

$$57. \lim_{x \rightarrow 0} \frac{\sin x}{5x} = \lim_{x \rightarrow 0} \left[\left(\frac{\sin x}{x} \right) \left(\frac{1}{5} \right) \right] = (1) \left(\frac{1}{5} \right) = \frac{1}{5}$$

$$58. \lim_{x \rightarrow 0} \frac{3(1 - \cos x)}{x} = \lim_{x \rightarrow 0} \left[3 \left(\frac{1 - \cos x}{x} \right) \right] = (3)(0) = 0$$

$$59. \lim_{\theta \rightarrow 0} \frac{\sec \theta - 1}{\theta \sec \theta} = \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta} = 0$$

$$60. \lim_{\theta \rightarrow 0} \frac{\cos \theta \tan \theta}{\theta} = \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$61. \lim_{x \rightarrow 0} \frac{\sin^2 x}{x} = \lim_{x \rightarrow 0} \left[\frac{\sin x}{x} \sin x \right] = (1) \sin 0 = 0$$

$$62. \lim_{x \rightarrow 0} \frac{\tan^2 x}{x} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x \cos^2 x} = \lim_{x \rightarrow 0} \left[\frac{\sin x}{x} \cdot \frac{\sin x}{\cos^2 x} \right] = (1)(0) = 0$$

$$63. \lim_{h \rightarrow 0} \frac{(1 - \cos h)^2}{h} = \lim_{h \rightarrow 0} \left[\frac{1 - \cos h}{h} (1 - \cos h) \right] = (0)(0) = 0$$

$$64. \lim_{\phi \rightarrow \pi} \phi \sec \phi = \pi(-1) = -\pi$$

$$65. \lim_{x \rightarrow \pi/2} \frac{\cos x}{\cot x} = \lim_{x \rightarrow \pi/2} \sin x = 1$$

$$\begin{aligned} 66. \lim_{x \rightarrow \pi/4} \frac{1 - \tan x}{\sin x - \cos x} &= \lim_{x \rightarrow \pi/4} \frac{\cos x - \sin x}{\sin x \cos x - \cos^2 x} \\ &= \lim_{x \rightarrow \pi/4} \frac{-(\sin x - \cos x)}{\cos x(\sin x - \cos x)} \\ &= \lim_{x \rightarrow \pi/4} \frac{-1}{\cos x} \\ &= \lim_{x \rightarrow \pi/4} (-\sec x) \\ &= -\sqrt{2} \end{aligned}$$

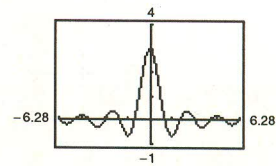
$$67. \lim_{t \rightarrow 0} \frac{\sin^2 t}{t^2} = \lim_{t \rightarrow 0} \left(\frac{\sin t}{t} \right)^2 = (1)^2 = 1$$

$$68. \lim_{x \rightarrow 0} \frac{\sin 2x}{\sin 3x} = \lim_{x \rightarrow 0} \left[2 \left(\frac{\sin 2x}{2x} \right) \left(\frac{1}{3} \right) \left(\frac{3x}{\sin 3x} \right) \right] = 2(1) \left(\frac{1}{3} \right) (1) = \frac{2}{3}$$

$$69. f(t) = \frac{\sin 3t}{t}$$

t	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(t)$	2.96	2.9996	3	?	3	2.9996	2.96

$$\text{Analytically, } \lim_{t \rightarrow 0} \frac{\sin 3t}{t} = \lim_{t \rightarrow 0} 3 \left(\frac{\sin 3t}{3t} \right) = 3(1) = 3.$$

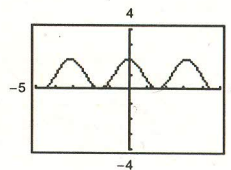


The limit appear to equal 3.

$$70. f(h) = (1 + \cos 2h)$$

h	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(h)$	1.98	1.9998	2	?	2	1.9998	1.98

$$\text{Analytically, } \lim_{h \rightarrow 0} (1 + \cos 2h) = 1 + \cos(0) = 1 + 1 = 2.$$

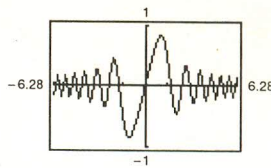


The limit appear to equal 2.

71. $f(x) = \frac{\sin x^2}{x}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	-0.099998	-0.01	-0.001	?	0.001	0.01	0.099998

Analytically, $\lim_{x \rightarrow 0} \frac{\sin x^2}{x} = \lim_{x \rightarrow 0} x \left(\frac{\sin x^2}{x^2} \right) = 0(1) = 0$.

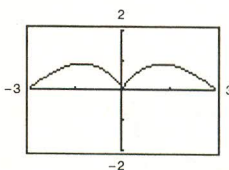


The limit appear to equal 0.

72. $f(x) = \frac{\sin x}{\sqrt[3]{x}}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$	0.215	0.0464	0.01	?	0.01	0.0464	0.215

Analytically, $\lim_{x \rightarrow 0} \frac{\sin x}{\sqrt[3]{x}} = \lim_{x \rightarrow 0} \sqrt[3]{x^2} \left(\frac{\sin x}{x} \right) = (0)(1) = 0$.



The limit appear to equal 0.

73. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{2(x+h) + 3 - (2x+3)}{h} = \lim_{h \rightarrow 0} \frac{2x+2h+3-2x-3}{h} = \lim_{h \rightarrow 0} \frac{2h}{h} = 2$

74. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$
 $= \lim_{h \rightarrow 0} \frac{x+h-x}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$

75. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h} = \lim_{h \rightarrow 0} \frac{4x - 4(x+h)}{(x+h)hx} = \lim_{h \rightarrow 0} \frac{-4}{(x+h)x} = \frac{-4}{x^2}$

76. $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - 4(x+h) - (x^2 - 4x)}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 4x - 4h - x^2 + 4x}{h}$
 $= \lim_{h \rightarrow 0} \frac{h(2x + h - 4)}{h} = \lim_{h \rightarrow 0} (2x + h - 4) = 2x - 4$

77. $\lim_{x \rightarrow 0} (4 - x^2) \leq \lim_{x \rightarrow 0} f(x) \leq \lim_{x \rightarrow 0} (4 + x^2)$

$4 \leq \lim_{x \rightarrow 0} f(x) \leq 4$

Therefore, $\lim_{x \rightarrow 0} f(x) = 4$.

78. $\lim_{x \rightarrow a} [b - |x - a|] \leq \lim_{x \rightarrow a} f(x) \leq \lim_{x \rightarrow a} [b + |x - a|]$

$b \leq \lim_{x \rightarrow a} f(x) \leq b$

Therefore, $\lim_{x \rightarrow a} f(x) = b$.