

3.  $p(x) = x$

$h(x) = \sqrt{x}$

$$V = 2\pi \int_0^4 x\sqrt{x} dx$$

$$= 2\pi \int_0^4 x^{3/2} dx$$

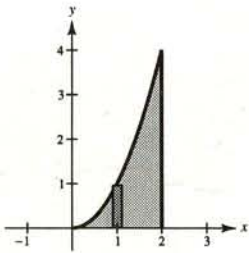
$$= \left[ \frac{4\pi}{5} x^{5/2} \right]_0^4 = \frac{128\pi}{5}$$

5.  $p(x) = x$

$h(x) = x^2$

$$V = 2\pi \int_0^2 x^3 dx$$

$$= \left[ \frac{\pi}{2} x^4 \right]_0^2 = 8\pi$$



4.  $p(x) = x$

$h(x) = 8 - (x^2 + 4) = 4 - x^2$

$$V = 2\pi \int_0^2 x(4 - x^2) dx$$

$$= 2\pi \int_0^2 (4x - x^3) dx$$

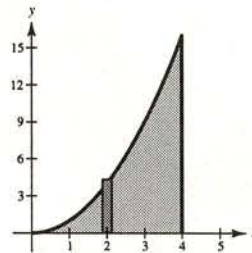
$$= 2\pi \left[ 2x^2 - \frac{x^4}{4} \right]_0^2 = 8\pi$$

6.  $p(x) = x$

$h(x) = x^2$

$$V = 2\pi \int_0^4 x^3 dx$$

$$= \left[ \frac{\pi}{2} x^4 \right]_0^4 = 128\pi$$



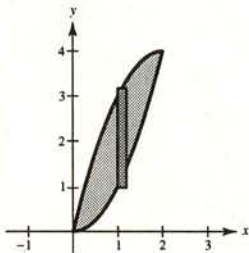
7.  $p(x) = x$

$h(x) = (4x - x^2) - x^2 = 4x - 2x^2$

$$V = 2\pi \int_0^2 x(4x - 2x^2) dx$$

$$= 4\pi \int_0^2 (2x^2 - x^3) dx$$

$$= 4\pi \left[ \frac{2}{3} x^3 - \frac{1}{4} x^4 \right]_0^2 = \frac{16\pi}{3}$$

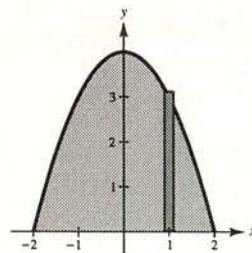


8.  $p(x) = x$

$h(x) = 4 - x^2$

$$V = 2\pi \int_0^2 (4x - x^3) dx$$

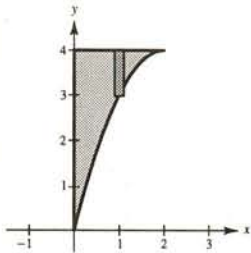
$$= 2\pi \left[ 2x^2 - \frac{1}{4} x^4 \right]_0^2 = 8\pi$$



9.  $p(x) = x$

$$h(x) = 4 - (4x - x^2) = x^2 - 4x + 4$$

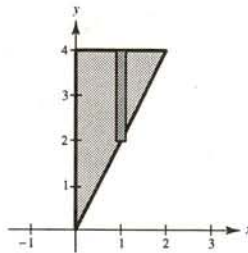
$$\begin{aligned}
 V &= 2\pi \int_0^2 (x^3 - 4x^2 + 4x) dx \\
 &= 2\pi \left[ \frac{x^4}{4} - \frac{4}{3}x^3 + 2x^2 \right]_0^2 = \frac{8\pi}{3}
 \end{aligned}$$



10.  $p(x) = x$

$$h(x) = 4 - 2x$$

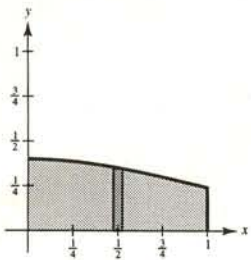
$$\begin{aligned}
 V &= 2\pi \int_0^2 x(4 - 2x) dx \\
 &= 2\pi \int_0^2 (4x - 2x^2) dx \\
 &= 2\pi \left[ 2x^2 - \frac{2}{3}x^3 \right]_0^2 = \frac{16\pi}{3}
 \end{aligned}$$



11.  $p(x) = x$

$$h(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

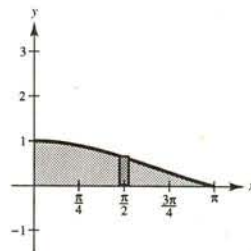
$$\begin{aligned}
 V &= 2\pi \int_0^1 x \left( \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \right) dx \\
 &= \sqrt{2\pi} \int_0^1 e^{-x^2/2} x dx \\
 &= \left[ -\sqrt{2\pi} e^{-x^2/2} \right]_0^1 = \sqrt{2\pi} \left( 1 - \frac{1}{\sqrt{e}} \right) \approx 0.986
 \end{aligned}$$



12.  $p(x) = x$

$$h(x) = \frac{\sin x}{x}$$

$$\begin{aligned}
 V &= 2\pi \int_0^\pi x \left[ \frac{\sin x}{x} \right] dx \\
 &= 2\pi \int_0^\pi \sin x dx = \left[ -2\pi \cos x \right]_0^\pi = 4\pi
 \end{aligned}$$



13.  $p(y) = y$

$$h(y) = 2 - y$$

$$\begin{aligned}
 V &= 2\pi \int_0^2 y(2 - y) dy \\
 &= 2\pi \int_0^2 (2y - y^2) dy \\
 &= 2\pi \left[ y^2 - \frac{y^3}{3} \right]_0^2 = \frac{8\pi}{3}
 \end{aligned}$$

14.  $p(y) = -y$  ( $p(y) \geq 0$  on  $[-2, 0]$ )

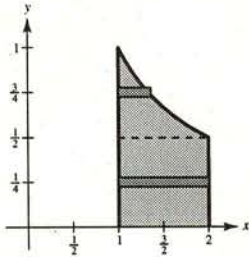
$$h(y) = 4 - (2 - y) = 2 + y$$

$$\begin{aligned}
 V &= 2\pi \int_{-2}^0 (-y)(2 + y) dy \\
 &= 2\pi \int_{-2}^0 (-2y - y^2) dy \\
 &= 2\pi \left[ -y^2 - \frac{y^3}{3} \right]_{-2}^0 = \frac{8\pi}{3}
 \end{aligned}$$

15.  $p(y) = y$  and  $h(y) = 1$  if  $0 \leq y < \frac{1}{2}$ .

$$p(y) = y \text{ and } h(y) = \frac{1}{y} - 1 \text{ if } \frac{1}{2} \leq y \leq 1.$$

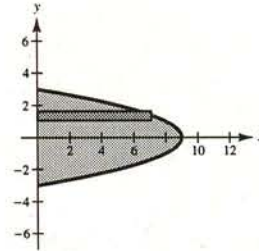
$$\begin{aligned} V &= 2\pi \int_0^{1/2} y \, dy + 2\pi \int_{1/2}^1 (1 - y) \, dy \\ &= 2\pi \left[ \frac{y^2}{2} \right]_0^{1/2} + 2\pi \left[ y - \frac{y^2}{2} \right]_{1/2}^1 = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} \end{aligned}$$



16.  $p(y) = y$

$$h(y) = 9 - y^2$$

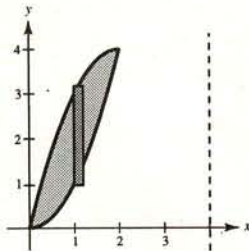
$$\begin{aligned} V &= 2\pi \int_0^3 y(9 - y^2) \, dy \\ &= 2\pi \int_0^3 (9y - y^3) \, dy \\ &= 2\pi \left[ \frac{9}{2}y^2 - \frac{1}{4}y^4 \right]_0^3 = \frac{81\pi}{2} \end{aligned}$$



17.  $p(x) = 4 - x$

$$h(x) = 4x - x^2 - x^2 = 4x - 2x^2$$

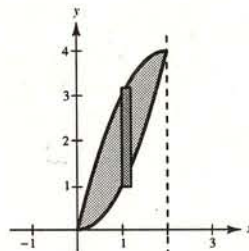
$$\begin{aligned} V &= 2\pi \int_0^2 (4 - x)(4x - 2x^2) \, dx \\ &= 2\pi(2) \int_0^2 (x^3 - 6x^2 + 8x) \, dx \\ &= 4\pi \left[ \frac{x^4}{4} - 2x^3 + 4x^2 \right]_0^2 = 16\pi \end{aligned}$$



18.  $p(x) = 2 - x$

$$h(x) = 4x - x^2 - x^2 = 4x - 2x^2$$

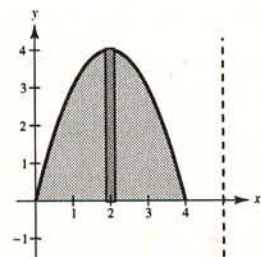
$$\begin{aligned} V &= 2\pi \int_0^2 (2 - x)(4x - 2x^2) \, dx \\ &= 2\pi \int_0^2 (8x - 8x^2 + 2x^3) \, dx \\ &= 2\pi \left[ 4x^2 - \frac{8}{3}x^3 + \frac{1}{2}x^4 \right]_0^2 = \frac{16\pi}{3} \end{aligned}$$



19.  $p(x) = 5 - x$

$$h(x) = 4x - x^2$$

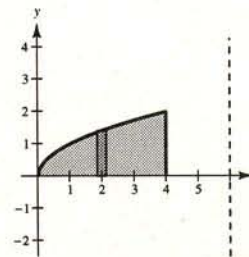
$$\begin{aligned} V &= 2\pi \int_0^4 (5 - x)(4x - x^2) \, dx \\ &= 2\pi \int_0^4 (x^3 - 9x^2 + 20x) \, dx \\ &= 2\pi \left[ \frac{x^4}{4} - 3x^3 + 10x^2 \right]_0^4 = 64\pi \end{aligned}$$



20.  $p(x) = 6 - x$

$$h(x) = \sqrt{x}$$

$$\begin{aligned} V &= 2\pi \int_0^4 (6 - x)\sqrt{x} \, dx \\ &= 2\pi \int_0^4 (6x^{1/2} - x^{3/2}) \, dx \\ &= 2\pi \left[ 4x^{3/2} - \frac{2}{5}x^{5/2} \right]_0^4 = \frac{192\pi}{5} \end{aligned}$$

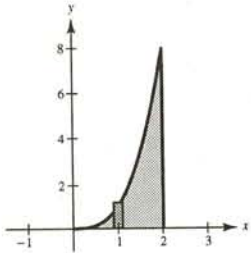


## 21. (a) Disc

$$R(x) = x^3$$

$$r(x) = 0$$

$$V = \pi \int_0^2 x^6 dx = \pi \left[ \frac{x^7}{7} \right]_0^2 = \frac{128\pi}{7}$$

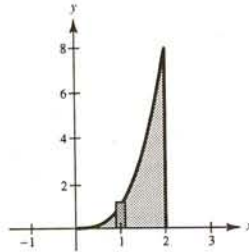


## (b) Shell

$$p(x) = x$$

$$h(x) = x^3$$

$$V = 2\pi \int_0^2 x^4 dx = 2\pi \left[ \frac{x^5}{5} \right]_0^2 = \frac{64\pi}{5}$$



## (c) Shell

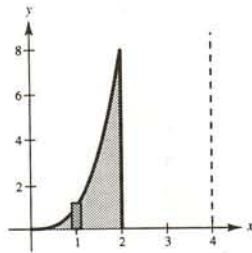
$$p(x) = 4 - x$$

$$h(x) = x^3$$

$$V = 2\pi \int_0^2 (4 - x)x^3 dx$$

$$= 2\pi \int_0^2 (4x^3 - x^4) dx$$

$$= 2\pi \left[ x^4 - \frac{1}{5}x^5 \right]_0^2 = \frac{96\pi}{5}$$



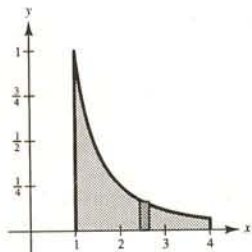
## 22. (a) Disc

$$R(x) = \frac{1}{x^2}$$

$$r(x) = 0$$

$$V = \pi \int_1^4 \left( \frac{1}{x^2} \right)^2 dx$$

$$= \pi \left[ -\frac{1}{3x^3} \right]_1^4 = \frac{21\pi}{64}$$



## (b) Shell

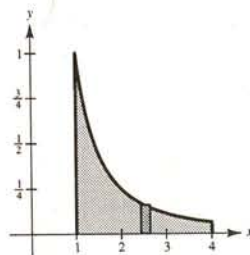
$$p(x) = x$$

$$h(x) = \frac{1}{x^2}$$

$$V = 2\pi \int_1^4 x \left( \frac{1}{x^2} \right) dx$$

$$= 2\pi \int_1^4 \frac{1}{x} dx$$

$$= 2\pi \left[ \ln|x| \right]_1^4 = 2\pi \ln 4$$



—CONTINUED—