5. $y = x^2 + 1$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = 2x\,\frac{dx}{dt}$$

(a) When $x = -1$,

$$\frac{dy}{dt} = 2(-1)(2) = -4 \text{ cm/sec.}$$

(b) When $x = 0$,

$$\frac{dy}{dt} = 2(0)(2) = 0 \text{ cm/sec.}$$

(c) When $x = 1$,

$$\frac{dy}{dt} = 2(1)(2) = 4 \text{ cm/sec.}$$

(d) When $x = 3$,

$$\frac{dy}{dt} = 2(3)(2) = 12 \text{ cm/sec.}$$

6. $y = \dfrac{1}{1 + x^2}$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \left[\frac{-2x}{(1 + x^2)^2}\right]\frac{dx}{dt}$$

(a) When $x = -2$,

$$\frac{dy}{dt} = \frac{-2(-2)(2)}{25} = \frac{8}{25} \text{ cm/sec.}$$

(b) When $x = 0$,

$$\frac{dy}{dt} = 0 \text{ cm/sec.}$$

(c) When $x = 2$,

$$\frac{dy}{dt} = \frac{-2(2)(2)}{25} = \frac{-8}{25} \text{ cm/sec.}$$

(d) When $x = 10$,

$$\frac{dy}{dt} = \frac{-2(10)(2)}{(101)^2} = \frac{-40}{10{,}201} \approx -0.0039 \text{ cm/sec.}$$

7. $y = \tan x$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \sec^2 x\,\frac{dx}{dt}$$

(a) When $x = -\pi/3$,

$$\frac{dy}{dt} = (2)^2(2) = 8 \text{ cm/sec.}$$

(b) When $x = -\pi/4$,

$$\frac{dy}{dt} = (\sqrt{2})^2(2) = 4 \text{ cm/sec.}$$

(c) When $x = 0$,

$$\frac{dy}{dt} = (1)^2(2) = 2 \text{ cm/sec.}$$

(d) When $x = 1$,

$$\frac{dy}{dt} = (\sec 1)^2 2 \approx 6.8510 \text{ cm/sec.}$$

8. $y = \sin x$

$$\frac{dx}{dt} = 2$$

$$\frac{dy}{dt} = \cos x\frac{dx}{dt}$$

(a) When $x = \pi/6$,

$$\frac{dy}{dt} = \left(\cos\frac{\pi}{6}\right)(2) = \sqrt{3} \text{ cm/sec.}$$

(b) When $x = \pi/4$,

$$\frac{dy}{dt} = \left(\cos\frac{\pi}{4}\right)(2) = \sqrt{2} \text{ cm/sec.}$$

(c) When $x = \pi/3$,

$$\frac{dy}{dt} = \left(\cos\frac{\pi}{3}\right)(2) = 1 \text{ cm/sec.}$$

(d) When $x = \pi/2$,

$$\frac{dy}{dt} = \left(\cos\frac{\pi}{2}\right)(2) = 0 \text{ cm/sec.}$$

9. (a) For increasing x, dy/dt decreases.

(b) For increasing y, dx/dt increases.

10. (a) For increasing x, dy/dt increases.

(b) For increasing y, dx/dt decreases.

11. $D = \sqrt{x^2 + y^2} = \sqrt{x^2 + (x^2 + 1)^2} = \sqrt{x^4 + 3x^2 + 1}$

$\dfrac{dx}{dt} = 2$

$$\frac{dD}{dt} = \frac{1}{2}(x^4 + 3x^2 + 1)^{-1/2}(4x^3 + 6x)\frac{dx}{dt} = \frac{2x^3 + 3x}{\sqrt{x^4 + 3x^2 + 1}}\frac{dx}{dt} = \frac{4x^3 + 6x}{\sqrt{x^4 + 3x^2 + 1}}$$

12. $D = \sqrt{x^2 + y^2} = \sqrt{x^2 + \sin^2 x}$

$\dfrac{dx}{dt} = 2$

$$\frac{dD}{dt} = \frac{1}{2}(x^2 + \sin^2 x)^{-1/2}(2x + 2\sin x \cos x)\frac{dx}{dt} = \frac{x + \sin x \cos x}{\sqrt{x^2 + \sin^2 x}}\frac{dx}{dt} = \frac{2 + 2\sin x \cos x}{\sqrt{x^2 + \sin^2 x}}$$

13. $A = \pi r^2$

$\dfrac{dr}{dt} = 2$

$\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$

(a) When $r = 6$,

$$\frac{dA}{dt} = 2\pi(6)(2) = 24\pi \text{ in}^2/\text{min}.$$

(b) When $r = 24$,

$$\frac{dA}{dt} = 2\pi(24)(2) = 96\pi \text{ in}^2/\text{min}.$$

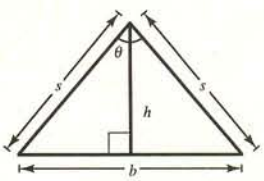
14. $A = \pi r^2$

$\dfrac{dA}{dt} = 2\pi r \dfrac{dr}{dt}$

If dr/dt is constant, dA/dt is not constant.

$\dfrac{dA}{dt}$ depends on r.

15. (a) $\sin\dfrac{\theta}{2} = \dfrac{(1/2)b}{s} \Longrightarrow b = 2s\sin\dfrac{\theta}{2}$



$\cos\dfrac{\theta}{2} = \dfrac{h}{s} \Longrightarrow h = s\cos\dfrac{\theta}{2}$

$$A = \frac{1}{2}bh = \frac{1}{2}\left(2s\sin\frac{\theta}{2}\right)\left(s\cos\frac{\theta}{2}\right)$$

$$= \frac{s^2}{2}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right) = \frac{s^2}{2}\sin\theta$$

(b) $\dfrac{dA}{dt} = \dfrac{s^2}{2}\cos\theta\dfrac{d\theta}{dt}$ where $\dfrac{d\theta}{dt} = \dfrac{1}{2}$ rad/min.

When $\theta = \dfrac{\pi}{6}$, $\dfrac{dA}{dt} = \dfrac{s^2}{2}\left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{\sqrt{3}s^2}{8}$ rad/min.

When $\theta = \dfrac{\pi}{3}$, $\dfrac{dA}{dt} = \dfrac{s^2}{2}\left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{s^2}{8}$ rad/min.

(c) If $d\theta/dt$ is constant, dA/dt is proportional to $\cos\theta$.

16. $V = \frac{4}{3}\pi r^3$

$\frac{dr}{dt} = 2$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

(a) When $r = 6$, $\frac{dV}{dt} = 4\pi(6)^2(2) = 288\pi$ in³/min.

When $r = 24$, $\frac{dV}{dt} = 4\pi(24)^2(2) = 4608\pi$ in³/min.

(b) If dr/dt is constant, dV/dt is proportional to r^2.

17. $V = \frac{4}{3}\pi r^3$, $\frac{dV}{dt} = 500$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{4\pi r^2}\left(\frac{dV}{dt}\right) = \frac{1}{4\pi r^2}(500)$

(a) When $r = 30$, $\frac{dr}{dt} = \frac{1}{4\pi(30)^2}(500) = \frac{5}{36\pi}$ cm/min.

(b) When $r = 60$, $\frac{dr}{dt} = \frac{1}{4\pi(60)^2}(500) = \frac{5}{144\pi}$ cm/min.

18. $V = x^3$

$\frac{dx}{dt} = 3$

$\frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

(a) When $x = 1$,

$\frac{dV}{dt} = 3(1)^2(3) = 9$ cm³/sec.

(b) When $x = 10$,

$\frac{dV}{dt} = 3(10)^2(3) = 900$ cm³/sec.

19. $s = 6x^2$

$\frac{dx}{dt} = 3$

$\frac{ds}{dt} = 12x \frac{dx}{dt}$

(a) When $x = 1$,

$\frac{ds}{dt} = 12(1)(3) = 36$ cm²/sec.

(b) When $x = 10$,

$\frac{ds}{dt} = 12(10)(3) = 360$ cm²/sec.

20. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2(3r) = \pi r^3$

$\frac{dr}{dt} = 2$

$\frac{dV}{dt} = 3\pi r^2 \frac{dr}{dt}$

(a) When $r = 6$,

$\frac{dV}{dt} = 3\pi(6)^2(2) = 216\pi$ in³/min.

(b) When $r = 24$,

$\frac{dV}{dt} = 3\pi(24)^2(2) = 3456\pi$ in³/min.

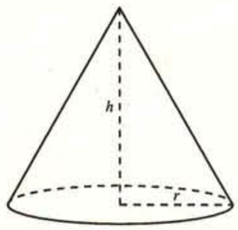
21. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{9}{4}h^2\right)h$ [since $2r = 3h$]

$= \frac{3\pi}{4}h^3$

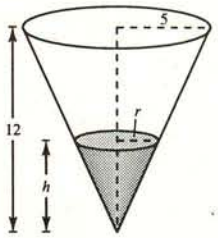
$\frac{dV}{dt} = 10$

$\frac{dV}{dt} = \frac{9\pi}{4}h^2 \frac{dh}{dt} \Longrightarrow \frac{dh}{dt} = \frac{4(dV/dt)}{9\pi h^2}$

When $h = 15$, $\frac{dh}{dt} = \frac{4(10)}{9\pi(15)^2} = \frac{8}{405\pi}$ ft/min.



22. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \frac{25}{144}h^3 = \frac{25\pi}{3(144)}h^3$



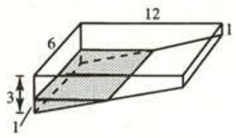
$\left(\text{By similar triangles, } \frac{r}{5} = \frac{h}{12} \Rightarrow r = \frac{5}{12}h.\right)$

$\frac{dV}{dt} = 10$

$\frac{dV}{dt} = \frac{25\pi}{144}h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \left(\frac{144}{25\pi h^2}\right)\frac{dV}{dt}$

When $h = 8$, $\frac{dh}{dt} = \frac{144}{25\pi(64)}(10) = \frac{9}{10\pi}$ ft/min.

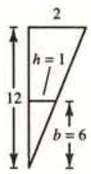
23.



(a) Total volume of pool $= \frac{1}{2}(2)(12)(6) + (1)(6)(12) = 144 \text{ m}^3$

Volume of 1m. of water $= \frac{1}{2}(1)(6)(6) = 18 \text{ m}^3$

(see similar triangle diagram)



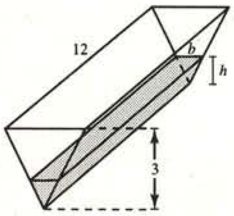
% pool filled $= \frac{18}{144}(100\%) = 12.5\%$

(b) Since for $0 \le h \le 2$, $b = 6h$, you have

$$V = \frac{1}{2}bh(6) = 3bh = 3(6h)h = 18h^2$$

$\frac{dV}{dt} = 36h\frac{dh}{dt} = \frac{1}{4} \Rightarrow \frac{dh}{dt} = \frac{1}{144h} = \frac{1}{144(1)} = \frac{1}{144}$ m/min.

24. $V = \left(\frac{1}{2}\right)bh(12) = 6bh = 6h^2$ (since $b = h$)

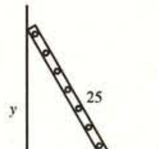


$\frac{dV}{dt} = 12h\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \left(\frac{1}{12h}\right)\frac{dV}{dt}$

When $h = 1$ and $dV/dt = 2$,

$\frac{dh}{dt} = \frac{1}{12}(2) = \frac{1}{6}$ ft/min $= 2$ in/min.

25. $x^2 + y^2 = 25^2$



$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-x}{y} \cdot \frac{dx}{dt} = \frac{-2x}{y}$ since $\frac{dx}{dt} = 2$.

(a) When $x = 7$, $y = \sqrt{576} = 24$, $\frac{dy}{dt} = \frac{-2(7)}{24} = \frac{-7}{12}$ ft/sec.

When $x = 15$, $y = \sqrt{400} = 20$, $\frac{dy}{dt} = \frac{-2(15)}{20} = \frac{-3}{2}$ ft/sec.

When $x = 24$, $y = 7$, $\frac{dy}{dt} = \frac{-2(24)}{7} = \frac{-48}{7}$ ft/sec.

—CONTINUED—

25. —CONTINUED—

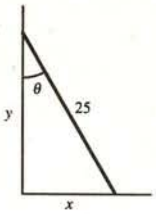
(b) $A = \frac{1}{2}xy$

$$\frac{dA}{dt} = \frac{1}{2}\left(x\frac{dy}{dt} + y\frac{dx}{dt}\right)$$

From part (a) we have $x = 7, y = 24, \frac{dx}{dt} = 2$, and $\frac{dy}{dt} = -\frac{7}{12}$.

Thus, $\frac{dA}{dt} = \frac{1}{2}\left[7\left(-\frac{7}{12}\right) + 24(2)\right] = \frac{527}{24} \approx 21.96 \text{ ft}^2/\text{sec}$.

(c) $\tan\theta = \frac{x}{y}$



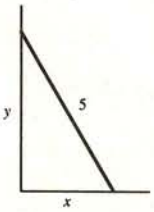
$$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta\left[\frac{1}{y}\cdot\frac{dx}{dt} - \frac{x}{y^2}\cdot\frac{dy}{dt}\right]$$

Using $x = 7, y = 24, \frac{dx}{dt} = 2, \frac{dy}{dt} = -\frac{7}{12}$ and $\cos\theta = \frac{24}{25}$, we have

$$\frac{d\theta}{dt} = \left(\frac{24}{25}\right)^2\left[\frac{1}{24}(2) - \frac{7}{(24)^2}\left(-\frac{7}{12}\right)\right] = \frac{1}{12} \text{ rad/sec.}$$

26. $x^2 + y^2 = 25$



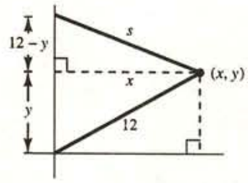
$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{y}{x}\cdot\frac{dy}{dt} = -\frac{0.15y}{x} \text{ since } \frac{dy}{dt} = 0.15.$$

When $x = 8$,

$$y = \sqrt{18.75}, \frac{dx}{dt} = -\frac{\sqrt{18.75}}{2.5}0.15 \approx -0.26 \text{ m/sec}$$

27. When $y = 6, x = \sqrt{12^2 - 6^2} = 6\sqrt{3}$, and



$$s = \sqrt{x^2 + (12 - y)^2}$$
$$= \sqrt{108 + 36} = 12.$$

$$x^2 + (12 - y)^2 = s^2$$

$$2x\frac{dx}{dt} + 2(12 - y)(-1)\frac{dy}{dt} = 2s\frac{ds}{dt}$$

$$x\frac{dx}{dt} + (y - 12)\frac{dy}{dt} = s\frac{ds}{dt}$$

Also, $x^2 + y^2 = 12^2$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}.$$

Thus, $x\frac{dx}{dt} + (y - 12)\left(\frac{-x}{y}\frac{dx}{dt}\right) = s\frac{ds}{dt}$

$$\frac{dx}{dt}\left[x - x + \frac{12x}{y}\right] = s\frac{ds}{dt} \Rightarrow \frac{dx}{dt} = \frac{sy}{12x}\cdot\frac{ds}{dt} = \frac{(12)(6)}{(12)(6\sqrt{3})}(-0.2) = \frac{-1}{5\sqrt{3}} = \frac{-\sqrt{3}}{15} \text{ m/sec (horizontal)}$$

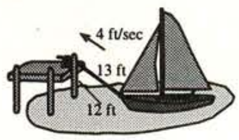
$$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt} = \frac{-6\sqrt{3}}{6}\cdot\frac{(-\sqrt{3})}{15} = \frac{1}{5} \text{ m/sec (vertical).}$$

28. Let L be the length of the rope.

$$L^2 = 144 + x^2$$

$$2L\frac{dL}{dt} = 2x\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{L}{x} \cdot \frac{dL}{dt} = -\frac{4L}{x} \text{ since } \frac{dL}{dt} = -4 \text{ ft/sec.}$$



When $L = 13$,

$$x = \sqrt{L^2 - 144} = \sqrt{169 - 144} = 5$$

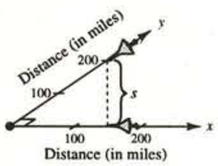
$$\frac{dx}{dt} = -\frac{4(13)}{5} = -\frac{52}{5} = -10.4 \text{ ft/sec.}$$

Speed of the boat increases as it approaches the dock.

29. (a) $s^2 = x^2 + y^2$

$$\frac{dx}{dt} = -450$$

$$\frac{dy}{dt} = -600$$



$$2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\frac{ds}{dt} = \frac{x(dx/dt) + y(dy/dt)}{s}$$

When $x = 150$ and $y = 200$, $s = 250$ and

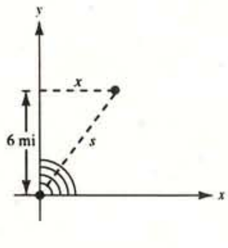
$$\frac{ds}{dt} = \frac{150(-450) + 200(-600)}{250} = -750 \text{ mph.}$$

(b) $t = \dfrac{250}{750} = \dfrac{1}{3}$ hr $= 20$ min

30. $x^2 + y^2 = s^2$

$$x = \sqrt{s^2 - y^2}$$

$$\frac{ds}{dt} = -240 \text{ mph}$$



$$y = 6 \text{ mi}$$

$$x = \sqrt{s^2 - 36}$$

$$\frac{dx}{dt} = \left(\frac{1}{2}\right)(s^2 - 36)^{-1/2}\left(2s\frac{ds}{dt}\right) = \frac{s}{\sqrt{s^2 - 36}} \cdot \frac{ds}{dt}$$

When $s = 10$, $\dfrac{dx}{dt} = \dfrac{10}{\sqrt{10^2 - 36}}(-240)$

$$= \frac{10}{8}(-240) = -300 \text{ mph.}$$

The speed of the plane is 300 mph.

31. $s^2 = 90^2 + x^2$

$x = 30$

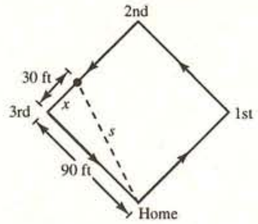
$\frac{dx}{dt} = -28$

$2s\frac{ds}{dt} = 2x\frac{dx}{dt} \Rightarrow \frac{ds}{dt} = \frac{x}{s} \cdot \frac{dx}{dt}$

When $x = 30$,

$s = \sqrt{90^2 + 30^2} = 30\sqrt{10}$

$\frac{ds}{dt} = \frac{30}{30\sqrt{10}}(-28) = \frac{-28}{\sqrt{10}} \approx -8.85$ ft/sec.



32. $s^2 = 90^2 + x^2$

$x = 60$

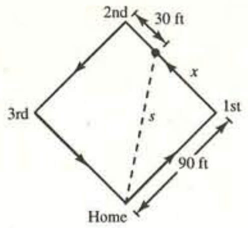
$\frac{dx}{dt} = 28$

$\frac{ds}{dt} = \frac{x}{s} \cdot \frac{dx}{dt}$

When $x = 60$,

$s = \sqrt{90^2 + 60^2} = 30\sqrt{13}$

$\frac{ds}{dt} = \frac{60}{30\sqrt{13}}(28) = \frac{56}{\sqrt{13}} \approx 15.53$ ft/sec.



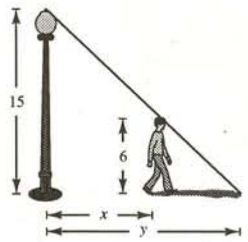
33. (a) $\frac{15}{6} = \frac{y}{y - x} \Rightarrow 15y - 15x = 6y$

$y = \frac{5}{3}x$

$\frac{dx}{dt} = 5$

$\frac{dy}{dt} = \frac{5}{3} \cdot \frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$ ft/sec

(b) $\frac{d(y - x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = \frac{25}{3} - 5 = \frac{10}{3}$ ft/sec



34. (a) $\frac{20}{6} = \frac{y}{y - x}$

$20y - 20x = 6y$

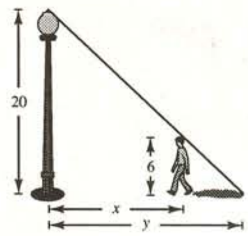
$14y = 20x$

$y = \frac{10}{7}x$

$\frac{dx}{dt} = -5$

$\frac{dy}{dt} = \frac{10}{7}\frac{dx}{dt} = \frac{10}{7}(-5) = \frac{-50}{7}$ ft/sec

(b) $\frac{d(y - x)}{dt} = \frac{dy}{dt} - \frac{dx}{dt} = \frac{-50}{7} - (-5) = \frac{-50}{7} + \frac{35}{7} = \frac{-15}{7}$ ft/sec



39.

$$pv^{1.3} = k$$

$$1.3\,pv^{0.3}\frac{dv}{dt} + v^{1.3}\frac{dp}{dt} = 0$$

$$v^{0.3}\left(1.3p\frac{dv}{dt} + v\frac{dp}{dt}\right) = 0$$

$$1.3p\frac{dv}{dt} = -v\frac{dp}{dt}$$

40.

$$rg\tan\theta = v^2$$

$32r\tan\theta = v^2$, r is a constant.

$$32r\sec^2\theta\frac{d\theta}{dt} = 2v\frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{16r}{v}\sec^2\theta\frac{d\theta}{dt}$$

Likewise, $\dfrac{d\theta}{dt} = \dfrac{v}{16r}\cos^2\theta\dfrac{dv}{dt}$.

41.

$$\tan\theta = \frac{y}{30}$$

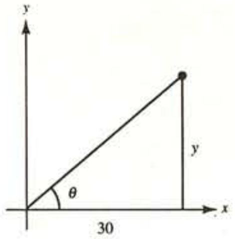
$$\frac{dy}{dt} = 3 \text{ m/sec.}$$

$$\sec^2\theta\cdot\frac{d\theta}{dt} = \frac{1}{30}\frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{30}\cos^2\theta\cdot\frac{dy}{dt}$$

When $y = 30$, $\theta = \pi/4$ and $\cos\theta = \sqrt{2}/2$. Thus,

$$\frac{d\theta}{dt} = \frac{1}{30}\left(\frac{1}{2}\right)(3) = \frac{1}{20}\text{ rad/sec.}$$



42.

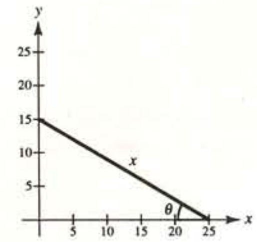
$$\sin\theta = \frac{15}{x}$$

$$\frac{dx}{dt} = -1 \text{ ft/sec}$$

$$\cos\theta\left(\frac{d\theta}{dt}\right) = \frac{-15}{x^2}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{-15}{x^2}(\sec\theta)\frac{dx}{dt}$$

$$= \frac{-15}{625}\left(\frac{25}{20}\right)(-1) = \frac{3}{100}\text{ rad/sec}$$



43.

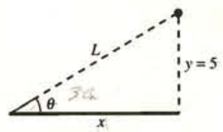
$$\tan\theta = \frac{y}{x}, y = 5$$

$$\frac{dx}{dt} = -600 \text{ mi/hr}$$

$$(\sec^2\theta)\frac{d\theta}{dt} = -\frac{5}{x^2}\cdot\frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \cos^2\theta\left(-\frac{5}{x^2}\right)\frac{dx}{dt} = \frac{x^2}{L^2}\left(-\frac{5}{x^2}\right)\frac{dx}{dt}$$

$$= \left(-\frac{5^2}{L^2}\right)\left(\frac{1}{5}\right)\frac{dx}{dt} = (-\sin^2\theta)\left(\frac{1}{5}\right)(-600) = 120\sin^2\theta$$



(a) When $\theta = 30°$, $\dfrac{d\theta}{dt} = \dfrac{120}{4} = 30$ rad/hr $= \dfrac{1}{2}$ rad/min.

(b) When $\theta = 60°$, $\dfrac{d\theta}{dt} = 120\left(\dfrac{3}{4}\right) = 90$ rad/hr $= \dfrac{3}{2}$ rad/min.

(c) When $\theta = 75°$, $\dfrac{d\theta}{dt} = 120\sin^2 75° \approx 111.96$ rad/hr ≈ 1.87 rad/min.