30. $\int 3\,dt = 3t + C$

Check: $\frac{d}{dt}(3t + C) = 3$

31. $\int (2\sin x + 3\cos x)\,dx = -2\cos x + 3\sin x + C$

Check: $\frac{d}{dx}(-2\cos x + 3\sin x + C) = 2\sin x + 3\cos x$

32. $\int (t^2 - \sin t)\,dt = \frac{1}{3}t^3 + \cos t + C$

Check: $\frac{d}{dt}\left(\frac{1}{3}t^3 + \cos t + C\right) = t^2 - \sin t$

33. $\int (1 - \csc t \cot t)\,dt = t + \csc t + C$

Check: $\frac{d}{dt}(t + \csc t + C) = 1 - \csc t \cot t$

34. $\int (\theta^2 + \sec^2\theta)\,d\theta = \frac{1}{3}\theta^3 + \tan\theta + C$

Check: $\frac{d}{d\theta}\left(\frac{1}{3}\theta^3 + \tan\theta + C\right) = \theta^2 + \sec^2\theta$

35. $\int (\sec^2\theta - \sin\theta)\,d\theta = \tan\theta + \cos\theta + C$

Check: $\frac{d}{d\theta}(\tan\theta + \cos\theta + C) = \sec^2\theta - \sin\theta$

36. $\int \sec y(\tan y - \sec y)\,dy = \int (\sec y \tan y - \sec^2 y)\,dy$

$= \sec y - \tan y + C$

Check: $\frac{d}{dy}(\sec y - \tan y + C) = \sec y \tan y - \sec^2 y$

$= \sec y(\tan y - \sec y)$

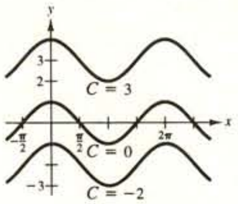
37. $\int (\tan^2 y + 1)\,dy = \int \sec^2 y\,dy = \tan y + C$

Check: $\frac{d}{dy}(\tan y + C) = \sec^2 y = \tan^2 y + 1$

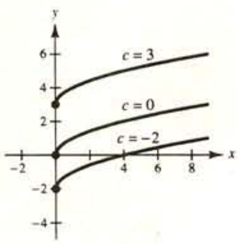
38. $\int \frac{\sin x}{1 - \sin^2 x}\,dx = \int \frac{\sin x}{\cos^2 x}\,dx = \int \left(\frac{1}{\cos}\right)\left(\frac{\sin x}{\cos x}\right)dx = \int \sec x \tan x\,dx = \sec x + C$

Check: $\frac{d}{dx}(\sec x + C) = \sec x \tan x = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{1 - \sin^2 x}$

39. $f(x) = \cos x$

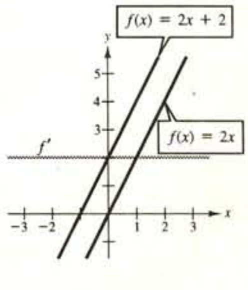


40. $f(x) = \sqrt{x}$



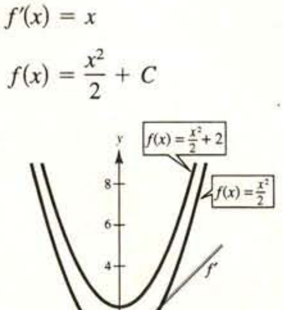
41. $f'(x) = 2$

$f(x) = 2x + C$



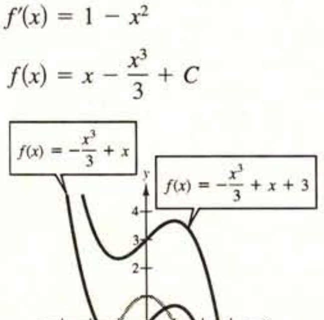
42. $f'(x) = x$

$f(x) = \frac{x^2}{2} + C$



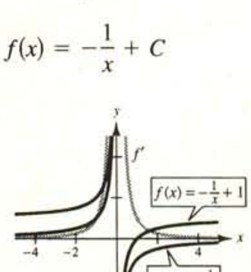
43. $f'(x) = 1 - x^2$

$f(x) = x - \frac{x^3}{3} + C$



44. $f'(x) = \frac{1}{x^2}$

$f(x) = -\frac{1}{x} + C$



45. $\frac{dy}{dx} = 2x - 1,\ (1, 1)$

$$y = \int (2x - 1)\, dx = x^2 - x + C$$

$$1 = (1)^2 - (1) + C \Rightarrow C = 1$$

$$y = x^2 - x + 1$$

46. $\frac{dy}{dx} = 2(x - 1) = 2x - 2,\ (3, 2)$

$$y = \int 2(x - 1)\, dx = x^2 - 2x + C$$

$$2 = (3)^2 - 2(3) + C \Rightarrow C = -1$$

$$y = x^2 - 2x - 1$$

47. $\frac{dy}{dx} = \cos x,\ (0, 4)$

$$y = \int \cos x\, dx = \sin x + C$$

$$4 = \sin 0 + C \Rightarrow C = 4$$

$$y = \sin x + 4$$

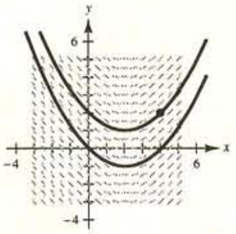
48. $\frac{dy}{dx} = -\frac{1}{x^2} = -x^{-2},\ (1,3)$

$$y = \int -x^{-2}\, dx = \frac{1}{x} + C$$

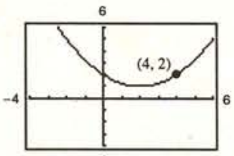
$$3 = \frac{1}{1} + C \Rightarrow C = 2$$

$$y = \frac{1}{x} + 2,\ x > 0$$

49. (a)



(b) $\frac{dy}{dx} = \frac{1}{2}x - 1,\ (4, 2)$



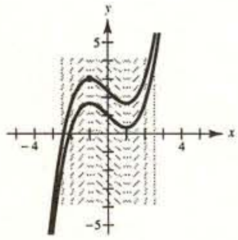
$$y = \frac{x^2}{4} - x + C$$

$$2 = \frac{4^2}{4} - 4 + C$$

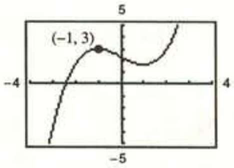
$$2 = C$$

$$y = \frac{x^2}{4} - x + 2$$

50. (a)



(b) $\frac{dy}{dx} = x^2 - 1,\ (-1, 3)$



$$y = \frac{x^3}{3} - x + C$$

$$3 = \frac{(-1)^3}{3} - (-1) + C$$

$$3 = -\frac{1}{3} + 1 + C$$

$$C = \frac{7}{3}$$

$$y = \frac{x^3}{3} - x + \frac{7}{3}$$

51. $f''(x) = 2$

$f'(2) = 5$

$f(2) = 10$

$$f'(x) = \int 2\, dx = 2x + C_1$$

$$f'(2) = 4 + C_1 = 5 \Rightarrow C_1 = 1$$

$$f'(x) = 2x + 1$$

$$f(x) = \int (2x + 1)\, dx = x^2 + x + C_2$$

$$f(2) = 6 + C_2 = 10 \Rightarrow C_2 = 4$$

$$f(x) = x^2 + x + 4$$

52. $f''(x) = x^2$

$f'(0) = 6$

$f(0) = 3$

$$f'(x) = \int x^2\, dx = \frac{1}{3}x^3 + C_1$$

$$f'(0) = 0 + C_1 = 6 \Rightarrow C_1 = 6$$

$$f'(x) = \frac{1}{3}x^3 + 6$$

$$f(x) = \int \left(\frac{1}{3}x^3 + 6\right) dx = \frac{1}{12}x^4 + 6x + C_2$$

$$f(0) = 0 + 0 + C_2 = 3 \Rightarrow C_2 = 3$$

$$f(x) = \frac{1}{12}x^4 + 6x + 3$$

53. $f''(x) = x^{-3/2}$

$f'(4) = 2$

$f(0) = 0$

$$f'(x) = \int x^{-3/2}\,dx = -2x^{-1/2} + C_1 = -\frac{2}{\sqrt{x}} + C_1$$

$$f'(4) = -\frac{2}{2} + C_1 = 2 \Rightarrow C_1 = 3$$

$$f'(x) = -\frac{2}{\sqrt{x}} + 3$$

$$f(x) = \int (-2x^{-1/2} + 3)\,dx = -4x^{1/2} + 3x + C_2$$

$$f(0) = 0 + 0 + C_2 = 0 \Rightarrow C_2 = 0$$

$$f(x) = -4x^{1/2} + 3x = -4\sqrt{x} + 3x$$

54. $f''(x) = \sin x$

$f'(0) = 1$

$f(0) = 6$

$$f'(x) = \int \sin x\,dx = -\cos x + C_1$$

$$f'(0) = -1 + C_1 = 1 \Rightarrow C_1 = 2$$

$$f'(x) = -\cos x + 2$$

$$f(x) = \int (-\cos x + 2)\,dx = -\sin x + 2x + C_2$$

$$f(0) = 0 + 0 + C_2 = 6 \Rightarrow C_2 = 6$$

$$f(x) = -\sin x + 2x + 6$$

55. (a) $h(t) = \int (1.5t + 5)\,dt = 0.75t^2 + 5t + C$

$$h(0) = 0 + 0 + C = 12 \Rightarrow C = 12$$

$$h(t) = 0.75t^2 + 5t + 12$$

(b) $h(6) = 0.75(6)^2 + 5(6) + 12 = 69$ cm

56. $\dfrac{dP}{dt} = k\sqrt{t},\ 0 \le t \le 10$

$$P(t) = \int kt^{1/2}\,dt = \frac{2}{3}kt^{3/2} + C$$

$$P(0) = 0 + C = 500 \Rightarrow C = 500$$

$$P(1) = \frac{2}{3}k + 500 = 600 \Rightarrow k = 150$$

$$P(t) = \frac{2}{3}(150)t^{3/2} + 500 = 100t^{3/2} + 500$$

$$P(7) = 100(7)^{3/2} + 500 \approx 2352 \text{ bacteria}$$

57. $a(t) = -32$ ft/sec^2

$$v(t) = \int -32\,dt = -32t + C_1$$

$$v(0) = 60 = C_1$$

$$v(t) = -32t + 60$$

$$s(t) = \int (-32t + 60)\,dt = -16t^2 + 60t + C_2$$

$$s(0) = 0 = C_2$$

$$s(t) = -16t^2 + 60t$$

The ball reaches its maximum height when

$$v(t) = 0:\ -32t + 60 = 0$$

$$32t = 60$$

$$t = \frac{15}{8}.$$

$$s\left(\frac{15}{8}\right) = -16\left(\frac{15}{8}\right)^2 + 60\left(\frac{15}{8}\right) = 56.26 \text{ feet}$$

58. $f''(t) = a(t) = -32$ ft/sec^2

$f'(0) = v_0$

$f(0) = s_0$

$$f'(t) = v(t) = \int -32\,dt = -32t + C_1$$

$$f'(0) = 0 + C_1 = v_0 \Rightarrow C_1 = v_0$$

$$f'(t) = -32t + v_0$$

$$f(t) = s(t) = \int (-32t + v_0)\,dt = -16t^2 + v_0t + C_2$$

$$f(0) = 0 + 0 + C_2 = s_0 \Rightarrow C_2 = s_0$$

$$f(t) = -16t^2 + v_0t + s_0$$

65. $a = -1.6$

$v(t) = \int -1.6\,dt = -1.6t + v_0 = -1.6t$, since the stone was dropped, $v_0 = 0$.

$s(t) = \int (-1.6t)\,dt = -0.8t^2 + s_0$

$s(20) = 0 \Rightarrow -0.8(20)^2 + s_0 = 0$

$s_0 = 320$

Thus, the height of the cliff is 320 meters.

$v(t) = -1.6t$

$v(20) = -32$ m/sec

66. $\int v\,dv = -GM \int \frac{1}{y^2}\,dy$

$\frac{1}{2}v^2 = \frac{GM}{y} + C$

When $y = R$, $v = v_0$.

$\frac{1}{2}v_0^2 = \frac{GM}{R} + C$

$C = \frac{1}{2}v_0^2 - \frac{GM}{R}$

$\frac{1}{2}v^2 = \frac{GM}{y} + \frac{1}{2}v_0^2 - \frac{GM}{R}$

$v^2 = \frac{2GM}{y} + v_0^2 - \frac{2GM}{R}$

$v^2 = v_0^2 + 2GM\left(\frac{1}{y} - \frac{1}{R}\right)$

67. $x(t) = t^3 - 6t^2 + 9t - 2 \qquad 0 \le t \le 5$

(a) $v(t) = x'(t) = 3t^2 - 12t + 9$

$= 3(t^2 - 4t + 3) = 3(t - 1)(t - 3)$

$a(t) = v'(t) = 6t - 12 = 6(t - 2)$

(b) $v(t) > 0$ when $0 < t < 1$ or $3 < t < 5$.

(c) $a(t) = 6(t - 2) = 0$ when $t = 2$.

$v(2) = 3(1)(-1) = -3$

68. $x(t) = (t - 1)(t - 3)^2 \qquad 0 \le t \le 5$

$= t^3 - 7t^2 + 15t - 9$

(a) $v(t) = x'(t) = 3t^2 - 14t + 15 = (3t - 5)(t - 3)$

$a(t) = v'(t) = 6t - 14$

(b) $v(t) > 0$ when $0 < t < \frac{5}{3}$ and $3 < t < 5$.

(c) $a(t) = 6t - 14 = 0$ when $t = \frac{7}{3}$.

$v\left(\frac{7}{3}\right) = \left(3\left(\frac{7}{3}\right) - 5\right)\left(\frac{7}{3} - 3\right) = 2\left(-\frac{2}{3}\right) = -\frac{4}{3}$

69. $v(t) = \frac{1}{\sqrt{t}} = t^{-1/2} \qquad t > 0$

$x(t) = \int v(t)\,dt = 2t^{1/2} + C$

$x(1) = 4 = 2(1) + C \Rightarrow C = 2$

$x(t) = 2t^{1/2} + 2$

$a(t) = v'(t) = -\frac{1}{2}t^{-3/2} = \frac{-1}{2t^{3/2}}$

70. (a) $a(t) = \cos t$

$$v(t) = \int a(t)\, dt = \int \cos t\, dt = \sin t + C_1 = \sin t \text{ (since } v_0 = 0)$$

$$f(t) = \int v(t)\, dt = \int \sin t\, dt = -\cos t + C_2$$

$$f(0) = 3 = -\cos(0) + C_2 = -1 + C_2 \Rightarrow C_2 = 4$$

$$f(t) = -\cos t + 4$$

(b) $v(t) = 0 = \sin t$ for $t = k\pi, k = 0, 1, 2, \ldots$

71. (a) $v(0) = 25 \text{ km/hr} = 25 \cdot \frac{1000}{3600} = \frac{250}{36} \text{ m/sec}$

$$v(13) = 80 \text{ km/hr} = 80 \cdot \frac{1000}{3600} = \frac{800}{36} \text{ m/sec}$$

$a(t) = a$ (constant acceleration)

$$v(t) = at + C$$

$$v(0) = \frac{250}{36} \Rightarrow v(t) = at + \frac{250}{36}$$

$$v(13) = \frac{800}{36} = 13a + \frac{250}{36}$$

$$\frac{550}{36} = 13a$$

$$a = \frac{550}{468} = \frac{275}{234} \approx 1.175 \text{ m/sec}^2$$

(b) $s(t) = a\frac{t^2}{2} + \frac{250}{36}t \quad (s(0) = 0)$

$$s(13) = \frac{275}{234}\frac{(13)^2}{2} + \frac{250}{36}(13) \approx 189.58 \text{ m}$$

72. $v(0) = 45 \text{ mph} = 66 \text{ ft/sec}$

$30 \text{ mph} = 44 \text{ ft/sec}$

$15 \text{ mph} = 22 \text{ ft/sec}$

$$a(t) = -a$$

$$v(t) = -at + 66$$

$s(t) = -\frac{a}{2}t^2 + 66t$ (Let $s(0) = 0$.)

$v(t) = 0$ after car moves 132 ft.

$-at + 66 = 0$ when $t = \frac{66}{a}$.

$$s\left(\frac{66}{a}\right) = -\frac{a}{2}\left(\frac{66}{a}\right)^2 + 66\left(\frac{66}{a}\right).$$

$$= 132 \text{ when } a = \frac{33}{2} = 16.5.$$

$$a(t) = -16.5$$

$$v(t) = -16.5t + 66$$

$$s(t) = -8.25t^2 + 66t$$

(a) $-16.5t + 66 = 44$

$$t = \frac{22}{16.5} \approx 1.333$$

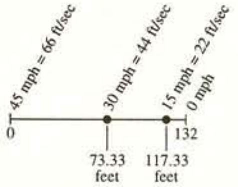
$$s\left(\frac{22}{16.5}\right) \approx 73.33 \text{ ft}$$

(b) $-16.5t + 66 = 22$

$$t = \frac{44}{16.5} \approx 2.667$$

$$s\left(\frac{44}{16.5}\right) \approx 117.33 \text{ ft}$$

(c)



It takes 1.333 seconds to reduce the speed from 45 mph to 30 mph, 1.333 seconds to reduce the speed from 30 mph to 15 mph, and 1.333 seconds to reduce the speed from 15 mph to 0 mph. Each time, less distance is needed to reach the next speed reduction.