AP Calculus Differentiation of Logarithmic Functions Critical Homework Answers

1b) $y = \ln(t+1)^2 = 2\ln(t+1)$

$y' = 2\dfrac{1}{t+1} = \dfrac{2}{t+1}$

1c) $f(x) = \ln\dfrac{\sqrt{4+x^2}}{x} = \dfrac{1}{2}\ln(4+x^2) - \ln x$

$f'(x) = \dfrac{x}{4+x^2} - \dfrac{1}{x} = \dfrac{-4}{x(x^2+4)}$

1d) $g'(t) = \dfrac{t^2(1/t) - 2t\ln t}{t^4} = \dfrac{1 - 2\ln t}{t^3}$

1e) $\dfrac{dy}{dx} = \dfrac{1}{\ln x^2}\dfrac{d}{dx}(\ln x^2) = \dfrac{(2x/x^2)}{\ln x^2} = \dfrac{2}{x\ln x^2} = \dfrac{1}{x\ln x}$

2b) $f(x) = \ln\sqrt{1+\sin^2 x}$

$= \dfrac{1}{2}\ln(1+\sin^2 x), \quad \left(\dfrac{\pi}{4}, \ln\sqrt{\dfrac{3}{2}}\right)$

$f'(x) = \dfrac{2\sin x\cos x}{2(1+\sin^2 x)} = \dfrac{\sin x\cos x}{1+\sin^2 x}$

$f'\left(\dfrac{\pi}{4}\right) = \dfrac{(\sqrt{2}/2)(\sqrt{2}/2)}{(3/2)} = \dfrac{1}{3}$

Tangent line: $y - \ln\sqrt{\dfrac{3}{2}} = \dfrac{1}{3}\left(x - \dfrac{\pi}{4}\right)$

$y = \dfrac{1}{3}x + \dfrac{1}{2}\ln\left(\dfrac{3}{2}\right) - \dfrac{\pi}{12}$

3b) $12x^2 + \dfrac{2}{y}y' + 2y' = 2$

$\left(\dfrac{2}{y} + 2\right)y' = 2 - 12x^2$

$y' = \dfrac{2 - 12x^2}{2/y + 2}$

$y' = \dfrac{y - 6yx^2}{1+y} = \dfrac{y(1-6x^2)}{1+y}$

4b) $1 + y' = \dfrac{2x + 2yy'}{x^2+y^2}$

$x^2 + y^2 + (x^2+y^2)y' = 2x + 2yy'$

At $(1, 0)$: $1 + y' = 2$

$y' = 1$

Tangent line: $y = x - 1$

5b) $\ln y = 2\ln x + \dfrac{1}{2}\ln(3x-2) - 2\ln(x+1)$

$\dfrac{1}{y}\left(\dfrac{dy}{dx}\right) = \dfrac{2}{x} + \dfrac{3}{2(3x-2)} - \dfrac{2}{x+1}$

$\dfrac{dy}{dx} = y\left[\dfrac{3x^2 + 15x - 8}{2x(3x-2)(x+1)}\right]$

$= \dfrac{3x^3 + 15x^2 - 8x}{2(x+1)^3\sqrt{3x-2}}$

5c)

$$\ln y = \frac{2}{x}\ln x$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{2}{x}\left(\frac{1}{x}\right) + \ln x\left(-\frac{2}{x^2}\right) = \frac{2}{x^2}(1 - \ln x)$$

$$\frac{dy}{dx} = \frac{2y}{x^2}(1 - \ln x) = 2x^{(2/x)-2}(1 - \ln x)$$