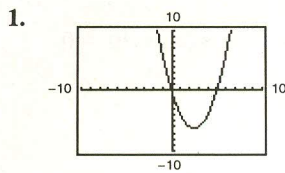


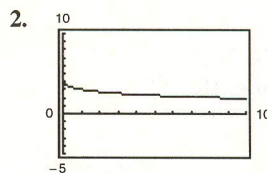
## Section 1.3 Evaluating Limits Analytically



$$h(x) = x^2 - 5x$$

(a)  $\lim_{x \rightarrow 5} h(x) = 0$

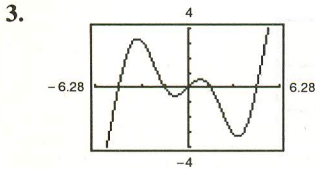
(b)  $\lim_{x \rightarrow -1} h(x) = 6$



$$g(x) = \frac{12(\sqrt{x} - 3)}{x - 9}$$

(a)  $\lim_{x \rightarrow 4} g(x) = 2.4$

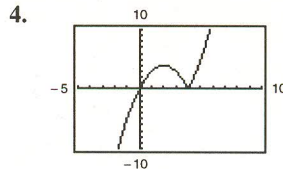
(b)  $\lim_{x \rightarrow 0} g(x) = 4$



$$f(x) = x \cos x$$

(a)  $\lim_{x \rightarrow 0} f(x) = 0$

(b)  $\lim_{x \rightarrow \pi/3} f(x) \approx 0.524$   
 $\left( = \frac{\pi}{6} \right)$



$$f(t) = t|t - 4|$$

(a)  $\lim_{t \rightarrow 4} f(t) = 0$

(b)  $\lim_{t \rightarrow -1} f(t) = -5$

5.  $\lim_{x \rightarrow 4} x^2 = 4^2 = 16$

6.  $\lim_{x \rightarrow -3} (3x + 2) = 3(-3) + 2 = 7$

7.  $\lim_{x \rightarrow 0} (2x - 1) = 2(0) - 1 = -1$

8.  $\lim_{x \rightarrow 1} (-x^2 + 1) = -(1)^2 + 1 = 0$

9.  $\lim_{x \rightarrow 2} (-x^2 + x - 2) = -(2)^2 + (2) - 2 = -4$

10.  $\lim_{x \rightarrow 1} (3x^3 - 2x^2 + 4) = 3(1)^3 - 2(1)^2 + 4 = 5$

11.  $\lim_{x \rightarrow 3} \sqrt{x+1} = \sqrt{3+1} = 2$

12.  $\lim_{x \rightarrow 4} \sqrt[3]{x+4} = \sqrt[3]{4+4} = 2$

13.  $\lim_{x \rightarrow -4} (x+3)^2 = (-4+3)^2 = 1$

14.  $\lim_{x \rightarrow 0} (2x - 1)^3 = [2(0) - 1]^3 = -1$

15.  $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$

16.  $\lim_{x \rightarrow -3} \frac{2}{x+2} = \frac{2}{-3+2} = -2$

17.  $\lim_{x \rightarrow -1} \frac{x^2 + 1}{x} = \frac{(-1)^2 + 1}{-1} = -2$

18.  $\lim_{x \rightarrow 3} \frac{\sqrt{x+1}}{x-4} = \frac{\sqrt{3+1}}{3-4} = -2$

19.  $\lim_{x \rightarrow \pi/2} \sin x = \sin \frac{\pi}{2} = 1$

20.  $\lim_{x \rightarrow \pi} \tan x = \tan \pi = 0$

21.  $\lim_{x \rightarrow 1} \cos \pi x = \cos \pi = -1$

22.  $\lim_{x \rightarrow 1} \sin \frac{\pi x}{2} = \sin \frac{\pi}{2} = 1$

23.  $\lim_{x \rightarrow 0} \sec 2x = \sec 0 = 1$

24.  $\lim_{x \rightarrow \pi} \cos 3x = \cos 3\pi = -1$

25.  $\lim_{x \rightarrow 5\pi/6} \sin x = \sin \frac{5\pi}{6} = \frac{1}{2}$

26.  $\lim_{x \rightarrow 5\pi/3} \cos x = \cos \frac{5\pi}{3} = \frac{1}{2}$

27.  $\lim_{x \rightarrow 3} \tan \left( \frac{\pi x}{4} \right) = \tan \frac{3\pi}{4} = -1$

28.  $\lim_{x \rightarrow 7} \sec \left( \frac{\pi x}{6} \right) = \sec \frac{7\pi}{6} = \frac{-2\sqrt{3}}{3}$

29. (a)  $\lim_{x \rightarrow c} [5g(x)] = 5 \lim_{x \rightarrow c} g(x) = 5(3) = 15$

(b)  $\lim_{x \rightarrow c} [f(x) + g(x)] = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x) = 2 + 3 = 5$

(c)  $\lim_{x \rightarrow c} [f(x)g(x)] = [\lim_{x \rightarrow c} f(x)][\lim_{x \rightarrow c} g(x)] = (2)(3) = 6$

(d)  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} = \frac{2}{3}$

31. (a)  $\lim_{x \rightarrow c} [f(x)]^3 = [\lim_{x \rightarrow c} f(x)]^3 = (4)^3 = 64$

(b)  $\lim_{x \rightarrow c} \sqrt{f(x)} = \sqrt{\lim_{x \rightarrow c} f(x)} = \sqrt{4} = 2$

(c)  $\lim_{x \rightarrow c} [3f(x)] = 3 \lim_{x \rightarrow c} f(x) = 3(4) = 12$

(d)  $\lim_{x \rightarrow c} [f(x)]^{3/2} = [\lim_{x \rightarrow c} f(x)]^{3/2} = (4)^{3/2} = 8$

33.  $f(x) = -2x + 1$  and  $g(x) = \frac{-2x^2 + x}{x}$  agree except at  $x = 0$ .

(a)  $\lim_{x \rightarrow 0} g(x) = \lim_{x \rightarrow 0} f(x) = 1$

(b)  $\lim_{x \rightarrow -1} g(x) = \lim_{x \rightarrow -1} f(x) = 3$

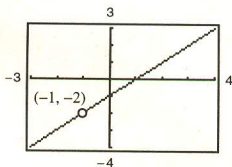
35.  $f(x) = x(x + 1)$  and  $g(x) = \frac{x^3 - x}{x - 1}$  agree except at  $x = 1$ .

(a)  $\lim_{x \rightarrow 1} g(x) = \lim_{x \rightarrow 1} f(x) = 2$

(b)  $\lim_{x \rightarrow -1} g(x) = \lim_{x \rightarrow -1} f(x) = 0$

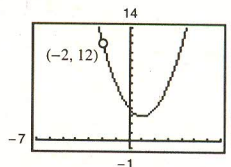
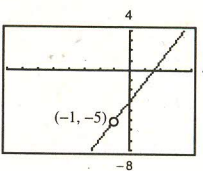
37.  $f(x) = \frac{x^2 - 1}{x + 1}$  and  $g(x) = x - 1$  agree except at  $x = -1$ .

$$\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} g(x) = -2$$



39.  $f(x) = \frac{x^3 + 8}{x + 2}$  and  $g(x) = x^2 - 2x + 4$  agree except at  $x = -2$ .

$$\lim_{x \rightarrow -2} f(x) = \lim_{x \rightarrow -2} g(x) = 12$$



30. (a)  $\lim_{x \rightarrow c} [4f(x)] = 4 \lim_{x \rightarrow c} f(x) = 4\left(\frac{3}{2}\right) = 6$

(b)  $\lim_{x \rightarrow c} [f(x) + g(x)] = \lim_{x \rightarrow c} f(x) + \lim_{x \rightarrow c} g(x) = \frac{3}{2} + \frac{1}{2} = 2$

(c)  $\lim_{x \rightarrow c} [f(x)g(x)] = [\lim_{x \rightarrow c} f(x)][\lim_{x \rightarrow c} g(x)] = \left(\frac{3}{2}\right)\left(\frac{1}{2}\right) = \frac{3}{4}$

(d)  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} g(x)} = \frac{3/2}{1/2} = 3$

32. (a)  $\lim_{x \rightarrow c} \sqrt[3]{f(x)} = \sqrt[3]{\lim_{x \rightarrow c} f(x)} = \sqrt[3]{27} = 3$

(b)  $\lim_{x \rightarrow c} \frac{f(x)}{18} = \frac{\lim_{x \rightarrow c} f(x)}{\lim_{x \rightarrow c} 18} = \frac{27}{18} = \frac{3}{2}$

(c)  $\lim_{x \rightarrow c} [f(x)]^2 = [\lim_{x \rightarrow c} f(x)]^2 = (27)^2 = 729$

(d)  $\lim_{x \rightarrow c} [f(x)]^{2/3} = [\lim_{x \rightarrow c} f(x)]^{2/3} = (27)^{2/3} = 9$

34.  $f(x) = x - 3$  and  $h(x) = \frac{x^2 - 3x}{x}$  agree except at  $x = 0$ .

(a)  $\lim_{x \rightarrow -2} h(x) = \lim_{x \rightarrow -2} f(x) = -5$

(b)  $\lim_{x \rightarrow 0} h(x) = \lim_{x \rightarrow 0} f(x) = -3$

36.  $g(x) = \frac{1}{x - 1}$  and  $f(x) = \frac{x}{x^2 - x}$  agree except at  $x = 0$ .

(a)  $\lim_{x \rightarrow 1} f(x)$  does not exist.

(b)  $\lim_{x \rightarrow 0} f(x) = -1$

38.  $f(x) = \frac{2x^2 - x - 3}{x + 1}$  and  $g(x) = 2x - 3$  agree except at  $x = -1$ .

$$\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} g(x) = -5$$

