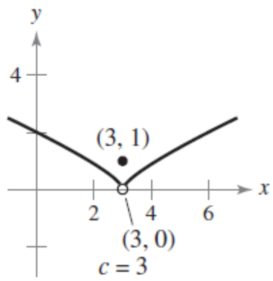
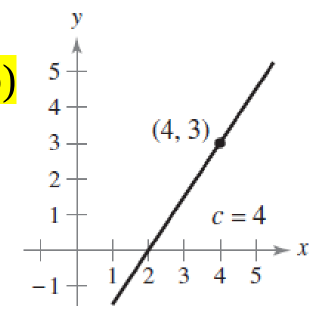
1) Use the graph to determine the limit and discuss the continuity of the function:

(a) $\lim\limits_{x \to c^+} f(x)$ (b) $\lim\limits_{x \to c^-} f(x)$ (c) $\lim\limits_{x \to c} f(x)$

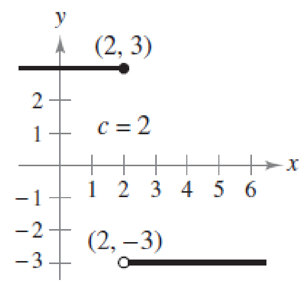
1a)



1b)



1c)



(a) $\lim\limits_{x \to 3^+} f(x) = 0$

(b) $\lim\limits_{x \to 3^-} f(x) = 0$

(c) $\lim\limits_{x \to 3} f(x) = 0$

The function is NOT continuous at $x = 3$

2a) $\lim\limits_{x \to 5^+} \dfrac{x - 5}{x^2 - 25}$

$= \lim\limits_{x \to 5^+} \dfrac{1}{x + 5} = \dfrac{1}{10}$

2b) $\lim\limits_{x \to 8^+} \dfrac{1}{x + 8}$

2c) $\lim\limits_{x \to -3^-} \dfrac{x}{\sqrt{x^2 - 9}}$

3a) $\lim\limits_{x \to 0^-} \dfrac{\lfloor x \rfloor}{x}$

$= \lim\limits_{x \to 0^-} \dfrac{-x}{x} = -1$

3b) $\lim\limits_{x \to 10^+} \dfrac{|x - 10|}{x - 10}$

4a) $\lim_{x\to 3^-} f(x)$, where $f(x) = \begin{cases} \dfrac{x+2}{2}, & x \le 3 \\ \dfrac{12-2x}{3}, & x > 3 \end{cases}$

$$= \lim_{x\to 3^-} \frac{x+2}{2} = \frac{5}{2}$$

4b) $\lim_{x\to 1^+} f(x)$, where $f(x) = \begin{cases} x, & x \le 1 \\ 1-x, & x > 1 \end{cases}$

4c) $\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^2 - 4x + 6, & x < 2 \\ -x^2 + 4x - 2, & x \ge 2 \end{cases}$

4d) $\lim_{x\to 1} f(x)$, where $f(x) = \begin{cases} x^3 + 1, & x < 1 \\ x + 1, & x \ge 1 \end{cases}$

Give intervals of continuity for each function.

5a) $f(x) = \begin{cases} 3 - x, & x < 1 \\ 3 + \frac{1}{2}x, & x \ge 1 \end{cases}$

5b) $g(x) = \dfrac{1}{x^2 - 4}$

5c) $f(x) = \begin{cases} 3 - x, & x \le 0 \\ 3 + \frac{1}{2}x, & x > 0 \end{cases}$

Not continuous at $x = 1$ since

$\lim_{x\to 1^-} f(x) = 2$

$\lim_{x\to 1^+} f(x) = 3.5$

so $\lim_{x\to 1^-} f(x) \ne \lim_{x\to 1^+} f(x)$

so.. continuous $(-\infty, 1) \cup (1, \infty)$

Find the x-values (if any) where $f(x)$ is not continuous. Which of the discontinuities are removable?

6a) $f(x) = \begin{cases} \frac{x}{2} + 1, & x \le 2 \\ 3 - x, & x > 2 \end{cases}$

6b) $f(x) = \frac{|x + 7|}{x + 7}$

6c) $f(x) = \frac{x + 2}{(x + 2)(x - 5)}$

has a **possible** discontinuity at $x = 2$.

1. $f(2) = \frac{2}{2} + 1 = 2$

2. $\left.\begin{aligned} \lim_{x \to 2^-} f(x) &= \lim_{x \to 2^-} \left(\frac{x}{2} + 1\right) = 2 \\ \lim_{x \to 2^+} f(x) &= \lim_{x \to 2^+} (3 - x) = 1 \end{aligned}\right\} \lim_{x \to 2} f(x)$ does not exist.

Therefore, f has a nonremovable discontinuity at $x = 2$.

6d) $f(x) = \frac{x}{x^2 + 1}$

6e) $f(x) = \begin{cases} x, & x < 1 \\ x^2, & x > 1 \end{cases}$

Find the constant "a" such that the function is continuous on the entire real line.

7a) $f(x) = \begin{cases} 3x^2, & x \ge 1 \\ ax - 4, & x < 1 \end{cases}$

7b) $f(x) = \begin{cases} x^3, & x \le 2 \\ ax^2, & x > 2 \end{cases}$

$f(1) = 3$

Find a so that $\lim_{x \to 1^-} (ax - 4) = 3$

$$a(1) - 4 = 3$$

$$a = 7.$$

8) Give an example of a function with:

A) non-removable discontinuity
B) removable discontinuity
C) both a removable and non-removable discontinuity

9) Make a graph of a function with the following characteristics:

$\lim_{x \to 3^+} f(x) = 1$ AND $\lim_{x \to 3^-} f(x) = 0$

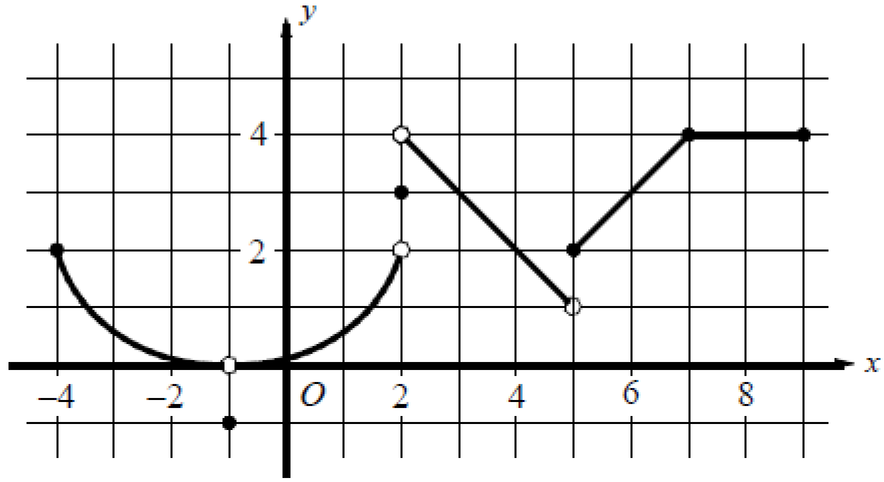
Is your function continuous? If not, what kind of discontinuity is it (removable or non-removable)? Why?

10) AP MULTIPLE CHOICE EXAMPLES

1) Let f be a function defined by $f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & \text{if } x \neq a \\ 4, & \text{if } x = a \end{cases}$. If f is continuous for all real numbers x, what is the value of a?

(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2

2)

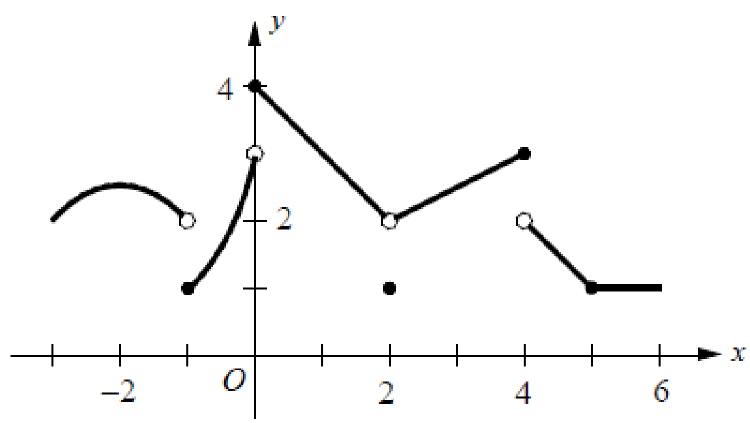


The figure above shows the graph of $y = f(x)$ on the closed interval $[-4, 9]$.

Find $\lim_{x \to 5^+} [x f(x)]$.

(A) DNE (B) 5 (C) 10 (D) 7

3)



The graph of a function f is shown above. If $\lim_{x \to a} f(x)$ exists and f is not continuous at $x = a$, then $a =$

(A) -1 (B) 0 (C) 2 (D) 4

4) $\lim_{x \to 1} \frac{|x-1|}{1-x} =$

(A) -2 (B) -1 (C) 1 (D) nonexistent