CHAPTER 6
Applications of Integration

Section 6.1 Area of a Region Between Two Curves

1. $A = \int_0^6 [0 - (x^2 - 6x)]\, dx = -\int_0^6 (x^2 - 6x)\, dx$

2. $A = \int_{-2}^2 [(2x + 5) - (x^2 + 2x + 1)]\, dx = \int_{-2}^2 (-x^2 + 4)\, dx$

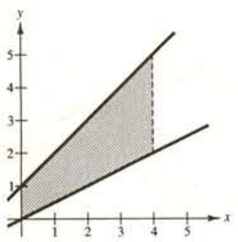
3. $A = \int_0^3 [(-x^2 + 2x + 3) - (x^2 - 4x + 3)]\, dx = \int_0^3 (-2x^2 + 6x)\, dx$

4. $A = \int_0^1 (x^2 - x^3)\, dx$

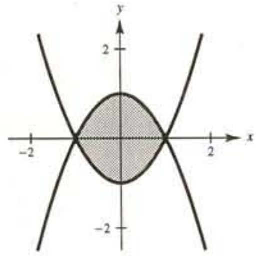
5. $A = 2\int_{-1}^0 3(x^3 - x)\, dx = 6\int_{-1}^0 (x^3 - x)\, dx$ or $-6\int_0^1 (x^3 - x)\, dx$

6. $A = 2\int_0^1 [(x - 1)^3 - (x - 1)]\, dx$

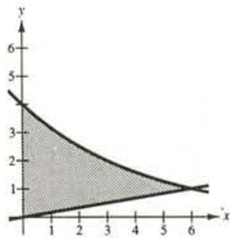
7. $\int_0^4 \left[(x + 1) - \frac{x}{2}\right] dx$



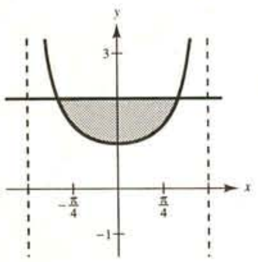
8. $\int_{-1}^1 [(1 - x^2) - (x^2 - 1)]\, dx$



9. $\int_0^6 \left[4(2^{-x/3}) - \frac{x}{6}\right] dx$



10. $\int_{-\pi/3}^{\pi/3} [2 - \sec x]\, dx$

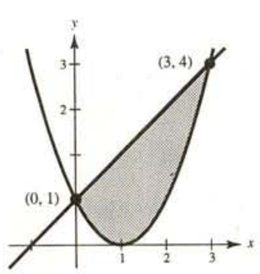


11. $f(x) = x + 1$

$g(x) = (x - 1)^2$

$A \approx 4$

Matches (d)

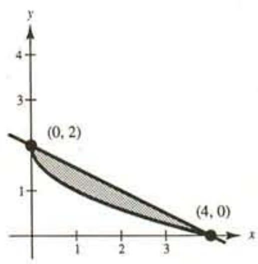


12. $f(x) = 2 - \frac{1}{2}x$

$g(x) = 2 - \sqrt{x}$

$A \approx 1$

Matches (a)



13. The points of intersection are given by:

$$x^2 - 4x = 0$$

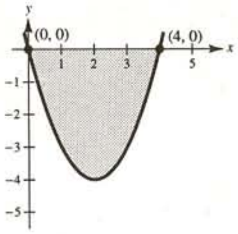
$$x(x - 4) = 0 \quad \text{when} \quad x = 0, 4$$

$$A = \int_0^4 [g(x) - f(x)]\, dx$$

$$= -\int_0^4 (x^2 - 4x)\, dx$$

$$= -\left[\frac{x^3}{3} - 2x^2\right]_0^4$$

$$= \frac{32}{3}$$



14. The points of intersection are given by:

$$3 - 2x - x^2 = 0$$

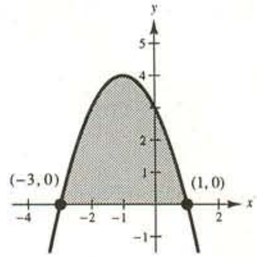
$$(3 + x)(1 - x) = 0 \quad \text{when} \quad x = -3, 1$$

$$A = \int_{-3}^1 [f(x) - g(x)]\, dx$$

$$= \int_{-3}^1 (3 - 2x - x^2)\, dx$$

$$= \left[3x - x^2 - \frac{x^3}{3}\right]_{-3}^1$$

$$= \frac{32}{3}$$



15. The points of intersection are given by:

$$x^2 + 2x + 1 = 3x + 3$$

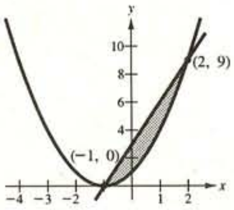
$$(x - 2)(x + 1) = 0 \quad \text{when} \quad x = -1, 2$$

$$A = \int_{-1}^2 [g(x) - f(x)]\, dx$$

$$= \int_{-1}^2 [(3x + 3) - (x^2 + 2x + 1)]\, dx$$

$$= \int_{-1}^2 (2 + x - x^2)\, dx$$

$$= \left[2x + \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^2 = \frac{9}{2}$$



16. The points of intersection are given by:

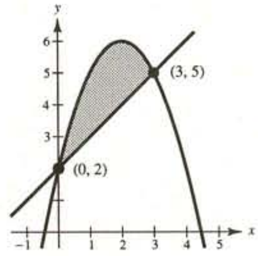
$$-x^2 + 4x + 2 = x + 2$$

$$x(3 - x) = 0 \quad \text{when} \quad x = 0, 3$$

$$A = \int_0^3 [f(x) - g(x)]\, dx$$

$$= \int_0^3 [(-x^2 + 4x + 2) - (x + 2)]\, dx$$

$$= \int_0^3 (-x^2 + 3x)\, dx = \left[\frac{-x^3}{3} + \frac{3}{2}x^2\right]_0^3 = \frac{9}{2}$$



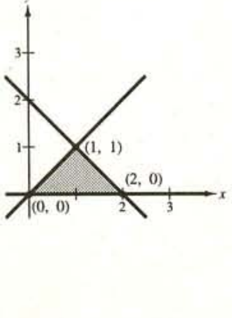
17. The points of intersection are given by:

$$x = 2 - x \quad \text{and} \quad x = 0 \quad \text{and} \quad 2 - x = 0$$

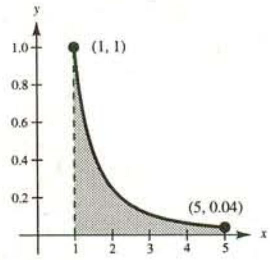
$$x = 1 \qquad\qquad x = 0 \qquad\qquad x = 2$$

$$A = \int_0^1 [(2 - y) - (y)]\, dy = \left[2y - y^2\right]_0^1 = 1$$

Note that if we integrate with respect to x, we need two integrals. Also, note that the region is a triangle.



18. $A = \int_1^5 \left(\frac{1}{x^2} - 0\right) dx = \left[-\frac{1}{x}\right]_1^5 = \frac{4}{5}$



19. The points of intersection are given by:

$$\sqrt{3x} + 1 = x + 1$$

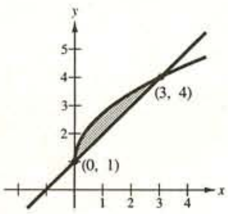
$$\sqrt{3x} = x \quad \text{when} \quad x = 0, 3$$

$$A = \int_0^3 [f(x) - g(x)]\,dx$$

$$= \int_0^3 \left[\left(\sqrt{3x} + 1\right) - (x + 1)\right] dx$$

$$= \int_0^3 [(3x)^{1/2} - x]\,dx$$

$$= \left[\frac{2}{9}(3x)^{3/2} - \frac{x^2}{2}\right]_0^3 = \frac{3}{2}$$



20. The points of intersection are given by:

$$\sqrt[3]{x} = x$$

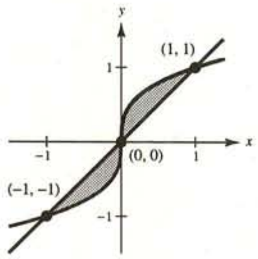
$$x = -1, 0, 1$$

$$A = 2\int_0^1 [f(x) - g(x)]\,dx$$

$$= 2\int_0^1 \left(\sqrt[3]{x} - x\right) dx$$

$$= 2\int_0^1 (x^{1/3} - x)\,dx$$

$$= 2\left[\frac{3}{4}x^{4/3} - \frac{1}{2}x^2\right]_0^1 = \frac{1}{2}$$



21. The points of intersection are given by:

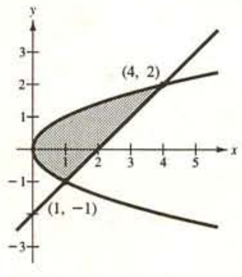
$$y^2 = y + 2$$

$$(y - 2)(y + 1) = 0 \quad \text{when} \quad y = -1, 2$$

$$A = \int_{-1}^2 [g(y) - f(y)]\,dy$$

$$= \int_{-1}^2 [(y + 2) - y^2]\,dy$$

$$= \left[2y + \frac{y^2}{2} - \frac{y^3}{3}\right]_{-1}^2 = \frac{9}{2}$$



22. The points of intersection are given by:

$$2y - y^2 = -y$$

$$y(y - 3) = 0 \quad \text{when} \quad y = 0, 3$$

$$A = \int_0^3 [f(y) - g(y)]\,dy$$

$$= \int_0^3 [(2y - y^2) - (-y)]\,dy$$

$$= \int_0^3 (3y - y^2)\,dy = \left[\frac{3}{2}y^2 - \frac{1}{3}y^3\right]_0^3 = \frac{9}{2}$$

23. $$A = \int_{-1}^2 [f(y) - g(y)]\,dy$$

$$= \int_{-1}^2 [(y^2 + 1) - 0]\,dy$$

$$= \left[\frac{y^3}{3} + y\right]_{-1}^2 = 6$$

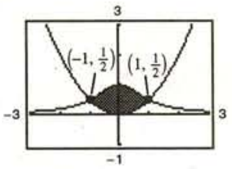
33. The points of intersection are given by:

$$\frac{1}{1+x^2} = \frac{x^2}{2}$$

$$x^4 + x^2 - 2 = 0$$

$$(x^2 + 2)(x^2 - 1) = 0$$

$$x = \pm 1$$



$$A = 2\int_0^1 [f(x) - g(x)]\,dx$$

$$= 2\int_0^1 \left[\frac{1}{1+x^2} - \frac{x^2}{2}\right] dx$$

$$= 2\left[\arctan x - \frac{x^3}{6}\right]_0^1$$

$$= 2\left(\frac{\pi}{4} - \frac{1}{6}\right) = \frac{\pi}{2} - \frac{1}{3} \approx 1.237$$

Numerical Approximation: 1.237

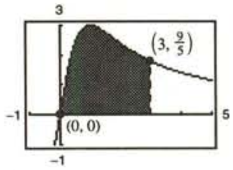
34. $A = \displaystyle\int_0^3 \left[\frac{6x}{x^2+1} - 0\right] dx$

$$= \left[3\ln(x^2+1)\right]_0^3$$

$$= 3\ln 10$$

$$\approx 6.908$$

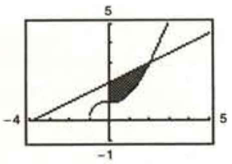
Numerical Approximation: 6.908



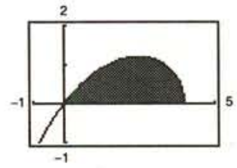
35. $\sqrt{1+x^3} \le \frac{1}{2}x + 2$ on $[0, 2]$

Numerical approximation: 1.759

$$A = \int_0^2 \left[\frac{1}{2}x + 2 - \sqrt{1+x^3}\right] dx \approx 1.759$$



36. $A = \displaystyle\int_0^4 x\sqrt{\frac{4-x}{4+x}}\,dx \approx 3.434$

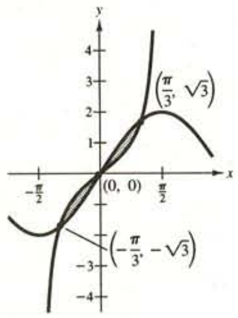


37. $A = 2\displaystyle\int_0^{\pi/3} [f(x) - g(x)]\,dx$

$$= 2\int_0^{\pi/3} (2\sin x - \tan x)\,dx$$

$$= 2\Big[-2\cos x + \ln|\cos x|\Big]_0^{\pi/3}$$

$$= 2(1 - \ln 2) \approx 0.614$$

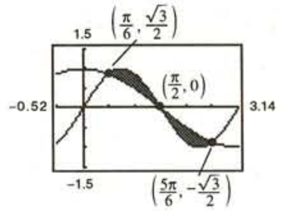


38. $A = 2\displaystyle\int_{\pi/6}^{\pi/2} [f(x) - g(x)]\,dx$

$$= 2\int_{\pi/6}^{\pi/2} [\sin 2x - \cos x]\,dx$$

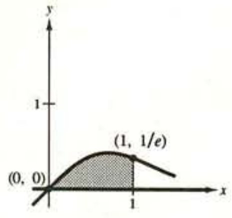
$$= 2\left[-\frac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2}$$

$$= \frac{1}{2}$$



39. $A = \int_0^1 [xe^{-x^2} - 0]\,dx$

$= \left[-\frac{1}{2}e^{-x^2}\right]_0^1 = \frac{1}{2}\left(1 - \frac{1}{e}\right) \approx 0.316$



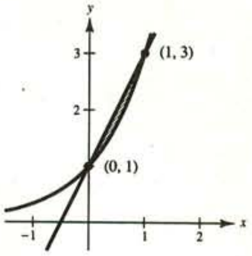
40. From the graph we see that f and g intersect twice at $x = 0$ and $x = 1$.

$A = \int_0^1 [g(x) - f(x)]\,dx$

$= \int_0^1 [(2x + 1) - 3^x]\,dx$

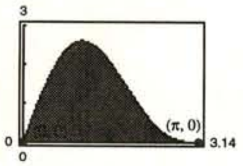
$= \left[x^2 + x - \frac{1}{\ln 3}(3^x)\right]_0^1$

$= 2\left(1 - \frac{1}{\ln 3}\right) \approx 0.180$



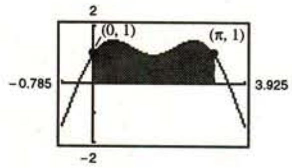
41. $A = \int_0^{\pi} [(2\sin x + \sin 2x) - 0]\,dx$

$= \left[-2\cos x - \frac{1}{2}\cos 2x\right]_0^{\pi} = 4.0$



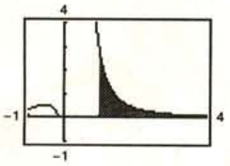
42. $A = \int_0^{\pi} [(2\sin x + \cos 2x) - 0]\,dx$

$= \left[-2\cos x + \frac{1}{2}\sin 2x\right]_0^{\pi} = 4$



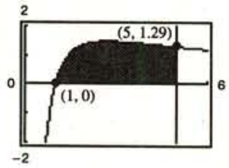
43. $A = \int_1^3 \left[\frac{1}{x^2}e^{1/x} - 0\right] dx$

$= \left[-e^{1/x}\right]_1^3 = e - e^{1/3} \approx 1.323$



44. $A = \int_1^5 \left[\frac{4\ln x}{x} - 0\right] dx$

$= \left[2(\ln x)^2\right]_1^5 = 2(\ln 5)^2 \approx 5.181$

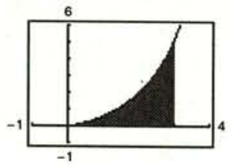


45. (a) $y = \sqrt{\frac{x^3}{4 - x}},\quad y = 0,\quad x = 3$

(b) $A = \int_0^3 \sqrt{\frac{x^3}{4 - x}}\,dx,$

No, it cannot be evaluated by hand.

(c) 4.7721



46. (a) $y = \sqrt{x}\,e^x,\ y = 0,\ x = 0,\ x = 1$

(b) $A = \int_0^1 \sqrt{x}\,e^x\,dx.$

No, it cannot be evaluated by hand.

(c) 1.2556