Section 4.5 Integration by Substitution

$\int f(g(x))g'(x)\,dx$	$u = g(x)$	$du = g'(x)\,dx$
1. $\int (5x^2+1)^2(10x)\,dx$	$5x^2+1$	$10x\,dx$
2. $\int x^2\sqrt{x^3+1}\,dx$	x^3+1	$3x^2\,dx$
3. $\int \frac{x}{\sqrt{x^2+1}}\,dx$	x^2+1	$2x\,dx$
4. $\int \sec 2x \tan 2x\,dx$	$2x$	$2\,dx$
5. $\int \tan^2 x \sec^2 x\,dx$	$\tan x$	$\sec^2 x\,dx$
6. $\int \frac{\cos x}{\sin^2 x}\,dx$	$\sin x$	$\cos x\,dx$

7. $\int (1+2x)^4\, 2\,dx = \frac{(1+2x)^5}{5} + C$

Check: $\frac{d}{dx}\left[\frac{(1+2x)^5}{5} + C\right] = 2(1+2x)^4$

8. $\int (x^2-1)^3\, 2x\,dx = \frac{(x^2-1)^4}{4} + C$

Check: $\frac{d}{dx}\left[\frac{(x^2-1)^4}{4} + C\right] = 2x(x^2-1)^3$

9. $\int (9-x^2)^{1/2}(-2x)\,dx = \frac{(9-x^2)^{3/2}}{3/2} + C = \frac{2}{3}(9-x^2)^{3/2} + C$

Check: $\frac{d}{dx}\left[\frac{2}{3}(9-x^2)^{3/2} + C\right] = \frac{2}{3}\cdot\frac{3}{2}(9-x^2)^{1/2}(-2x) = \sqrt{9-x^2}(-2x)$

10. $\int (1-2x^2)^3(-4x)\,dx = \frac{(1-2x^2)^4}{4} + C$

Check: $\frac{d}{dx}\left[\frac{(1-2x^2)^4}{4} + C\right] = \frac{4(1-2x^2)^3(-4x)}{4} = (1-2x^2)^3(-4x)$

11. $\int x^2(x^3-1)^4\,dx = \frac{1}{3}\int (x^3-1)^4(3x^2)\,dx = \frac{1}{3}\left[\frac{(x^3-1)^5}{5}\right] + C = \frac{(x^3-1)^5}{15} + C$

Check: $\frac{d}{dx}\left[\frac{(x^3-1)^5}{15} + C\right] = \frac{5(x^3-1)^4(3x^2)}{15} = x^2(x^3-1)^4$

12. $\int x(4x^2+3)^3\,dx = \frac{1}{8}\int (4x^2+3)^3(8x)\,dx = \frac{1}{8}\left[\frac{(4x^2+3)^4}{4}\right] + C = \frac{(4x^2+3)^4}{32} + C$

Check: $\frac{d}{dx}\left[\frac{(4x^2+3)^4}{32} + C\right] = \frac{4(4x^2+3)^3(8x)}{32} = x(4x^2+3)^3$

13. $\int 5x(1-x^2)^{1/3}\,dx = -\frac{5}{2}\int (1-x^2)^{1/3}(-2x)\,dx = -\frac{5}{2}\cdot\frac{(1-x^2)^{4/3}}{4/3} + C = -\frac{15}{8}(1-x^2)^{4/3} + C$

Check: $\frac{d}{dx}\left[-\frac{15}{8}(1-x^2)^{4/3} + C\right] = -\frac{15}{8}\cdot\frac{4}{3}(1-x^2)^{1/3}(-2x) = 5x(1-x^2)^{1/3} = 5x\sqrt[3]{1-x^2}$

14. $\int u^3\sqrt{u^4+2}\,du = \frac{1}{4}\int (u^4+2)^{1/2}(4u^3)\,du = \frac{1}{4}\left[\frac{(u^4+2)^{3/2}}{3/2}\right] + C = \frac{1}{6}(u^4+2)^{3/2} + C$

Check: $\frac{d}{dx}\left[\frac{1}{6}(u^4+2)^{3/2} + C\right] = \frac{1}{6}\cdot\frac{3}{2}(u^4+2)^{1/2}(4u^3) = u^3\sqrt{u^4+2}$

15. $\int \frac{x^2}{(1+x^3)^2}\,dx = \frac{1}{3}\int (1+x^3)^{-2}(3x^2)\,dx = \frac{1}{3}\left[\frac{(1+x^3)^{-1}}{-1}\right] + C = -\frac{1}{3(1+x^3)} + C$

Check: $\frac{d}{dx}\left[-\frac{1}{3(1+x^3)} + C\right] = -\frac{1}{3}(-1)(1+x^3)^{-2}(3x^2) = \frac{x^2}{(1+x^3)^2}$

16. $\int \frac{x^2}{(16-x^3)^2}\,dx = -\frac{1}{3}\int (16-x^3)^{-2}(-3x^2)\,dx = -\frac{1}{3}\left[\frac{(16-x^3)^{-1}}{-1}\right] + C = \frac{1}{3(16-x^3)} + C$

Check: $\frac{d}{dx}\left[\frac{1}{3(16-x^3)} + C\right] = \frac{1}{3}(-1)(16-x^3)^{-2}(3x^2) = \frac{x^2}{(16-x^3)^2}$

17. $\int \left(1+\frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt = -\int \left(1+\frac{1}{t}\right)^3\left(-\frac{1}{t^2}\right)dt = -\frac{[1+(1/t)]^4}{4} + C$

Check: $\frac{d}{dt}\left[-\frac{[1+(1/t)]^4}{4} + C\right] = -\frac{1}{4}(4)\left(1+\frac{1}{t}\right)^3\left(-\frac{1}{t^2}\right) = \frac{1}{t^2}\left(1+\frac{1}{t}\right)^3$

18. $\int \left[x^2 + \frac{1}{(3x)^2}\right]dx = \int \left(x^2 + \frac{1}{9}x^{-2}\right)dx = \frac{x^3}{3} + \frac{1}{9}\left(\frac{x^{-1}}{-1}\right) + C = \frac{x^3}{3} - \frac{1}{9x} + C = \frac{3x^4-1}{9x} + C$

Check: $\frac{d}{dx}\left[\frac{1}{3}x^3 - \frac{1}{9}x^{-1} + C\right] = x^2 + \frac{1}{9}x^{-2} = x^2 + \frac{1}{(3x)^2}$

19. $\int \frac{1}{\sqrt{2x}}\,dx = \frac{1}{2}\int (2x)^{-1/2}\,2\,dx = \frac{1}{2}\left[\frac{(2x)^{1/2}}{1/2}\right] + C = \sqrt{2x} + C$

Check: $\frac{d}{dx}\left[\sqrt{2x} + C\right] = \frac{1}{2}(2x)^{-1/2}(2) = \frac{1}{\sqrt{2x}}$

20. $\int \frac{1}{2\sqrt{x}}\,dx = \frac{1}{2}\int x^{-1/2}\,dx = \frac{1}{2}\left(\frac{x^{1/2}}{1/2}\right) + C = \sqrt{x} + C$

Check: $\frac{d}{dx}\left[\sqrt{x} + C\right] = \frac{1}{2\sqrt{x}}$

21. $\int \frac{x^2+3x+7}{\sqrt{x}}\,dx = \int (x^{3/2} + 3x^{1/2} + 7x^{-1/2})\,dx = \frac{2}{5}x^{5/2} + 2x^{3/2} + 14x^{1/2} + C = \frac{2}{5}\sqrt{x}(x^2+5x+35) + C$

Check: $\frac{d}{dx}\left[\frac{2}{5}x^{5/2} + 2x^{3/2} + 14x^{1/2} + C\right] = \frac{x^2+3x+7}{\sqrt{x}}$

22. $\int \frac{t+2t^2}{\sqrt{t}}\,dt = \int (t^{1/2} + 2t^{3/2})\,dt = \frac{2}{3}t^{3/2} + \frac{4}{5}t^{5/2} + C = \frac{2}{15}t^{3/2}(5+6t) + C$

Check: $\frac{d}{dt}\left[\frac{2}{3}t^{3/2} + \frac{4}{5}t^{5/2} + C\right] = t^{1/2} + 2t^{3/2} = \frac{t+2t^2}{\sqrt{t}}$

23. $\int t^2\left(t - \frac{2}{t}\right) dt = \int (t^3 - 2t)\, dt = \frac{1}{4}t^4 - t^2 + C$

Check: $\frac{d}{dt}\left[\frac{1}{4}t^4 - t^2 + C\right] = t^3 - 2t = t^2\left(t - \frac{2}{t}\right)$

24. $\int \left(\frac{t^3}{3} + \frac{1}{4t^2}\right) dt = \int \left(\frac{1}{3}t^3 + \frac{1}{4}t^{-2}\right) dt = \frac{1}{3}\left(\frac{t^4}{4}\right) + \frac{1}{4}\left(\frac{t^{-1}}{-1}\right) + C = \frac{1}{12}t^4 - \frac{1}{4t} + C$

Check: $\frac{d}{dt}\left[\frac{1}{12}t^4 - \frac{1}{4t} + C\right] = \frac{1}{3}t^3 + \frac{1}{4t^2}$

25. $\int (9 - y)\sqrt{y}\, dy = \int (9y^{1/2} - y^{3/2})\, dy = 9\left(\frac{2}{3}y^{3/2}\right) - \frac{2}{5}y^{5/2} + C = \frac{2}{5}y^{3/2}(15 - y) + C$

Check: $\frac{d}{dy}\left[\frac{2}{5}y^{3/2}(15 - y) + C\right] = \frac{d}{dy}\left[6y^{3/2} - \frac{2}{5}y^{5/2} + C\right] = 9y^{1/2} - y^{3/2} = (9 - y)\sqrt{y}$

26. $\int 2\pi y(8 - y^{3/2})\, dy = 2\pi \int (8y - y^{5/2})\, dy = 2\pi\left(4y^2 - \frac{2}{7}y^{7/2}\right) + C = \frac{4\pi y^2}{7}(14 - y^{3/2}) + C$

Check: $\frac{d}{dy}\left[\frac{4\pi y^2}{7}(14 - y^{3/2}) + C\right] = \frac{d}{dy}\left[2\pi\left(4y^2 - \frac{2}{7}y^{7/2}\right) + C\right] = 16\pi y - 2\pi y^{5/2} = (2\pi y)(8 - y^{3/2})$

27. $y = \int \left[4x + \frac{4x}{\sqrt{16 - x^2}}\right] dx$

$= 4\int x\, dx - 2\int (16 - x^2)^{-1/2}(-2x)\, dx$

$= 4\left(\frac{x^2}{2}\right) - 2\left[\frac{(16 - x^2)^{1/2}}{1/2}\right] + C$

$= 2x^2 - 4\sqrt{16 - x^2} + C$

28. $y = \int \frac{10x^2}{\sqrt{1 + x^3}}\, dx$

$= \frac{10}{3}\int (1 + x^3)^{-1/2}(3x^2)\, dx$

$= \frac{10}{3}\left[\frac{(1 + x^3)^{1/2}}{1/2}\right] + C$

$= \frac{20}{3}\sqrt{1 + x^3} + C$

29. $y = \int \frac{x + 1}{(x^2 + 2x - 3)^2}\, dx$

$= \frac{1}{2}\int (x^2 + 2x - 3)^{-2}(2x + 2)\, dx$

$= \frac{1}{2}\left[\frac{(x^2 + 2x - 3)^{-1}}{-1}\right] + C$

$= -\frac{1}{2(x^2 + 2x - 3)} + C$

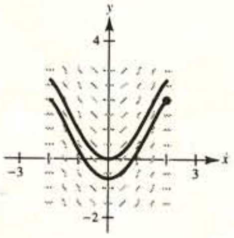
30. $y = \int \frac{x - 4}{\sqrt{x^2 - 8x + 1}}\, dx$

$= \frac{1}{2}\int (x^2 - 8x + 1)^{-1/2}(2x - 8)\, dx$

$= \frac{1}{2}\left[\frac{(x^2 - 8x + 1)^{1/2}}{1/2}\right] + C$

$= \sqrt{x^2 - 8x + 1} + C$

31. (a)



(b) $\frac{dy}{dx} = x\sqrt{4 - x^2}$, $(2, 2)$

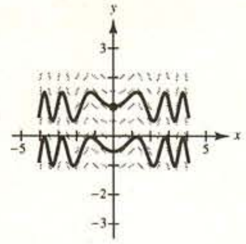
$y = \int x\sqrt{4 - x^2}\, dx = -\frac{1}{2}\int (4 - x^2)^{1/2}(-2x\, dx)$

$= -\frac{1}{2} \cdot \frac{2}{3}(4 - x^2)^{3/2} + C = -\frac{1}{3}(4 - x^2)^{3/2} + C$

$(2, 2)$: $2 = -\frac{1}{3}(4 - 2^2)^{3/2} + C \Rightarrow C = 2$

$y = -\frac{1}{3}(4 - x^2)^{3/2} + 2$

32. (a)



(b) $\frac{dy}{dx} = x\cos x^2, (0, 1)$

$$y = \int x\cos x^2\,dx = \frac{1}{2}\int \cos(x^2)2x\,dx$$

$$= \frac{1}{2}\sin(x^2) + C$$

$$(0, 1):\ 1 = \frac{1}{2}\sin(0) + C \implies C = 1$$

$$y = \frac{1}{2}\sin(x^2) + 1$$

33. $\int \sin 2x\,dx = \frac{1}{2}\int (\sin 2x)(2x)\,dx = -\frac{1}{2}\cos 2x + C$

34. $\int x\sin x^2\,dx = \frac{1}{2}\int (\sin x^2)(2x)\,dx = -\frac{1}{2}\cos x^2 + C$

35. $\int \frac{1}{\theta^2}\cos\frac{1}{\theta}\,d\theta = -\int \cos\frac{1}{\theta}\left(-\frac{1}{\theta^2}\right)d\theta = -\sin\frac{1}{\theta} + C$

36. $\int \cos 6x\,dx = \frac{1}{6}\int (\cos 6x)(6)\,dx = \frac{1}{6}\sin 6x + C$

37. $\int \sin 2x\cos 2x\,dx = \frac{1}{2}\int (\sin 2x)(2\cos 2x)\,dx = \frac{1}{2}\frac{(\sin 2x)^2}{2} + C = \frac{1}{4}\sin^2 2x + C$ OR

$\int \sin 2x\cos 2x\,dx = -\frac{1}{2}\int (\cos 2x)(-2\sin 2x)\,dx = -\frac{1}{2}\frac{(\cos 2x)^2}{2} + C_1 = -\frac{1}{4}\cos^2 2x + C_1$ OR

$\int \sin 2x\cos 2x\,dx = \frac{1}{2}\int 2\sin 2x\cos 2x\,dx = \frac{1}{2}\int \sin 4x\,dx = \frac{1}{8}\cos 4x + C_2$

38. $\int \sec(1-x)\tan(1-x)\,dx = -\int [\sec(1-x)\tan(1-x)](-1)\,dx = -\sec(1-x) + C$

39. $\int \tan^4 x\sec^2 x\,dx = \frac{\tan^5 x}{5} + C = \frac{1}{5}\tan^5 x + C$

40. $\int \sqrt{\cot x}\csc^2 x\,dx = -\int (\cot x)^{1/2}(-\csc^2 x)\,dx = -\frac{2}{3}(\cot x)^{3/2} + C$

41. $\int \frac{\csc^2 x}{\cot^3 x}\,dx = -\int (\cot x)^{-3}(-\csc^2 x)\,dx$

$$= -\frac{(\cot x)^{-2}}{-2} + C = \frac{1}{2\cot^2 x} + C = \frac{1}{2}\tan^2 x + C = \frac{1}{2}(\sec^2 x - 1) + C = \frac{1}{2}\sec^2 x + C_1$$

42. $\int \frac{\sin x}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right)dx = \int \sec x\tan x\,dx = \sec x + C$

43. $\int \cot^2 x\,dx = \int (\csc^2 x - 1)\,dx = -\cot x - x + C$

44. $\int \csc^2\left(\frac{x}{2}\right)dx = 2\int \csc^2\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)dx = -2\cot\left(\frac{x}{2}\right) + C$

45. $f(x) = \int \cos\frac{x}{2}\,dx = 2\sin\frac{x}{2} + C$

Since $f(0) = 3 = 2\sin 0 + C$, $C = 3$. Thus,

$$f(x) = 2\sin\frac{x}{2} + 3.$$

46. $f(x) = \int \pi\sec\pi x\tan\pi x\,dx = \sec\pi x + C$

Since $f(1/3) = 1 = \sec(\pi/3) + C$, $C = -1$. Thus

$$f(x) = \sec\pi x - 1.$$

47. $u = x + 2, x = u - 2, dx = du$

$$\int x\sqrt{x+2}\,dx = \int (u-2)\sqrt{u}\,du$$
$$= \int (u^{3/2} - 2u^{1/2})\,du$$
$$= \frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2} + C$$
$$= \frac{2u^{3/2}}{15}(3u - 10) + C$$
$$= \frac{2}{15}(x+2)^{3/2}[3(x+2) - 10] + C$$
$$= \frac{2}{15}(x+2)^{3/2}(3x - 4) + C$$

48. $u = 2x + 1, x = \frac{1}{2}(u - 1), dx = \frac{1}{2}\,du$

$$\int x\sqrt{2x+1}\,dx = \int \frac{1}{2}(u-1)\sqrt{u}\,\frac{1}{2}\,du$$
$$= \frac{1}{4}\int (u^{3/2} - u^{1/2})\,du$$
$$= \frac{1}{4}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$
$$= \frac{u^{3/2}}{30}(3u - 5) + C$$
$$= \frac{1}{30}(2x+1)^{3/2}[3(2x+1) - 5] + C$$
$$= \frac{1}{30}(2x+1)^{3/2}(6x - 2) + C$$
$$= \frac{1}{15}(2x+1)^{3/2}(3x - 1) + C$$

49. $u = 1 - x, x = 1 - u, dx = -du$

$$\int x^2\sqrt{1-x}\,dx = -\int (1-u)^2\sqrt{u}\,du$$
$$= -\int (u^{1/2} - 2u^{3/2} + u^{5/2})\,du$$
$$= -\left(\frac{2}{3}u^{3/2} - \frac{4}{5}u^{5/2} + \frac{2}{7}u^{7/2}\right) + C$$
$$= -\frac{2u^{3/2}}{105}(35 - 42u + 15u^2) + C$$
$$= -\frac{2}{105}(1-x)^{3/2}[35 - 42(1-x) + 15(1-x)^2] + C$$
$$= -\frac{2}{105}(1-x)^{3/2}(15x^2 + 12x + 8) + C$$

50. $u = 2 - x, x = 2 - u, dx = -du$

$$\int (x+1)\sqrt{2-x}\,dx = -\int (3-u)\sqrt{u}\,du$$
$$= -\int (3u^{1/2} - u^{3/2})\,du$$
$$= -\left(2u^{3/2} - \frac{2}{5}u^{5/2}\right) + C$$
$$= -\frac{2u^{3/2}}{5}(5 - u) + C$$
$$= -\frac{2}{5}(2-x)^{3/2}[5 - (2-x)] + C$$
$$= -\frac{2}{5}(2-x)^{3/2}(x + 3) + C$$

51. $u = 2x - 1, x = \frac{1}{2}(u + 1), dx = \frac{1}{2}\,du$

$$\int \frac{x^2 - 1}{\sqrt{2x - 1}}\,dx = \int \frac{[(1/2)(u + 1)]^2 - 1}{\sqrt{u}} \frac{1}{2}\,du$$

$$= \frac{1}{8}\int u^{-1/2}[(u^2 + 2u + 1) - 4]\,du$$

$$= \frac{1}{8}\int (u^{3/2} + 2u^{1/2} - 3u^{-1/2})\,du$$

$$= \frac{1}{8}\left(\frac{2}{5}u^{5/2} + \frac{4}{3}u^{3/2} - 6u^{1/2}\right) + C$$

$$= \frac{u^{1/2}}{60}(3u^2 + 10u - 45) + C$$

$$= \frac{\sqrt{2x - 1}}{60}[3(2x - 1)^2 + 10(2x - 1) - 45] + C$$

$$= \frac{1}{60}\sqrt{2x - 1}(12x^2 + 8x - 52) + C$$

$$= \frac{1}{15}\sqrt{2x - 1}(3x^2 + 2x - 13) + C$$

52. $u = x + 3, x = u - 3, dx = du$

$$\int \frac{2x - 1}{\sqrt{x + 3}}\,dx = \int \frac{2(u - 3) - 1}{\sqrt{u}}\,du$$

$$= \int (2u^{1/2} - 7u^{-1/2})\,du$$

$$= \frac{4}{3}u^{3/2} - 14u^{1/2} + C$$

$$= \frac{2u^{1/2}}{3}(2u - 21) + C$$

$$= \frac{2}{3}\sqrt{x + 3}[2(x + 3) - 21] + C$$

$$= \frac{2}{3}\sqrt{x + 3}(2x - 15) + C$$

53. $u = x + 1, x = u - 1, dx = du$

$$\int \frac{-x}{(x + 1) - \sqrt{x + 1}}\,dx = \int \frac{-(u - 1)}{u - \sqrt{u}}\,du$$

$$= -\int \frac{(\sqrt{u} + 1)(\sqrt{u} - 1)}{\sqrt{u}(\sqrt{u} - 1)}\,du$$

$$= -\int (1 + u^{-1/2})\,du$$

$$= -(u + 2u^{1/2}) + C$$

$$= -u - 2\sqrt{u} + C$$

$$= -(x + 1) - 2\sqrt{x + 1} + C$$

$$= -x - 2\sqrt{x + 1} - 1 + C$$

$$= -\left(x + 2\sqrt{x + 1}\right) + C_1$$

where $C_1 = -1 + C$.

54. $u = t - 4, t = u + 4, dt = du$

$$\int t\sqrt[3]{t - 4}\,dt = \int (u + 4)u^{1/3}\,du$$

$$= \int (u^{4/3} + 4u^{1/3})\,du$$

$$= \frac{3}{7}u^{7/3} + 3u^{4/3} + C$$

$$= \frac{3u^{4/3}}{7}(u + 7) + C$$

$$= \frac{3}{7}(t - 4)^{4/3}[(t - 4) + 7] + C$$

$$= \frac{3}{7}(t - 4)^{4/3}(t + 3) + C$$

55. Let $u = x^2 + 1$, $du = 2x\,dx$.

$$\int_{-1}^{1} x(x^2+1)^3\,dx = \frac{1}{2}\int_{-1}^{1}(x^2+1)^3(2x)\,dx = \left[\frac{1}{8}(x^2+1)^4\right]_{-1}^{1} = 0$$

56. Let $u = 1 - x^2$, $du = -2x\,dx$.

$$\int_0^1 x\sqrt{1-x^2}\,dx = -\frac{1}{2}\int_0^1 (1-x^2)^{1/2}(-2x)\,dx = \left[-\frac{1}{3}(1-x^2)^{3/2}\right]_0^1 = 0 + \frac{1}{3} = \frac{1}{3}$$

57. Let $u = 2x + 1$, $du = 2\,dx$.

$$\int_0^4 \frac{1}{\sqrt{2x+1}}\,dx = \frac{1}{2}\int_0^4 (2x+1)^{-1/2}(2)\,dx = \left[\sqrt{2x+1}\right]_0^4 = \sqrt{9} - \sqrt{1} = 2$$

58. Let $u = 1 + 2x^2$, $du = 4x\,dx$.

$$\int_0^2 \frac{x}{\sqrt{1+2x^2}}\,dx = \frac{1}{4}\int_0^2 (1+2x^2)^{-1/2}(4x)\,dx = \left[\frac{1}{2}\sqrt{1+2x^2}\right]_0^2 = \frac{3}{2} - \frac{1}{2} = 1$$

59. Let $u = 1 + \sqrt{x}$, $du = \dfrac{1}{2\sqrt{x}}\,dx$.

$$\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx = 2\int_1^9 (1+\sqrt{x})^{-2}\left(\frac{1}{2\sqrt{x}}\right)dx = \left[-\frac{2}{1+\sqrt{x}}\right]_1^9 = -\frac{1}{2} + 1 = \frac{1}{2}$$

60. Let $u = 4 + x^2$, $du = 2x\,dx$.

$$\int_0^2 x\sqrt[3]{4+x^2}\,dx = \frac{1}{2}\int_0^2 (4+x^2)^{1/3}(2x)\,dx = \left[\frac{3}{8}(4+x^2)^{4/3}\right]_0^2 = \frac{3}{8}(8^{4/3} - 4^{4/3}) = 6 - \frac{3}{2}\sqrt[3]{4} \approx 3.619$$

61. $u = 2 - x$, $x = 2 - u$, $dx = -du$

When $x = 1$, $u = 1$. When $x = 2$, $u = 0$.

$$\int_1^2 (x-1)\sqrt{2-x}\,dx = \int_1^0 -[(2-u)-1]\sqrt{u}\,du = \int_1^0 (u^{3/2} - u^{1/2})\,du = \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_1^0 = -\left[\frac{2}{5} - \frac{2}{3}\right] = \frac{4}{15}$$

62. $u = 2x + 1$, $x = \frac{1}{2}(u-1)$, $dx = \frac{1}{2}\,du$

When $x = 0$, $u = 1$. When $x = 4$, $u = 9$.

$$\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx = \int_1^9 \frac{(1/2)(u-1)}{\sqrt{u}}\frac{1}{2}\,du = \frac{1}{4}\int_1^9 (u^{1/2} - u^{-1/2})\,du = \frac{1}{4}\left[\frac{2}{3}u^{3/2} - 2u^{1/2}\right]_1^9 = \frac{1}{4}(18-6) - \frac{1}{4}\left(\frac{2}{3} - 2\right) = \frac{10}{3}$$

63. $$\int_0^{\pi/2} \cos\left(\frac{2}{3}x\right)dx = \left[\frac{3}{2}\sin\left(\frac{2}{3}x\right)\right]_0^{\pi/2} = \frac{3}{2}\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{4}$$

64. $$\int_{\pi/3}^{\pi/2} (x + \cos x)\,dx = \left[\frac{x^2}{2} + \sin x\right]_{\pi/3}^{\pi/2} = \left(\frac{\pi^2}{8} + 1\right) - \left(\frac{\pi^2}{18} + \frac{\sqrt{3}}{2}\right) = \frac{5\pi^2}{72} + \frac{2-\sqrt{3}}{2}$$

65. $u = x + 1, x = u - 1, dx = du$

When $x = 0, u = 1$. When $x = 7, u = 8$.

$$\text{Area} = \int_0^7 x\sqrt[3]{x+1}\,dx = \int_1^8 (u-1)\sqrt[3]{u}\,du$$

$$= \int_1^8 (u^{4/3} - u^{1/3})\,du = \left[\frac{3}{7}u^{7/3} - \frac{3}{4}u^{4/3}\right]_1^8 = \left(\frac{384}{7} - 12\right) - \left(\frac{3}{7} - \frac{3}{4}\right) = \frac{1209}{28}$$

66. $u = x + 2, x = u - 2, dx = du$

When $x = -2, u = 0$. When $x = 6, u = 8$.

$$\text{Area} = \int_{-2}^6 x^2\sqrt[3]{x+2}\,dx = \int_0^8 (u-2)^2\sqrt[3]{u}\,du = \int_0^8 (u^{7/3} - 4u^{4/3} + 4u^{1/3})\,du = \left[\frac{3}{10}u^{10/3} - \frac{12}{7}u^{7/3} + 3u^{4/3}\right]_0^8 = \frac{4752}{35}$$

67. $A = \int_0^\pi (2\sin x + \sin 2x)\,dx = -\left[2\cos x + \frac{1}{2}\cos 2x\right]_0^\pi = 4$

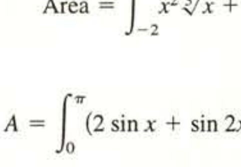
68. $A = \int_0^\pi (\sin x + \cos 2x)\,dx = \left[-\cos x + \frac{1}{2}\sin 2x\right]_0^\pi = 2$

69. $\text{Area} = \int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)dx = 2\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)\left(\frac{1}{2}\right)dx = \left[2\tan\left(\frac{x}{2}\right)\right]_{\pi/2}^{2\pi/3} = 2\left(\sqrt{3} - 1\right)$

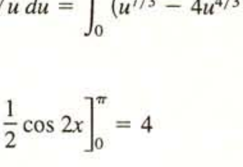
70. Let $u = 2x, du = 2\,dx$.

$$\text{Area} = \int_{\pi/12}^{\pi/4} \csc 2x \cot 2x\,dx = \frac{1}{2}\int_{\pi/12}^{\pi/4} \csc 2x \cot 2x(2)\,dx = \left[-\frac{1}{2}\csc 2x\right]_{\pi/12}^{\pi/4} = \frac{1}{2}$$

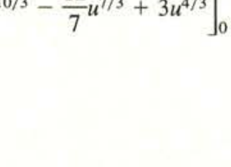
71. $\int_0^4 \frac{x}{\sqrt{2x+1}}\,dx \approx 3.333 = \frac{10}{3}$



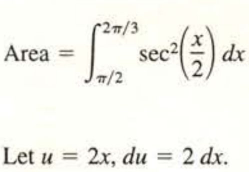
72. $\int_0^2 x^3\sqrt{x+2}\,dx \approx 7.581$



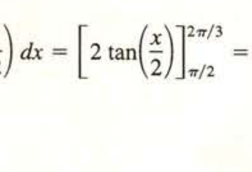
73. $\int_3^7 x\sqrt{x-3}\,dx \approx 28.8$



74. $\int_1^5 x^2\sqrt{x-1}\,dx \approx 67.505$



75. $\int_0^3 \left(\theta + \cos\frac{\theta}{6}\right)d\theta \approx 7.377$



76. $\int_0^{\pi/2} \sin 2x\,dx \approx 1.0$

77. $\int (2x-1)^2\,dx = \frac{1}{2}\int (2x-1)^2\,2\,dx = \frac{1}{6}(2x-1)^3 + C_1 = \frac{4}{3}x^3 - 2x^2 + x - \frac{1}{6} + C_1$

$\int (2x-1)^2\,dx = \int (4x^2 - 4x + 1)\,dx = \frac{4}{3}x^3 - 2x^2 + x + C_2$

They differ by a constant: $C_2 = C_1 - \frac{1}{6}$.