99. $f(x) = x|x| = \begin{cases} x^2, & \text{if } x \geq 0 \\ -x^2, & \text{if } x < 0 \end{cases}$

$f'(x) = \begin{cases} 2x, & \text{if } x \geq 0 \\ -2x, & \text{if } x < 0 \end{cases} = 2|x|$

$f''(x) = \begin{cases} 2, & \text{if } x > 0 \\ -2, & \text{if } x < 0 \end{cases}$

$f''(0)$ does not exist since the left and right derivatives are not equal.

100. (a) $(fg' - f'g)' = fg'' + f'g' - f'g' - f''g$

$= fg'' - f''g$ True

(b) $(fg)'' = (fg' + f'g)'$

$= fg'' + f'g' + f'g' + f''g$

$= fg'' + 2f'g' + f''g$

$\neq fg'' + f''g$ False

Section 2.4 The Chain Rule

$y = f(g(x))$	$u = g(x)$	$y = f(u)$
1. $y = (6x - 5)^4$	$u = 6x - 5$	$y = u^4$
2. $y = \dfrac{1}{\sqrt{x+1}}$	$u = x + 1$	$y = u^{-1/2}$
3. $y = \sqrt{x^2 - 1}$	$u = x^2 - 1$	$y = \sqrt{u}$
4. $y = \tan(\pi x + 1)$	$u = \pi x + 1$	$y = \tan u$
5. $y = \csc^3 x$	$u = \csc x$	$y = u^3$
6. $y = \cos \dfrac{3x}{2}$	$u = \dfrac{3x}{2}$	$y = \cos u$

7. $y = (2x - 7)^3$

$y' = 3(2x - 7)^2(2) = 6(2x - 7)^2$

8. $y = (3x^2 + 1)^4$

$y' = 4(3x^2 + 1)^3(6x) = 24x(3x^2 + 1)^3$

9. $g(x) = 3(4 - 9x)^4$

$g'(x) = 12(4 - 9x)^3(-9) = -108(4 - 9x)^3$

10. $f(x) = 2(1 - x^2)^3$

$f'(x) = 6(1 - x^2)^2(-2x) = -12x(1 - x^2)^2$

11. $f(x) = (9 - x^2)^{2/3}$

$f'(x) = \frac{2}{3}(9 - x^2)^{-1/3}(-2x) = -\dfrac{4x}{3(9 - x^2)^{1/3}}$

12. $f(t) = (9t + 2)^{2/3}$

$f'(t) = \frac{2}{3}(9t + 2)^{-1/3}(9) = \dfrac{6}{\sqrt[3]{9t + 2}}$

13. $f(t) = (1 - t)^{1/2}$

$f'(t) = \frac{1}{2}(1 - t)^{-1/2}(-1) = -\dfrac{1}{2\sqrt{1 - t}}$

14. $g(x) = (3 - 2x)^{1/2}$

$g'(x) = \frac{1}{2}(3 - 2x)^{-1/2}(-2) = -\dfrac{1}{\sqrt{3 - 2x}}$

15. $y = (9x^2 + 4)^{1/3}$

$y' = \frac{1}{3}(9x^2 + 4)^{-2/3}(18x) = \dfrac{6x}{(9x^2 + 4)^{2/3}}$

16. $g(x) = \sqrt{x^2 - 2x + 1} = \sqrt{(x - 1)^2} = |x - 1|$

$g'(x) = \begin{cases} 1, & x > 1 \\ -1, & x < 1 \end{cases}$

17. $y = 2(4 - x^2)^{1/2}$

$$y' = (4 - x^2)^{-1/2}(-2x) = -\frac{2x}{\sqrt{4 - x^2}}$$

18. $f(x) = -3(2 - 9x)^{1/4}$

$$f'(x) = -\frac{3}{4}(2 - 9x)^{-3/4}(-9) = \frac{27}{4(2 - 9x)^{3/4}}$$

19. $y = (x - 2)^{-1}$

$$y' = -1(x - 2)^{-2}(1) = \frac{-1}{(x - 2)^2}$$

20. $s(t) = (t^2 + 3t - 1)^{-1}$

$$s'(t) = -1(t^2 + 3t - 1)^{-2}(2t + 3)$$
$$= \frac{-(2t + 3)}{(t^2 + 3t - 1)^2}$$

21. $f(t) = (t - 3)^{-2}$

$$f'(t) = -2(t - 3)^{-3} = \frac{-2}{(t - 3)^3}$$

22. $y = -4(t + 2)^{-2}$

$$y' = 8(t + 2)^{-3} = \frac{8}{(t + 2)^3}$$

23. $y = (x + 2)^{-1/2}$

$$\frac{dy}{dx} = -\frac{1}{2}(x + 2)^{-3/2} = -\frac{1}{2(x + 2)^{3/2}}$$

24. $g(t) = (t^2 - 2)^{-1/2}$

$$g'(t) = -\frac{1}{2}(t^2 - 2)^{-3/2}(2t) = -\frac{t}{(t^2 - 2)^{3/2}}$$

25. $f(x) = x^2(x - 2)^4$

$$f'(x) = x^2[4(x - 2)^3(1)] + (x - 2)^4(2x)$$
$$= 2x(x - 2)^3[2x + (x - 2)]$$
$$= 2x(x - 2)^3(3x - 2)$$

26. $f(x) = x(3x - 9)^3$

$$f'(x) = x[3(3x - 9)^2(3)] + (3x - 9)^3(1)$$
$$= (3x - 9)^2[9x + 3x - 9]$$
$$= 27(x - 3)^2(4x - 3)$$

27. $y = x\sqrt{1 - x^2} = x(1 - x^2)^{1/2}$

$$y' = x\left[\frac{1}{2}(1 - x^2)^{-1/2}(-2x)\right] + (1 - x^2)^{1/2}(1)$$
$$= -x^2(1 - x^2)^{-1/2} + (1 - x^2)^{1/2}$$
$$= (1 - x^2)^{-1/2}[-x^2 + (1 - x^2)]$$
$$= \frac{1 - 2x^2}{\sqrt{1 - x^2}}$$

28. $y = x^2\sqrt{9 - x^2} = x^2(9 - x^2)^{1/2}$

$$y' = x^2\left[\frac{1}{2}(9 - x^2)^{-1/2}(-2x)\right] + (9 - x^2)^{1/2}(2x)$$
$$= -x^3(9 - x^2)^{-1/2} + 2x(9 - x^2)^{1/2}$$
$$= x(9 - x^2)^{-1/2}[-x^2 + 2(9 - x^2)]$$
$$= \frac{x(18 - 3x^2)}{\sqrt{9 - x^2}}$$
$$= \frac{3x(6 - x^2)}{\sqrt{9 - x^2}}$$

29. $y = \dfrac{x}{\sqrt{x^2 + 1}} = x(x^2 + 1)^{-1/2}$

$$y' = x\left[-\frac{1}{2}(x^2 + 1)^{-3/2}(2x)\right] + (x^2 + 1)^{-1/2}(1)$$
$$= -x^2(x^2 + 1)^{-3/2} + (x^2 + 1)^{-1/2}$$
$$= (x^2 + 1)^{-3/2}[-x^2 + (x^2 + 1)]$$
$$= \frac{1}{(x^2 + 1)^{3/2}}$$

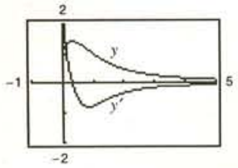
30. $y = \dfrac{x^2}{(x^9 + 9)^{1/2}}$

$$y' = \frac{(x^9 + 9)^{1/2}(2x) - x^2(1/2)(x^9 + 9)^{-1/2}\,9x^8}{x^9 + 9}$$
$$= \frac{(x^9 + 9)^{-1/2}x[2(x^9 + 9) - (9/2)x^9]}{x^9 + 9}$$
$$= \frac{x[18 - (5/2)x^9]}{(x^9 + 9)^{3/2}} = \frac{x(36 - 5x^9)}{2(x^9 + 9)^{3/2}}$$

31. $y = \dfrac{\sqrt{x} + 1}{x^2 + 1}$

$y' = \dfrac{1 - 3x^2 - 4x^{3/2}}{2\sqrt{x}(x^2 + 1)^2}$

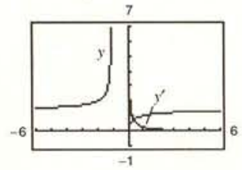
The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



32. $y = \sqrt{\dfrac{2x}{x + 1}}$

$y' = \dfrac{1}{\sqrt{2x}(x + 1)^{3/2}}$

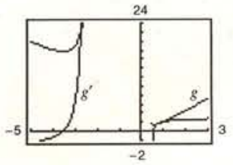
y' has no zeros.



33. $g(t) = \dfrac{3t^2}{\sqrt{t^2 + 2t - 1}}$

$g'(t) = \dfrac{3t(t^2 + 3t - 2)}{(t^2 + 2t - 1)^{3/2}}$

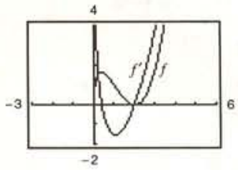
The zeros of g' correspond to the points on the graph of g where the tangent lines are horizontal.



34. $f(x) = \sqrt{x}(2 - x)^2$

$f'(x) = \dfrac{(x - 2)(5x - 2)}{2\sqrt{x}}$

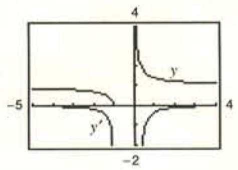
The zeros of f' correspond to the points on the graph of f where the tangent lines are horizontal.



35. $y = \sqrt{\dfrac{x + 1}{x}}$

$y' = -\dfrac{\sqrt{(x + 1)/x}}{2x(x + 1)}$

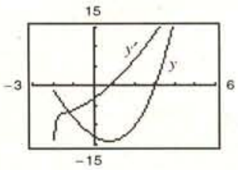
y' has no zeros.



36. $y = (t^2 - 9)\sqrt{t + 2}$

$y' = \dfrac{5t^2 + 8t - 9}{2\sqrt{t + 2}}$

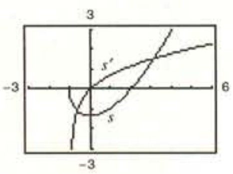
The zero of y' corresponds to the point on the graph of y where the tangent line is horizontal.



37. $s(t) = \dfrac{-2(2 - t)\sqrt{1 + t}}{3}$

$s'(t) = \dfrac{t}{\sqrt{1 + t}}$

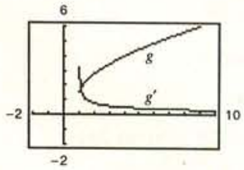
The zero of $s'(t)$ corresponds to the point on the graph of $s(t)$ where the tangent line is horizontal.



38. $g(x) = \sqrt{x-1} + \sqrt{x+1}$

$$g'(x) = \frac{1}{2\sqrt{x-1}} + \frac{1}{2\sqrt{x+1}}$$

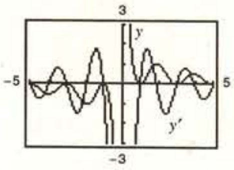
g' has no zeros.



39. $y = \dfrac{\cos \pi x + 1}{x}$

$$\frac{dy}{dx} = \frac{-\pi x \sin \pi x - \cos \pi x - 1}{x^2}$$

$$= -\frac{\pi x \sin \pi x + \cos \pi x + 1}{x^2}$$

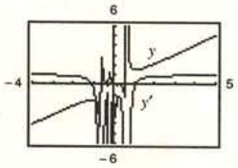


The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.

40. $y = x^2 \tan \dfrac{1}{x}$

$$\frac{dy}{dx} = 2x \tan \frac{1}{x} - \sec^2 \frac{1}{x}$$

The zeros of y' correspond to the points on the graph of y where the tangent lines are horizontal.



41. (a) $y = \sin x$

$y' = \cos x$

$y'(0) = 1$

1 cycle in $[0, 2\pi]$

(b) $y = \sin 2x$

$y' = 2 \cos 2x$

$y'(0) = 2$

2 cycles in $[0, 2\pi]$

42. (a) $y = \sin 3x$

$y' = 3 \cos 3x$

$y'(0) = 3$

3 cycles in $[0, 2\pi]$

(b) $y = \sin\left(\dfrac{x}{2}\right)$

$y' = \left(\dfrac{1}{2}\right)\cos\left(\dfrac{x}{2}\right)$

$y'(0) = \dfrac{1}{2}$

Half cycle in $[0, 2\pi]$

43. $y = \cos 3x$

$$\frac{dy}{dx} = -3 \sin 3x$$

44. $y = \sin \pi x$

$$\frac{dy}{dx} = \pi \cos \pi x$$

45. $g(x) = 3\tan 4x$

$g'(x) = 12\sec^2 4x$

46. $h(x) = \sec(x^2)$

$h'(x) = 2x\sec(x^2)\tan(x^2)$

47. $f(\theta) = \frac{1}{4}\sin^2 2\theta = \frac{1}{4}(\sin 2\theta)^2$

$f'(\theta) = 2(\frac{1}{4})(\sin 2\theta)(\cos 2\theta)(2)$

$= \sin 2\theta\cos 2\theta = \frac{1}{2}\sin 4\theta$

48. $g(t) = 5\cos^2 \pi t = 5(\cos \pi t)^2$

$g'(t) = 10\cos \pi t(-\sin \pi t)(\pi)$

$= -10\pi(\sin \pi t)(\cos \pi t) = -5\pi\sin 2\pi t$

49. $y = \sqrt{x} + \dfrac{1}{4}\sin(2x)^2$

$= \sqrt{x} + \dfrac{1}{4}\sin(4x^2)$

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{-1/2} + \dfrac{1}{4}\cos(4x^2)(8x)$

$= \dfrac{1}{2\sqrt{x}} + 2x\cos(2x)^2$

50. $y = 3x - 5\cos(\pi x)^2$

$= 3x - 5\cos(\pi^2 x^2)$

$\dfrac{dy}{dx} = 3 + 5\sin(\pi^2 x^2)(2\pi^2 x)$

$= 3 + 10\pi^2 x\sin(\pi x)^2$

51. $y = \sin(\cos x)$

$\dfrac{dy}{dx} = \cos(\cos x)\cdot(-\sin x)$

$= -\sin x\cos(\cos x)$

52. $y = \sin\sqrt{x} + \sqrt{\sin x}$

$\dfrac{dy}{dx} = \cos\sqrt{x}\cdot\dfrac{1}{2}x^{-1/2} + \dfrac{1}{2}(\sin x)^{-1/2}\cdot\cos x$

$= \dfrac{\cos\sqrt{x}}{2\sqrt{x}} + \dfrac{\cos x}{2\sqrt{\sin x}}$

53. $s(t) = (t^2 + 2t + 8)^{1/2}$, $(2, 4)$

$s'(t) = \dfrac{1}{2}(t^2 + 2t + 8)^{-1/2}(2t + 2)$

$= \dfrac{t + 1}{\sqrt{t^2 + 2t + 8}}$

$s'(2) = \dfrac{3}{4}$

54. $y = (3x^3 + 4x)^{1/5}$, $(2, 2)$

$y' = \dfrac{1}{5}(3x^3 + 4x)^{-4/5}(9x^2 + 4)$

$= \dfrac{9x^2 + 4}{5(3x^3 + 4x)^{4/5}}$

$y'(2) = \dfrac{1}{2}$

55. $f(x) = \dfrac{3}{x^3 - 4} = 3(x^3 - 4)^{-1}$, $\left(-1, -\dfrac{3}{5}\right)$

$f'(x) = -3(x^3 - 4)^{-2}(3x^2) = -\dfrac{9x^2}{(x^3 - 4)^2}$

$f'(-1) = -\dfrac{9}{25}$

56. $f(x) = \dfrac{1}{(x^2 - 3x)^2} = (x^2 - 3x)^{-2}$, $\left(4, \dfrac{1}{16}\right)$

$f'(x) = -2(x^2 - 3x)^{-3}(2x - 3) = \dfrac{-2(2x - 3)}{(x^2 - 3x)^3}$

$f'(4) = -\dfrac{5}{32}$

57. $f(t) = \dfrac{3t + 2}{t - 1}$, $(0, -2)$

$f'(t) = \dfrac{(t - 1)(3) - (3t + 2)(1)}{(t - 1)^2} = \dfrac{-5}{(t - 1)^2}$

$f'(0) = -5$

58. $f(x) = \dfrac{x+1}{2x-3},\ (2, 3)$

$$f'(x) = \frac{(2x-3)(1) - (x+1)(2)}{(2x-3)^2} = \frac{-5}{(2x-3)^2}$$

$$f'(2) = -5$$

59. $y = 37 - \sec^3(2x),\ (0, 36)$

$$y' = -3\sec^2(2x)[2\sec(2x)\tan(2x)]$$
$$= -6\sec^3(2x)\tan(2x)$$
$$y'(0) = 0$$

60. $y = \dfrac{1}{x} + \sqrt{\cos x},\ \left(\dfrac{\pi}{2}, \dfrac{2}{\pi}\right)$

$$y' = -\frac{1}{x^2} - \frac{\sin x}{2\sqrt{\cos x}}$$

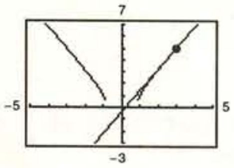
$y'(\pi/2)$ is undefined.

61. (a) $f(x) = \sqrt{3x^2 - 2},\ (3, 5)$

$$f'(x) = \frac{1}{2}(3x^2 - 2)^{-1/2}(6x)$$
$$= \frac{3x}{\sqrt{3x^2 - 2}}$$
$$f'(3) = \frac{9}{5}$$

Tangent line:

$$y - 5 = \frac{9}{5}(x - 3) \Rightarrow 9x - 5y - 2 = 0$$

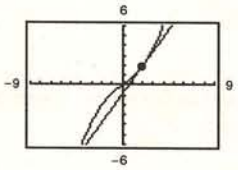
(b) 

62. (a) $f(x) = \dfrac{1}{3}x\sqrt{x^2 + 5},\ (2, 2)$

$$f'(x) = \frac{1}{3}x\left[\frac{1}{2}(x^2 + 5)^{-1/2}(2x)\right] + \frac{1}{3}(x^2 + 5)^{1/2}$$
$$= \frac{x^2}{3\sqrt{x^2 + 5}} + \frac{1}{3}\sqrt{x^2 + 5}$$
$$f'(2) = \frac{4}{3(3)} + \frac{1}{3}(3) = \frac{13}{9}$$

Tangent line:

$$y - 2 = \frac{13}{9}(x - 2) \Rightarrow 13x - 9y - 8 = 0$$

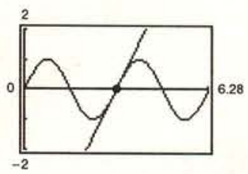
(b) 

63. (a) $f(x) = \sin 2x,\ (\pi, 0)$

$$f'(x) = 2\cos 2x$$
$$f'(\pi) = 2$$

Tangent line:

$$y = 2(x - \pi) \Rightarrow 2x - y - 2\pi = 0$$

(b) 

64. (a) $f(x) = \tan^2 x, \left(\frac{\pi}{4}, 1\right)$

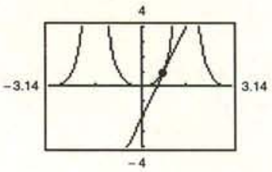
$f'(x) = 2 \tan x \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2(1)(2) = 4$

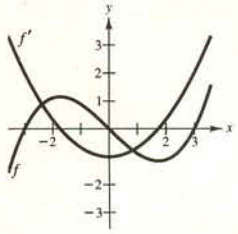
Tangent line:

$y - 1 = 4\left(x - \frac{\pi}{4}\right) \Rightarrow 4x - y + (1 - \pi) = 0$

(b)

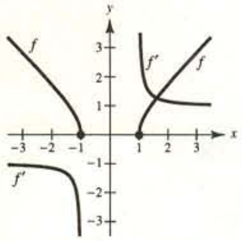


65.



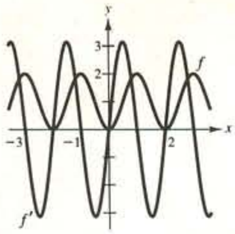
The zeros of f' correspond to the points where the graph of f has horizontal tangents.

66.



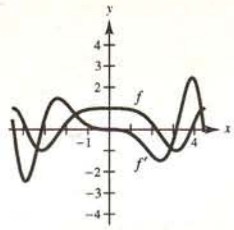
f is decreasing on $(-\infty, -1)$ so f' must be negative there. f is increasing on $(1, \infty)$ so f' must be postive there.

67.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

68.



The zeros of f' correspond to the points where the graph of f has horizontal tangents.

When $v = 30, f' \approx -1.016$.

76. $y = \frac{1}{3} \cos 12t - \frac{1}{4} \sin 12t$

$v = y' = \frac{1}{3}[-12 \sin 12t] - \frac{1}{4}[12 \cos 12t]$

$= -4 \sin 12t - 3 \cos 12t$

When $t = \pi/8$, $y = 0.25$ feet and $v = 4$ feet per second.

77. $\theta = 0.2 \cos 8t$

The maximum angular displacement is $\theta = 0.2$ (since $-1 \le \cos 8t \le 1$).

$$\frac{d\theta}{dt} = 0.2[-8 \sin 8t] = -1.6 \sin 8t$$

When $t = 3$, $d\theta/dt = -1.6 \sin 24 \approx 1.4489$ radians per second.