AP Calculus Continuity and One-Sided Limits Critical Homework (Answers)

1b) (a) $\lim_{x\to 4^+} f(x) = 3$

(b) $\lim_{x\to 4^-} f(x) = 3$

(c) $\lim_{x\to 4} f(x) = 3$

The function is continuous at $x = 4$ and is continuous on $(-\infty, \infty)$.

1c) (a) $\lim_{x\to 2^+} f(x) = -3$

(b) $\lim_{x\to 2^-} f(x) = 3$

(c) $\lim_{x\to 2} f(x)$ does not exist

The function is NOT continuous at $x = 2$.

2b) $\lim_{x\to 8^+} \frac{1}{x+8}$

$= \frac{1}{8+8} = \frac{1}{16}$

2c) DNE

3b) $\lim_{x\to 10^+} \frac{|x-10|}{x-10}$

$= \frac{x-10}{x-10}$

$= 1$

4b) $= \lim_{x\to 1^+} (1-x)$

$= 0$

4c) $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 - 4x + 6 = 2$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} -x^2 + 4x - 2 = 2$

$\lim_{x\to 2} f(x) = \lim_{x\to 2} f(x) = 2$

4d) $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} (x+1) = 2$

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} (x^3+1) = 2$

$\lim_{x\to 1} f(x) = 2$

5b) $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

5c) $(-\infty,\infty)$

6b) $f(x) = \frac{|x+7|}{x+7}$

has a nonremovable discontinuity at $x = -7$ because $\lim_{x\to -7} f(x)$ does not exist.

6c) $f(x) = \frac{x+2}{(x+2)(x-5)}$

has a nonremovable discontinuity at $x = 5$ because $\lim_{x\to 5} f(x)$ does not exist, and has a removable discontinuity at $x = -2$ because

$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{1}{x-5} = -\frac{1}{7}$.

6d) Continuous on all real numbers

6e)
$$\left.\begin{aligned}\lim_{x\to1^-} f(x) &= \lim_{x\to1^-} x = 1\\ \lim_{x\to1^+} f(x) &= \lim_{x\to1^+} x^2 = 1\end{aligned}\right\}\lim_{x\to1} f(x) = 1$$

Therefore f has a removable discontinuity at $x = 1$.

7b) $f(2) = 8$

Find a so that $\lim_{x\to2^+} ax^2 = 8$

$$a = \frac{8}{2^2} = 2.$$

8) Give an example of a function with:

A) non-removable discontinuity
B) removable discontinuity
C) both a removable and non-removable discontinuity

A) Asymptote

$f(x) = \frac{1}{x-2}$

N.R. disc. at $x = 2$

B) Hole in graph

$f(x) = \begin{cases} x+1, & x>0 \\ x+1, & x<0 \end{cases}$

Rem. disc. at $x = 0$

C) Both hole in graph & Asymptote

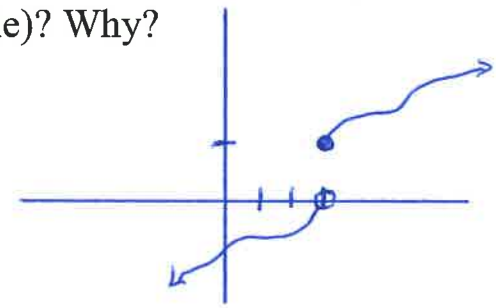
$f(x) = \frac{x-1}{x^2-1}$

N.R. disc. at $x = -1$
Rem. disc. at $x = 1$

9) Make a graph of a function with the following characteristics:

$\lim_{x \to 3^+} f(x) = 1$ AND $\lim_{x \to 3^-} f(x) = 0$

Is your function continuous? If not, what kind of discontinuity is it (removable or non-removable)? Why?



NOT continuous at $x = 3$
NON Removable as it represents a jump (more than one point needed to make it continuous)

10) AP MULTIPLE CHOICE EXAMPLES

1) Let f be a function defined by $f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a}, & \text{if } x \neq a \\ 4, & \text{if } x = a \end{cases}$. If f is continuous for all real numbers x, what is the value of a?

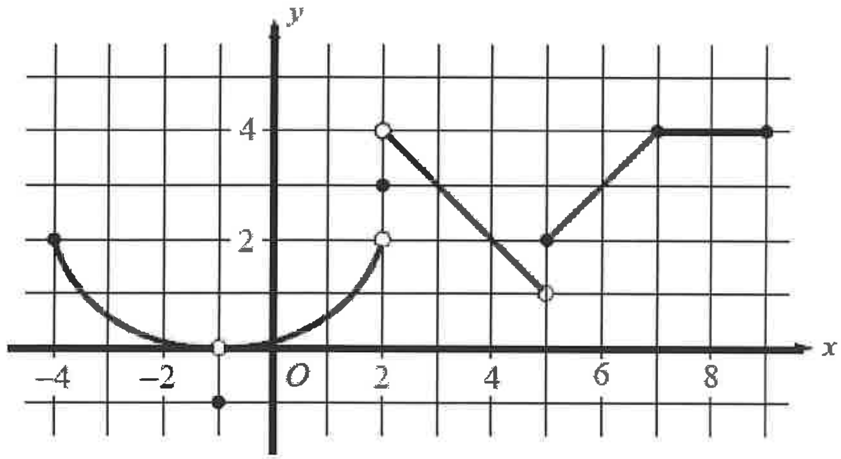
(A) $\frac{1}{2}$ (B) 0 (C) 1 (D) 2

When $x = a$ the y-value (closed dot) is "4"
So... the top expression needs to have a hole at $(a, 4)$ which happens when $a = 2$

$\frac{x^2 - 4}{x - 2} = \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$

when "2" is plugged into "x" for $x+2$ the answer $= 4$

2)

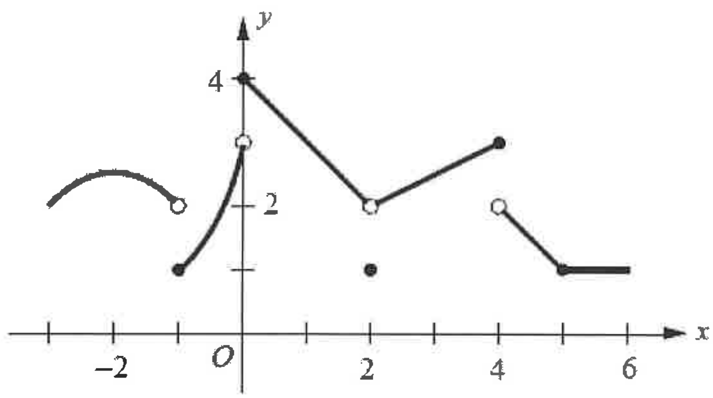


The figure above shows the graph of $y = f(x)$ on the closed interval $[-4, 9]$.

Find $\lim\limits_{x\to 5^+} [x f(x)]$. $= \lim\limits_{x\to 5^+} x \cdot \lim\limits_{x\to 5^+} f(x) = 5 \cdot 2 = 10$

(A) DNE (B) 5 (C) 10 (D) 7

3)



The graph of a function f is shown above. If $\lim\limits_{x\to a} f(x)$ exists and f is not continuous at $x = a$, then $a =$

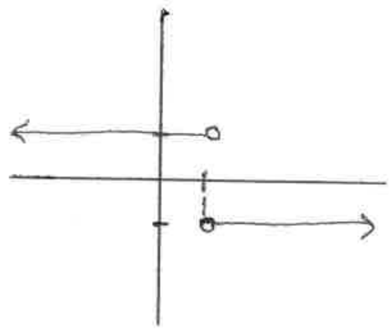
approaches same y-value from left & right

must be a hole in the graph if limit exists

(A) −1 (B) 0 (C) 2 (D) 4

4) $\lim\limits_{x \to 1} \frac{|x-1|}{1-x} =$

(A) -2 (B) -1 (C) 1 (D) nonexistent



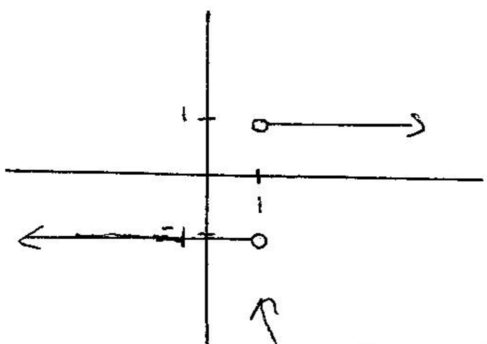
$$\lim_{x \to 1^-} \frac{|x-1|}{1-x} \neq \lim_{x \to 1^+} \frac{|x-1|}{1-x}$$

4) $\lim_{x \to 1} \frac{|x-1|}{1-x} =$

(A) -2 (B) -1 (C) 1 (D) nonexistent

$$= \lim_{x \to 1} \frac{|x-1|}{-(x-1)}$$

$$= -\lim_{x \to 1} \frac{|x-1|}{x-1}$$



$$\lim_{x \to 1^+} \frac{|x-1|}{x-1} = 1$$

$$\lim_{x \to 1^-} \frac{|x-1|}{x-1} = -1$$

so … $\lim_{x \to 1} \frac{|x-1|}{x-1}$ DNE