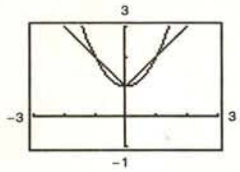
74.



As you zoom in, the graph of $y_1 = x^2 + 1$ appears to be locally the graph of a horizontal line, whereas the graph of $y_2 = |x| + 1$ always has a sharp corner at (0, 1). y_2 is not differentiable at (0, 1).

Section 2.2 Basic Differentiation Rules and Rates of Change

1. (a) $y = x^{1/2}$, $y' = \frac{1}{2}x^{-1/2}$, $y'(1) = \frac{1}{2}$

(b) $y = x^{3/2}$, $y' = \frac{3}{2}x^{1/2}$, $y'(1) = \frac{3}{2}$

(c) $y = x^2$, $y' = 2x$, $y'(1) = 2$

(d) $y = x^3$, $y' = 3x^2$, $y'(1) = 3$

2. (a) $y = x^{-1/2}$, $y' = -\frac{1}{2}x^{-3/2}$, $y'(1) = -\frac{1}{2}$

(b) $y = x^{-1}$, $y' = -x^{-2}$, $y'(1) = -1$

(c) $y = x^{-3/2}$, $y' = -\frac{3}{2}x^{-5/2}$, $y'(1) = -\frac{3}{2}$

(d) $y = x^{-2}$, $y' = -2x^{-3}$, $y'(1) = -2$

3. $y = 3$
$y' = 0$

4. $f(x) = -2$
$f'(x) = 0$

5. $f(x) = x + 1$
$f'(x) = 1$

6. $g(x) = 3x - 1$
$g'(x) = 3$

7. $g(x) = x^2 + 4$
$g'(x) = 2x$

8. $y = t^2 + 2t - 3$
$y' = 2t + 2$

9. $f(t) = -2t^2 + 3t - 6$
$f'(t) = -4t + 3$

10. $y = x^3 - 9$
$y' = 3x^2$

11. $s(t) = t^3 - 2t + 4$
$s'(t) = 3t^2 - 2$

12. $f(x) = 2x^3 - x^2 + 3x$
$f'(x) = 6x^2 - 2x + 3$

13. $y = x^2 - \frac{1}{2}\cos x$
$y' = 2x + \frac{1}{2}\sin x$

14. $y = 5 + \sin x$
$y' = \cos x$

15. $y = \dfrac{1}{x} - 3\sin x$
$y' = -\dfrac{1}{x^2} - 3\cos x$

16. $g(t) = \pi\cos t$
$g'(t) = -\pi\sin t$

	Function	*Rewrite*	*Derivative*	*Simplify*
17.	$y = \dfrac{1}{3x^3}$	$y = \dfrac{1}{3}x^{-3}$	$y' = -x^{-4}$	$y' = -\dfrac{1}{x^4}$
18.	$y = \dfrac{2}{3x^2}$	$y = \dfrac{2}{3}x^{-2}$	$y' = -\dfrac{4}{3}x^{-3}$	$y' = -\dfrac{4}{3x^3}$
19.	$y = \dfrac{1}{(3x)^3}$	$y = \dfrac{1}{27}x^{-3}$	$y' = -\dfrac{1}{9}x^{-4}$	$y' = -\dfrac{1}{9x^4}$
20.	$y = \dfrac{\pi}{(3x)^2}$	$y = \dfrac{\pi}{9}x^{-2}$	$y' = -\dfrac{2\pi}{9}x^{-3}$	$y' = -\dfrac{2\pi}{9x^3}$

—CONTINUED—

—CONTINUED—

Function	Rewrite	Derivative	Simplify
21. $y = \frac{\sqrt{x}}{x}$	$y = x^{-1/2}$	$y' = -\frac{1}{2}x^{-3/2}$	$y' = -\frac{1}{2x^{3/2}}$
22. $y = \frac{4}{x^{-3}}$	$y = 4x^3$	$y' = 12x^2$	$y' = 12x^2$

23. $f(x) = \frac{1}{x}, \ (1, 1)$

$f'(x) = -\frac{1}{x^2}$

$f'(1) = -1$

24. $f(t) = 3 - \frac{3}{5t}, \ \left(\frac{3}{5}, 2\right)$

$f'(t) = \frac{3}{5t^2}$

$f'\left(\frac{3}{5}\right) = \frac{5}{3}$

25. $f(x) = -\frac{1}{2} + \frac{7}{5}x^3, \ \left(0, -\frac{1}{2}\right)$

$f'(x) = \frac{21}{5}x^2$

$f'(0) = 0$

26. $y = 3x\left(x^2 - \frac{2}{x}\right), \ (2, 18)$

$= 3x^3 - 6$

$y' = 9x^2$

$y'(2) = 36$

27. $y = (2x + 1)^2, \ (0, 1)$

$= 4x^2 + 4x + 1$

$y' = 8x + 4$

$y'(0) = 4$

28. $f(x) = 3(5 - x)^2, \ (5, 0)$

$= 3x^2 - 30x + 75$

$f'(x) = 6x - 30$

$f'(5) = 0$

29. $f(\theta) = 4 \sin \theta - \theta, \ (0, 0)$

$f'(\theta) = 4 \cos \theta - 1$

$f'(0) = 4(1) - 1 = 3$

30. $g(t) = 2 + 3 \cos t, \ (\pi, -1)$

$g'(t) = -3 \sin t$

$g'(\pi) = 0$

31. $f(x) = x^3 - 3x - 2x^{-4}$

$f'(x) = 3x^2 - 3 + 8x^{-5}$

$= 3x^2 - 3 + \frac{8}{x^5}$

32. $f(x) = x^2 - 3x - 3x^{-2}$

$f'(x) = 2x - 3 + 6x^{-3}$

$= 2x - 3 + \frac{6}{x^3}$

33. $g(t) = t^2 - 4t^{-1}$

$g'(t) = 2t + 4t^{-2}$

$= 2t + \frac{4}{t^2}$

34. $f(x) = x + x^{-2}$

$f'(x) = 1 - 2x^{-3}$

$= 1 - \frac{2}{x^3}$

35. $f(x) = \frac{x^3 - 3x^2 + 4}{x^2} = x - 3 + 4x^{-2}$

$f'(x) = 1 - \frac{8}{x^3} = \frac{x^3 - 8}{x^3}$

36. $h(x) = \frac{2x^2 - 3x + 1}{x} = 2x - 3 + x^{-1}$

$h'(x) = 2 - \frac{1}{x^2} = \frac{2x^2 - 1}{x^2}$

37. $y = x(x^2 + 1) = x^3 + x$

$y' = 3x^2 + 1$

38. $f(x) = \sqrt[3]{x} + \sqrt[5]{x} = x^{1/3} + x^{1/5}$

$f'(x) = \frac{1}{3}x^{-2/3} + \frac{1}{5}x^{-4/5} = \frac{1}{3x^{2/3}} + \frac{1}{5x^{4/5}}$

39. $h(s) = s^{4/5}$

$h'(s) = \frac{4}{5}s^{-1/5} = \frac{4}{5s^{1/5}}$

40. $f(t) = t^{1/3} - 1$

$f'(t) = \frac{1}{3}t^{-2/3} = \frac{1}{3t^{2/3}}$

41. $f(x) = 4\sqrt{x} + 3\cos x = 4x^{1/2} + 3\cos x$

$f'(x) = 2x^{-1/2} - 3\sin x$

$= \frac{2}{\sqrt{x}} - 3\sin x$

42. $f(x) = 2\sin x + 3\cos x$

$f'(x) = 2\cos x - 3\sin x$

43. (a) $y = x^4 - 3x^2 + 2$

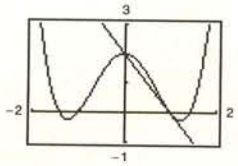
$y' = 4x^3 - 6x$

At $(1, 0)$: $y' = 4(1)^3 - 6(1) = -2$.

Tangent line: $y - 0 = -2(x - 1)$

$2x + y - 2 = 0$

(b)



44. (a) $y = x^3 + x$

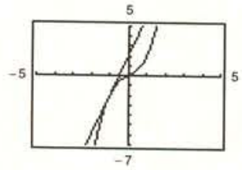
$y' = 3x^2 + 1$

At $(-1, -2)$: $y' = 3(-1)^2 + 1 = 4$.

Tangent line: $y + 2 = 4(x + 1)$

$4x - y + 2 = 0$

(b)



45. (a) $f(x) = \frac{1}{\sqrt[3]{x^2}} = x^{-2/3}$

$f'(x) = -\frac{2}{3}x^{-5/3} = -\frac{2}{3\sqrt[3]{x^5}}$

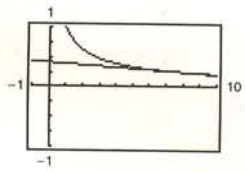
At $\left(8, \frac{1}{4}\right)$: $y' = -\frac{2}{3(\sqrt[3]{8})^5} = -\frac{1}{48}$.

Tangent line: $y - \frac{1}{4} = -\frac{1}{48}(x - 8)$

$-48y + 12 = x - 8$

$0 = x + 48y - 20$

(b)



46. (a) $y = (x^2 + 2x)(x + 1)$

$= x^3 + 3x^2 + 2x$

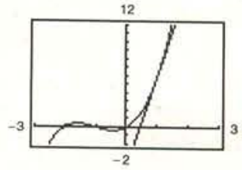
$y' = 3x^2 + 6x + 2$

At $(1, 6)$: $y' = 3(1)^2 + 6(1) + 2 = 11$.

Tangent line: $y - 6 = 11(x - 1)$

$0 = 11x - y - 5$

(b)



47. $y = x^4 - 8x^2 + 2$

$y' = 4x^3 - 16x$

$= 4x(x^2 - 4)$

$= 4x(x - 2)(x + 2)$

$y' = 0 \Rightarrow x = 0, \pm 2$

Horizontal tangents: $(0, 2), (2, -14), (-2, -14)$

48. $y = x^3 + x$

$y' = 3x^2 + 1 > 0$ for all x.

Therefore, there are no horizontal tangents.

49. $y = \dfrac{1}{x^2} = x^{-2}$

$y' = -2x^{-3} = \dfrac{-2}{x^3}$ cannot equal zero.

Therefore, there are no horizontal tangents.

50. $y = x^2 + 1$

$y' = 2x = 0 \Rightarrow x = 0$

At $x = 0, y = 1$.

Horizontal tangent: $(0, 1)$

51. $y = x + \sin x, 0 \le x < 2\pi$

$y' = 1 + \cos x = 0$

$\cos x = -1 \Rightarrow x = \pi$

At $x = \pi, y = \pi$.

Horizontal tangent: (π, π)

52. $y = \sqrt{3}x + 2\cos x,\ 0 \le x < 2\pi$

$y' = \sqrt{3} - 2\sin x = 0$

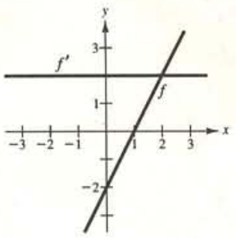
$\sin x = \dfrac{\sqrt{3}}{2} \Rightarrow x = \dfrac{\pi}{3}$ or $\dfrac{2\pi}{3}$

At $x = \dfrac{\pi}{3}, y = \dfrac{\sqrt{3}\pi + 3}{3}$.

At $x = \dfrac{2\pi}{3}, y = \dfrac{2\sqrt{3}\pi - 3}{3}$.

Horizontal tangents: $\left(\dfrac{\pi}{3}, \dfrac{\sqrt{3}\pi + 3}{3}\right), \left(\dfrac{2\pi}{3}, \dfrac{2\sqrt{3}\pi - 3}{3}\right)$

53.

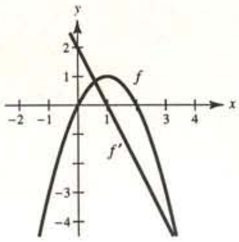


If f is linear then its derivative is a constant function.

$f(x) = ax + b$

$f'(x) = a$

54.



If f is quadratic its derivative is a linear function.

$f(x) = ax^2 + bx + c$

$f'(x) = 2ax + b$

91. $f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$

f must be continuous at $x = 2$ to be differentiable at $x = 2$.

$$\left.\begin{aligned} \lim_{x\to 2^-} f(x) &= \lim_{x\to 2^-} ax^3 = 8a \\ \lim_{x\to 2^+} f(x) &= \lim_{x\to 2^+} (x^2 + b) = 4 + b \end{aligned}\right\} \quad \begin{aligned} 8a &= 4 + b \\ 8a - 4 &= b \end{aligned}$$

$f'(x) = \begin{cases} 3ax^2, & x < 2 \\ 2x, & x > 2 \end{cases}$

For f to be differentiable at $x = 2$, the left derivative must equal the right derivative.

$3a(2)^2 = 2(2)$

$12a = 4$

$a = \frac{1}{3}$

$b = 8a - 4 = -\frac{4}{3}$