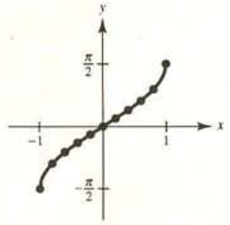
Section 5.8 Inverse Trigonometric Functions and Differentiation

1. $y = \arcsin x$

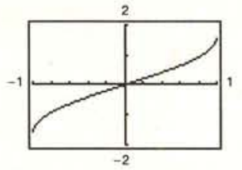
(a)

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
y	-1.571	-0.927	-0.644	-0.412	-0.201	0	0.201	0.412	0.644	0.927	1.571

(b)



(c)



(d) Symmetric about origin:
$\arcsin(-x) = -\arcsin x$

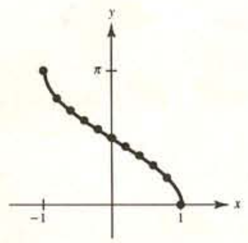
Intercept: $(0, 0)$

2. $y = \arccos x$

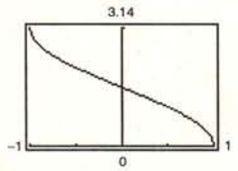
(a)

x	-1	-0.8	-0.6	-0.4	-0.2	0	0.2	0.4	0.6	0.8	1
y	3.142	2.499	2.214	1.982	1.772	1.571	1.369	1.159	0.927	0.634	0

(b)



(c)



(d) Intercepts: $\left(0, \frac{\pi}{2}\right)$ and $(1, 0)$

No symmetry

3. False.

$$\arccos \frac{1}{2} = \frac{\pi}{3}$$

since the range is $[0, \pi]$.

4. $\left(\_\_, \frac{\pi}{4}\right) = \left(1, \frac{\pi}{4}\right)$

$\left(\_\_, -\frac{\pi}{6}\right) = \left(-\frac{\sqrt{3}}{3}, -\frac{\pi}{6}\right)$

$\left(-\sqrt{3}, \_\_\right) = \left(-\sqrt{3}, -\frac{\pi}{3}\right)$

5. $\arcsin \frac{1}{2} = \frac{\pi}{6}$

6. $\arcsin 0 = 0$

7. $\arccos \frac{1}{2} = \frac{\pi}{3}$

8. $\arccos 0 = \frac{\pi}{2}$

9. $\arctan \frac{\sqrt{3}}{3} = \frac{\pi}{6}$

10. $\operatorname{arccot}(-1) = \frac{3\pi}{4}$

11. $\operatorname{arccsc}(-\sqrt{2}) = -\frac{\pi}{4}$

12. $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$

13. $\arccos(-0.8) \approx 2.50$

14. $\arcsin(-0.39) \approx -0.40$

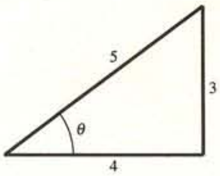
15. $\operatorname{arcsec}(1.269) = \arccos\left(\frac{1}{1.269}\right)$
≈ 0.66

16. $\arctan(-3) \approx -1.25$

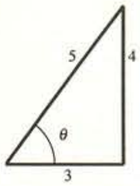
17. $\arctan 0 = 0$; π is not in the range of $y = \arctan x$.

18. $y = \arcsin x$ represents the inverse of the restricted sine function where $-1 \le x \le 1$ and $-\pi/2 \le y \le \pi/2$.

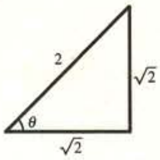
19. (a) $\sin\left(\arctan \frac{3}{4}\right) = \frac{3}{5}$



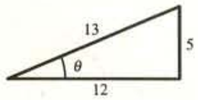
(b) $\sec\left(\arcsin \frac{4}{5}\right) = \frac{5}{3}$



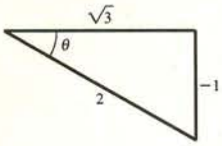
20. (a) $\tan\left(\arccos \dfrac{\sqrt{2}}{2}\right) = \tan\left(\dfrac{\pi}{4}\right) = 1$



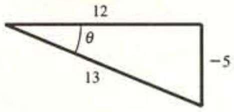
(b) $\cos\left(\arcsin \dfrac{5}{13}\right) = \dfrac{12}{13}$



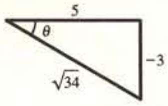
21. (a) $\cot\left[\arcsin\left(-\dfrac{1}{2}\right)\right] = \cot\left(-\dfrac{\pi}{6}\right) = -\sqrt{3}$



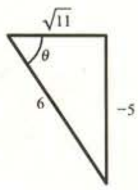
(b) $\csc\left[\arctan\left(-\dfrac{5}{12}\right)\right] = -\dfrac{13}{5}$



22. (a) $\sec\left[\arctan\left(-\dfrac{3}{5}\right)\right] = \dfrac{\sqrt{34}}{5}$



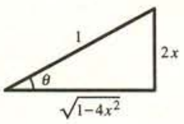
(b) $\tan\left[\arcsin\left(-\dfrac{5}{6}\right)\right] = -\dfrac{5\sqrt{11}}{11}$



23. $y = \cos(\arcsin 2x)$

$\theta = \arcsin 2x$

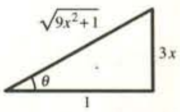
$y = \cos\theta = \sqrt{1 - 4x^2}$



24. $y = \sec(\arctan 3x)$

$\theta = \arctan 3x$

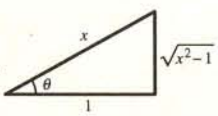
$y = \sec\theta = \sqrt{9x^2 + 1}$



25. $y = \sin(\operatorname{arcsec} x)$

$\theta = \operatorname{arcsec} x,\ 0 \le \theta \le \pi,\ \theta \ne \dfrac{\pi}{2}$

$y = \sin\theta = \dfrac{\sqrt{x^2 - 1}}{|x|}$

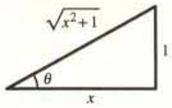


The absolute value bars on x are necessary because of the restriction $0 \le \theta \le \pi$, $\theta \ne \pi/2$, and $\sin\theta$ for this domain must always be nonnegative.

26. $y = \cos(\operatorname{arccot} x)$

$\theta = \operatorname{arccot} x$

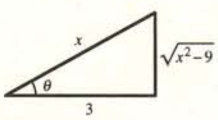
$y = \cos\theta = \dfrac{x}{\sqrt{x^2 + 1}}$



27. $y = \tan\left(\operatorname{arcsec} \dfrac{x}{3}\right)$

$\theta = \operatorname{arcsec} \dfrac{x}{3}$

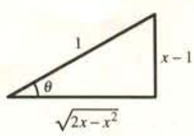
$y = \tan\theta = \dfrac{\sqrt{x^2 - 9}}{3}$



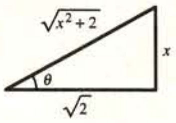
28. $y = \sec[\arcsin(x - 1)]$

$\theta = \arcsin(x - 1)$

$y = \sec\theta = \dfrac{1}{\sqrt{2x - x^2}}$



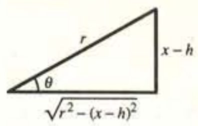
29. $y = \csc\left(\arctan \dfrac{x}{\sqrt{2}}\right)$



$\theta = \arctan \dfrac{x}{\sqrt{2}}$

$y = \csc \theta = \dfrac{\sqrt{x^2 + 2}}{x}$

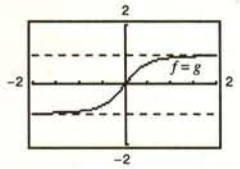
30. $y = \cos\left(\arcsin \dfrac{x - h}{r}\right)$



$\theta = \arcsin \dfrac{x - h}{r}$

$y = \cos \theta = \dfrac{\sqrt{r^2 - (x - h)^2}}{r}$

31. $\sin(\arctan 2x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

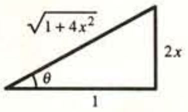


Asymptotes: $y = \pm 1$

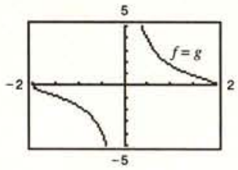
$\arctan 2x = \theta$

$\tan \theta = 2x$

$\sin \theta = \dfrac{2x}{\sqrt{1 + 4x^2}}$



32.

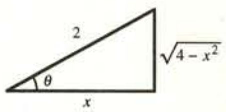


Asymptote: $x = 0$

$\arccos \dfrac{x}{2} = \theta$

$\cos \theta = \dfrac{x}{2}$

$\tan \theta = \dfrac{\sqrt{4 - x^2}}{x}$



33. (a) $\operatorname{arccsc} x = \arcsin \dfrac{1}{x}, |x| \geq 1$

Let $y = \operatorname{arccsc} x$. Then for

$$-\frac{\pi}{2} \leq y < 0 \text{ and } 0 < y \leq \frac{\pi}{2},$$

$\csc y = x \implies \sin y = 1/x$. Thus, $y = \arcsin(1/x)$. Therefore, $\operatorname{arccsc} x = \arcsin(1/x)$.

(b) $\arctan x + \arctan \dfrac{1}{x} = \dfrac{\pi}{2}, x > 0$

Let $y = \arctan x + \arctan(1/x)$. Then,

$$\tan y = \frac{\tan(\arctan x) + \tan[\arctan(1/x)]}{1 - \tan(\arctan x)\tan[\arctan(1/x)]}$$

$$= \frac{x + (1/x)}{1 - x(1/x)}$$

$$= \frac{x + (1/x)}{0} \text{ (which is undefined).}$$

Thus, $y = \pi/2$. Therefore, $\arctan x + \arctan(1/x) = \pi/2$.

34. (a) $\arcsin(-x) = -\arcsin x, |x| \leq 1.$

Let $y = \arcsin(-x)$. Then,

$$-x = \sin y \implies x = -\sin y \implies x = \sin(-y).$$

Thus, $-y = \arcsin x \implies y = -\arcsin x$. Therefore, $\arcsin(-x) = -\arcsin x$.

(b) $\arccos(-x) = \pi - \arccos x, |x| \leq 1.$

Let $y = \arccos(-x)$. Then,

$$-x = \cos y \implies x = -\cos y \implies x = \cos(\pi - y).$$

Thus, $\pi - y = \arccos x \implies y = \pi - \arccos x$. Therefore, $\arccos(-x) = \pi - \arccos x$.

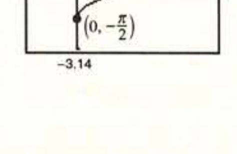
35. $f(x) = \arcsin(x - 1)$

$x - 1 = \sin y$

$x = 1 + \sin y$

Domain: $[0, 2]$

Range: $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$



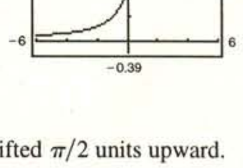
$f(x)$ is the graph of $\arcsin x$ shifted 1 unit to the right.

36. $f(x) = \arctan x + \dfrac{\pi}{2}$

$x = \tan\left(y - \dfrac{\pi}{2}\right)$

Domain: $(-\infty, \infty)$

Range: $(0, \pi)$



$f(x)$ is the graph of $\arctan x$ shifted $\pi/2$ units upward.

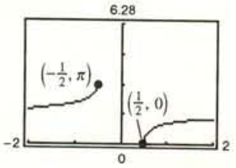
37. $f(x) = \operatorname{arcsec} 2x$

$2x = \sec y$

$x = \dfrac{1}{2}\sec y$

Domain: $\left(-\infty, -\dfrac{1}{2}\right], \left[\dfrac{1}{2}, \infty\right)$

Range: $\left[0, \dfrac{\pi}{2}\right), \left(\dfrac{\pi}{2}, \pi\right]$



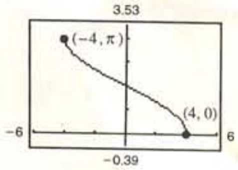
38. $f(x) = \arccos\left(\dfrac{x}{4}\right)$

$\dfrac{x}{4} = \cos y$

$x = 4\cos y$

Domain: $[-4, 4]$

Range: $[0, \pi]$



39. $\arcsin(3x - \pi) = \frac{1}{2}$

$3x - \pi = \sin\left(\frac{1}{2}\right)$

$x = \frac{1}{3}\left[\sin\left(\frac{1}{2}\right) + \pi\right] \approx 1.207$

40. $\arctan(2x) = -1$

$2x = \tan(-1)$

$x = \frac{1}{2}\tan(-1) \approx -0.779$

41. $\arcsin\sqrt{2x} = \arccos\sqrt{x}$

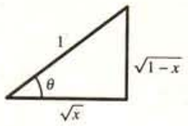
$\sqrt{2x} = \sin\left(\arccos\sqrt{x}\right)$

$\sqrt{2x} = \sqrt{1-x}, 0 \le x \le 1$

$2x = 1 - x$

$3x = 1$

$x = \dfrac{1}{3}$



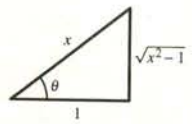
42. $\arccos x = \operatorname{arcsec} x$

$x = \cos(\operatorname{arcsec} x)$

$x = \dfrac{1}{x}$

$x^2 = 1$

$x = \pm 1$



43. $f(x) = 2\arcsin(x - 1)$

$f'(x) = \dfrac{2}{\sqrt{1-(x-1)^2}} = \dfrac{2}{\sqrt{2x - x^2}}$

44. $f(t) = \arcsin t^2$

$f'(t) = \dfrac{2t}{\sqrt{1-t^4}}$

45. $g(x) = 3\arccos\dfrac{x}{2}$

$g'(x) = \dfrac{-3(1/2)}{\sqrt{1-(x^2/4)}} = \dfrac{-3}{\sqrt{4-x^2}}$

46. $f(x) = \operatorname{arcsec} 2x$

$f'(x) = \dfrac{2}{|2x|\sqrt{4x^2-1}} = \dfrac{1}{|x|\sqrt{4x^2-1}}$

47. $f(x) = \arctan\dfrac{x}{a}$

$f'(x) = \dfrac{1/a}{1+(x^2/a^2)} = \dfrac{a}{a^2+x^2}$

48. $f(x) = \arctan\sqrt{x}$

$f'(x) = \left(\dfrac{1}{1+x}\right)\left(\dfrac{1}{2\sqrt{x}}\right) = \dfrac{1}{2\sqrt{x}(1+x)}$

49. $g(x) = \dfrac{\arcsin 3x}{x}$

$g'(x) = \dfrac{x\left(3/\sqrt{1-9x^2}\right) - \arcsin 3x}{x^2}$

$= \dfrac{3x - \sqrt{1-9x^2}\arcsin 3x}{x^2\sqrt{1-9x^2}}$

50. $h(x) = x\arctan x$

$h'(x) = \dfrac{x}{1+x^2} + \arctan x$

51. $h(t) = \sin(\arccos t) = \sqrt{1-t^2}$

$h'(t) = \dfrac{1}{2}(1-t^2)^{-1/2}(-2t) = \dfrac{-t}{\sqrt{1-t^2}}$

52. $f(x) = \arcsin x + \arccos x = \dfrac{\pi}{2}$

$f'(x) = 0$