + 1)} \right] = \frac{2x^2 + 1}{\sqrt{x^2 + 1}}$$

6) AP MULTIPLE CHOICE EXAMPLES

1) The slope of the line tangent to the graph of $y = \ln(x^2)$ at $x = e^2$ is

- (A) $\frac{1}{e^2}$ (B) $\frac{2}{e^2}$ (C) $\frac{4}{e^2}$ (D) $\frac{1}{e^4}$ (E) $\frac{4}{e^4}$

2) The slope of the line tangent to the graph of $y = \ln\left(\frac{x}{2}\right)$ at $x = 4$ is

- (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 1 (E) 4

3) If $y = \frac{\ln x}{x}$, then $\frac{dy}{dx} =$

- (A) $\frac{1}{x}$ (B) $\frac{1}{x^2}$ (C) $\frac{\ln x - 1}{x^2}$ (D) $\frac{1 - \ln x}{x^2}$ (E) $\frac{1 + \ln x}{x^2}$

4) $\frac{d}{dx}(x^{\ln x}) =$

- (A) $x^{\ln x}$ (B) $(\ln x)^x$ (C) $\frac{2}{x}(\ln x)(x^{\ln x})$ (D) $(\ln x)(x^{\ln x-1})$ (E) $2(\ln x)(x^{\ln x})$

5) The slope of the line normal to the graph of $y = 2 \ln(\sec x)$ at $x = \frac{\pi}{4}$ is

- (A) -2
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 2
(E) nonexistent

6) If $f(x) = e^{3\ln(x^2)}$, then $f'(x) =$

- (A) $e^{3\ln(x^2)}$ (B) $\frac{3}{x^2}e^{3\ln(x^2)}$ (C) $6(\ln x)e^{3\ln(x^2)}$ (D) $5x^4$ (E) $6x^5$