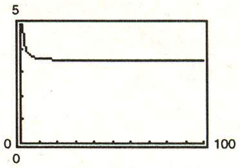
4. $\lim_{x\to\infty} \dfrac{6x}{\sqrt{3x^2 - 2x}} \approx 3.4641 \left(\text{exact: } \dfrac{6}{\sqrt{3}}\right)$

x	1	10	10^2	10^3	10^4	10^5
$f(x)$	6	3.5857	3.4757	3.4653	3.4642	3.4641



5. (a) $\lim_{x\to 3} \dfrac{2(x-3)}{x^2-9} = \lim_{x\to 3} \dfrac{2(x-3)}{(x+3)(x-3)} = \lim_{x\to 3} \dfrac{2}{x+3} = \dfrac{1}{3}$

(b) $\lim_{x\to 3} \dfrac{2(x-3)}{x^2-9} = \lim_{x\to 3} \dfrac{(d/dx)[2(x-3)]}{(d/dx)[x^2-9]} = \lim_{x\to 3} \dfrac{2}{2x} = \dfrac{2}{6} = \dfrac{1}{3}$

6. (a) $\lim_{x\to -1} \dfrac{2x^2 - x - 3}{x+1} = \lim_{x\to -1} \dfrac{(2x-3)(x+1)}{x+1} = \lim_{x\to -1} (2x-3) = -5$

(b) $\lim_{x\to -1} \dfrac{2x^2 - x - 3}{x+1} = \lim_{x\to -1} \dfrac{(d/dx)[2x^2 - x - 3]}{(d/dx)[x+1]} = \lim_{x\to -1} \dfrac{4x-1}{1} = -5$

7. (a) $\lim_{x\to 3} \dfrac{\sqrt{x+1} - 2}{x-3} = \lim_{x\to 3} \dfrac{\sqrt{x+1} - 2}{x-3} \cdot \dfrac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2} = \lim_{x\to 3} \dfrac{(x+1) - 4}{(x-3)\left[\sqrt{x+1} + 2\right]} = \lim_{x\to 3} \dfrac{1}{\sqrt{x+1} + 2} = \dfrac{1}{4}$

(b) $\lim_{x\to 3} \dfrac{\sqrt{x+1} - 2}{x-3} = \lim_{x\to 3} \dfrac{(d/dx)\left[\sqrt{x+1} - 2\right]}{(d/dx)[x-3]} = \lim_{x\to 3} \dfrac{1/\left(2\sqrt{x+1}\right)}{1} = \dfrac{1}{4}$

8. (a) $\lim_{x\to 0} \dfrac{\sin 4x}{2x} = \lim_{x\to 0} 2\left(\dfrac{\sin 4x}{4x}\right) = 2(1) = 2$

(b) $\lim_{x\to 0} \dfrac{\sin 4x}{2x} = \lim_{x\to 0} \dfrac{(d/dx)[\sin 4x]}{(d/dx)[2x]} = \lim_{x\to 0} \dfrac{4\cos 4x}{2} = 2$

9. (a) $\lim_{x\to\infty} \dfrac{5x^2 - 3x + 1}{3x^2 - 5} = \lim_{x\to\infty} \dfrac{5 - (3/x) + (1/x^2)}{3 - (5/x^2)} = \dfrac{5}{3}$

(b) $\lim_{x\to\infty} \dfrac{5x^2 - 3x + 1}{3x^2 - 5} = \lim_{x\to\infty} \dfrac{(d/dx)[5x^2 - 3x + 1]}{(d/dx)[3x^2 - 5]} = \lim_{x\to\infty} \dfrac{10x - 3}{6x} = \lim_{x\to\infty} \dfrac{(d/dx)[10x - 3]}{(d/dx)[6x]} = \lim_{x\to\infty} \dfrac{10}{6} = \dfrac{5}{3}$

10. (a) $\lim_{x\to\infty} \dfrac{2x+1}{4x^2 + x} = \lim_{x\to\infty} \dfrac{(2/x) + (1/x^2)}{4 + (1/x)} = \dfrac{0}{4} = 0$

(b) $\lim_{x\to\infty} \dfrac{2x+1}{4x^2 + x} = \lim_{x\to\infty} \dfrac{(d/dx)[2x+1]}{(d/dx)[4x^2 + x]} = \lim_{x\to\infty} \dfrac{2}{8x+1} = 0$

11. $\lim_{x\to 2} \dfrac{x^2 - x - 2}{x-2} = \lim_{x\to 2} \dfrac{2x-1}{1} = 3$

12. $\lim_{x\to -1} \dfrac{x^2 - x - 2}{x+1} = \lim_{x\to -1} \dfrac{2x-1}{1} = -3$

13. $\lim_{x\to 0} \dfrac{\sqrt{4-x^2} - 2}{x} = \lim_{x\to 0} \dfrac{-x/\sqrt{4-x^2}}{1} = 0$

14. $\lim_{x\to 2^-} \dfrac{\sqrt{4-x^2}}{x-2} = \lim_{x\to 2^-} \dfrac{-x/\sqrt{4-x^2}}{1}$

$= \lim_{x\to 2^-} \dfrac{-x}{\sqrt{4-x^2}} = -\infty$

15. $\lim_{x\to 0} \dfrac{e^x - (1-x)}{x} = \lim_{x\to 0} \dfrac{e^x + 1}{1} = 2$

16. $\lim_{x\to 0^+} \dfrac{e^x - (1+x)}{x^3} = \lim_{x\to 0^+} \dfrac{e^x - 1}{3x^2}$

$= \lim_{x\to 0^+} \dfrac{e^x}{6x} = \infty$

17. Case 1: $n = 1$

$$\lim_{x\to 0^+} \frac{e^x - (1+x)}{x} = \lim_{x\to 0^+} \frac{e^x - 1}{1} = 0$$

Case 2: $n = 2$

$$\lim_{x\to 0^+} \frac{e^x - (1+x)}{x^2} = \lim_{x\to 0^+} \frac{e^x - 1}{2x} = \lim_{x\to 0^+} \frac{e^x}{2} = \frac{1}{2}$$

Case 3: $n \geq 3$

$$\lim_{x\to 0^+} \frac{e^x - (1+x)}{x^n} = \lim_{x\to 0^+} \frac{e^x - 1}{nx^{n-1}} = \lim_{x\to 0^+} \frac{e^x}{n(n-1)x^{n-2}} = \infty$$

18. $$\lim_{x\to 1} \frac{\ln x}{x^2 - 1} = \lim_{x\to 1} \frac{(1/x)}{2x} = \lim_{x\to 1} \frac{1}{2x^2} = \frac{1}{2}$$

19. $$\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{2\cos 2x}{3\cos 3x} = \frac{2}{3}$$

20. $$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{a\cos ax}{b\cos bx} = \frac{a}{b}$$

21. $$\lim_{x\to 0} \frac{\arcsin x}{x} = \lim_{x\to 0} \frac{1/\sqrt{1-x^2}}{1} = 1$$

22. $$\lim_{x\to 1} \frac{\arctan x - (\pi/4)}{x - 1} = \lim_{x\to 1} \frac{1/(1+x^2)}{1} = \frac{1}{2}$$

23. $$\lim_{x\to\infty} \frac{3x^2 - 2x + 1}{2x^2 + 3} = \lim_{x\to\infty} \frac{6x - 2}{4x} = \lim_{x\to\infty} \frac{6}{4} = \frac{3}{2}$$

24. $$\lim_{x\to\infty} \frac{x - 1}{x^2 + 2x + 3} = \lim_{x\to\infty} \frac{1}{2x + 2} = 0$$

25. $$\lim_{x\to\infty} \frac{x^2 + 2x + 3}{x - 1} = \lim_{x\to\infty} \frac{2x + 2}{1} = \infty$$

26. $$\lim_{x\to\infty} \frac{x^2}{e^x} = \lim_{x\to\infty} \frac{2x}{e^x} = \lim_{x\to\infty} \frac{2}{e^x} = 0$$

27. $$\lim_{x\to\infty} \frac{x}{\sqrt{x^2 + 1}} = \lim_{x\to\infty} \frac{1}{\sqrt{1 + (1/x^2)}} = 1$$

Note: L'Hôpital's Rule does not work on this limit. See Exercise 67.

28. $$\lim_{x\to\infty} \frac{\sin x}{x - \pi} = 0$$

Note: Use the Squeeze Theorem for $x > \pi$.

$$-\frac{1}{x - \pi} \leq \frac{\sin x}{x - \pi} \leq \frac{1}{x - \pi}$$

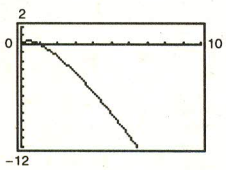
29. $$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = 0$$

30. $$\lim_{x\to\infty} \frac{e^x}{x} = \lim_{x\to\infty} \frac{e^x}{1} = \infty$$

31. (a) $\lim_{x\to 0^+} (-x\ln x) = (-0)(-\infty) = (0)(\infty)$

(b) $$\lim_{x\to 0^+} (-x\ln x) = \lim_{x\to 0^+} \frac{\ln x}{-1/x} = \lim_{x\to 0^+} \frac{1/x}{1/x^2} = \lim_{x\to 0^+} x = 0$$

(c)



32. (a) $\lim_{x\to 0^+} x^2 \cot x = (0)(\infty)$

(b) $$\lim_{x\to 0^+} x^2 \cot x = \lim_{x\to 0^+} \frac{x^2}{\tan x} = \lim_{x\to 0^+} \frac{2x}{\sec^2 x} = 0$$

(c)