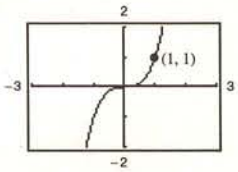
**60.** (a) Nearby point: (1.0073138, 1.0221024)

Secant line: $y - 1 = \frac{1.0221024 - 1}{1.0073138 - 1}(x - 1)$

$y = 3.022(x - 1) + 1$

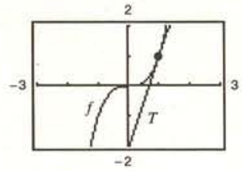
(Answers will vary.)



(b) $f'(x) = 3x^2$

$T(x) = 3(x - 1) + 1 = 3x - 2$

(c) The accuracy worsens at you move away from (1, 1).



(d)

| $\Delta x$ | $-3$ | $-2$ | $-1$ | $-0.5$ | $-0.1$ | 0 | 0.1 | 0.5 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $-8$ | $-1$ | 0 | 0.125 | 0.729 | 1 | 1.331 | 3.375 | 8 | 27 | 64 |
| $T(x)$ | $-8$ | $-5$ | $-2$ | $-0.5$ | 0.7 | 1 | 1.3 | 2.5 | 4 | 7 | 10 |

The accuracy decreases more rapidly than in Exercise 59 because $y = x^3$ is less "linear" than $y = x^{3/2}$.

**61.** False. Let $f(x) = x^2$ and $g(x) = x^2 + 4$. Then $f'(x) = g'(x) = 2x$, but $f(x) \neq g(x)$.

**62.** True. If $f(x) = g(x) + c$, then $f'(x) = g'(x) + 0 = g'(x)$.

**63.** False. If $y = \pi^2$, then $dy/dx = 0$. ($\pi^2$ is a constant.)

**64.** True. If $y = x/\pi = (1/\pi) \cdot x$, then $dy/dx = 1/\pi(1) = 1/\pi$.

**65.** $f(t) = 2t + 7, [1, 2]$

$f'(t) = 2$

Instantaneous rate of change is the constant 2.
Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{[2(2) + 7] - [2(1) + 7]}{1} = 2$$

(These are the same because $f$ is a line of slope 2.)

**66.** $f(t) = t^2 - 3, [2, 2.1]$

$f'(t) = 2t$

Instantaneous rate of change:

$(2, 1) \Rightarrow f'(2) = 2(2) = 4$

$(2.1, 1.41) \Rightarrow f'(2.1) = 4.2$

Average rate of change:

$$\frac{f(2.1) - f(2)}{2.1 - 2} = \frac{1.41 - 1}{0.1} = 4.1$$

**67.** $f(x) = -\frac{1}{x}, [1, 2]$

$f'(x) = \frac{1}{x^2}$

Instantaneous rate of change:

$(1, -1) \Rightarrow f'(1) = 1$

$\left(2, -\frac{1}{2}\right) \Rightarrow f'(2) = \frac{1}{4}$

Average rate of change:

$$\frac{f(2) - f(1)}{2 - 1} = \frac{(-1/2) - (-1)}{2 - 1} = \frac{1}{2}$$

**68.** $f(x) = \sin x, \left[0, \frac{\pi}{6}\right]$

$f'(x) = \cos x$

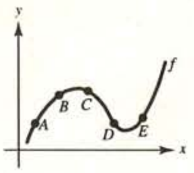
Instantaneous rate of change:

$(0, 0) \Rightarrow f'(0) = 1$

$\left(\frac{\pi}{6}, \frac{1}{2}\right) \Rightarrow f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \approx 0.866$

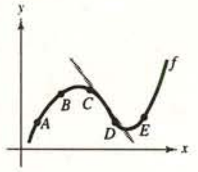
Average rate of change:

$$\frac{f(\pi/6) - f(0)}{(\pi/6) - 0} = \frac{(1/2) - 0}{(\pi/6) - 0} = \frac{3}{\pi} \approx 0.955$$

**69.** 

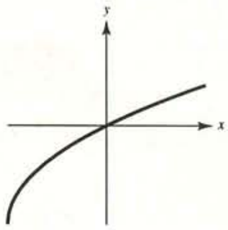

(a) The slope appears to be steepest between $A$ and $B$.

(b) The average rate of change between $A$ and $B$ is **greater** than the instantaneous rate of change at $B$.

(c) 


(d) The average rates of change are approximately equal between $B$ and $C$, and between $D$ and $E$.

**70.** The graph of a function $f$ such that $f' > 0$ for all $x$ and the rate of change the function is decreasing (i.e. $f'' < 0$)would, in general, look like the graph at the right.




**71.** (a) $s(t) = -16t^2 + 1362$

$v(t) = -32t$

(b) $\dfrac{s(2) - s(1)}{2 - 1} = 1298 - 1346 = -48$ ft/sec

(c) $v(t) = s'(t) = -32t$

When $t = 1$: $v(1) = -32$ ft/sec.

When $t = 2$: $v(2) = -64$ ft/sec.

(d) $-16t^2 + 1362 = 0$

$$t^2 = \frac{1362}{16} \Rightarrow t = \frac{\sqrt{1362}}{4} \approx 9.226 \text{ sec}$$

(e) $$v\left(\frac{\sqrt{1362}}{4}\right) = -32\left(\frac{\sqrt{1362}}{4}\right)$$

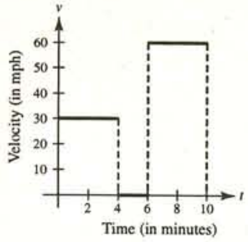
$$= -8\sqrt{1362} \approx -295.242 \text{ ft/sec}$$

**72.**
$$s(t) = -16t^2 - 22t + 220$$
$$v(t) = -32t - 22$$
$$v(3) = -118 \text{ ft/sec}$$
$$s(t) = -16t^2 - 22t + 220$$
$$= 112 \text{ (height after falling 108 ft)}$$
$$-16t^2 - 22t + 108 = 0$$
$$-2(t - 2)(8t + 27) = 0$$
$$t = 2$$
$$v(2) = -32(2) - 22$$
$$= -86 \text{ ft/sec}$$

**73.**
$$s(t) = -4.9t^2 + v_0t + s_0$$
$$= -4.9t^2 + 120t$$
$$v(t) = -9.8t + 120$$
$$v(5) = -9.8(5) + 120 = 71 \text{ m/sec}$$
$$v(10) = -9.8(10) + 120 = 22 \text{ m/sec}$$

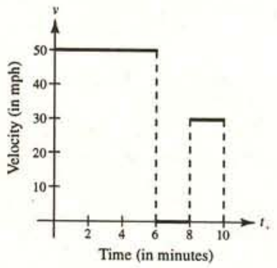
**74.**
$$s(t) = -4.9t^2 + v_0t + s_0$$
$$= -4.9t^2 + s_0 = 0 \text{ when } t = 6.8.$$
$$s_0 = 4.9t^2 = 4.9(6.8)^2 = 226.6 \text{ m}$$

**75.**




(The velocity has been converted to miles per hour)

**76.**




(The velocity has been converted to miles per hour)

**77.** $v = 40 \text{ mph} = \frac{2}{3} \text{ mi/min}$

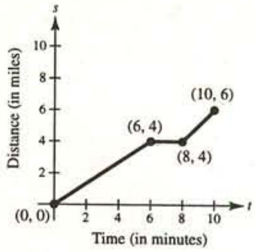
$\left(\frac{2}{3} \text{ mi/min}\right)(6 \text{ min}) = 4 \text{ mi}$

$v = 0 \text{ mph} = 0 \text{ mi/min}$

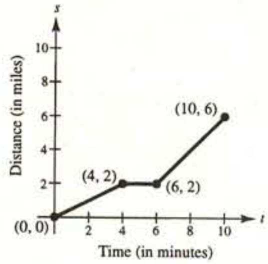
$(0 \text{ mi/min})(2 \text{ min}) = 0 \text{ mi}$

$v = 60 \text{ mph} = 1 \text{mi/min}$

$(1 \text{ mi/min})(2 \text{ min}) = 2 \text{ mi}$




**78.** This graph corresponds with Exercise 75.




**79.** (a) Using a graphing utility, you obtain

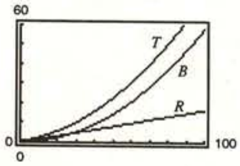
$$R = 0.167v - 0.02.$$

(b) Using a graphing utility, you obtain

$$B = 0.00586v^2 - 0.0239v + 0.46.$$

(c) $T = R + B = 0.00586v^2 + 0.1431v + 0.44$

(d)




(e) $\dfrac{dT}{dv} = 0.01172v + 0.1431$

For $v = 40$, $T'(40) \approx 0.612$.

For $v = 80$, $T'(80) \approx 1.081$.

For $v = 100$, $T'(100) \approx 1.315$.

(f) For increasing speeds, the total stopping distance increases.

**80.** $s(t) = -\dfrac{1}{2}at^2 + c$ and $s'(t) = -at$.

Average velocity:
$$\frac{s(t_0 + \Delta t) - s(t_0 - \Delta t)}{(t_0 + \Delta t) - (t_0 - \Delta t)} = \frac{[-(1/2)a(t_0 + \Delta t)^2 + c] - [-(1/2)a(t_0 - \Delta t)^2 + c)]}{2\Delta t}$$
$$= \frac{-(1/2)a(t_0^2 + 2t_0\Delta t + (\Delta t)^2) + (1/2)a(t_0^2 - 2t_0\Delta t + (\Delta t)^2)}{2\Delta t}$$
$$= \frac{-2at_0\Delta t}{2\Delta t}$$
$$= -at_0$$
$$= s'(t_0) \quad \text{Instantaneous velocity at } t = t_0$$

**81.** $A = s^2, \dfrac{dA}{ds} = 2s$

When $s = 4$ m, $\dfrac{dA}{ds} = 8 \text{ m}^2$.

**82.** $V = s^3, \dfrac{dV}{ds} = 3s^2$

When $s = 4$ cm, $\dfrac{dV}{ds} = 48 \text{ cm}^3$.

**83.** $$C = \frac{1{,}008{,}000}{Q} + 6.3Q$$

$$\frac{dC}{dQ} = -\frac{1{,}008{,}000}{Q^2} + 6.3$$

$C(351) - C(350) \approx 5083.095 - 5085 \approx -\$1.91$

When $Q = 350$, $\frac{dC}{dQ} \approx -\$1.93$.

**84.** $C =$ (gallons of fuel used)(cost per gallon)

$$= \left(\frac{15{,}000}{x}\right)(1.25) = \frac{18{,}750}{x}$$

$$\frac{dC}{dx} = -\frac{18{,}750}{x^2}$$

| $x$ | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
|---|---|---|---|---|---|---|---|
| $C$ | 1875 | 1250 | 537.5 | 750 | 625 | 535.71 | 468.75 |
| $\frac{dC}{dx}$ | −187.5 | −83.333 | −46.875 | −30 | −20.833 | −15.306 | −11.719 |

The driver who gets 15 miles per gallon would benefit more from a 1 mile per gallon increase in fuel efficiency. The rate of change is larger when $x = 15$.

**85.** The runner was safe. Explanations will vary.

**86.** $\frac{dT}{dt} = K(T - T_a)$

**87.** $y = ax^2 + bx + c$

Since the parabola passes through (0, 1) and (1, 0), we have

$(0, 1)$: $1 = a(0)^2 + b(0) + c \Rightarrow c = 1$

$(1, 0)$: $0 = a(1)^2 + b(1) + 1 \Rightarrow b = -a - 1$.

Thus, $y = ax^2 + (-a - 1)x + 1$. From the tangent line $y = x - 1$, we know that the derivative is 1 at the point (1, 0).

$y' = 2ax + (-a - 1)$

$1 = 2a(1) + (-a - 1)$

$1 = a - 1$

$a = 2$

$b = -a - 1 = -3$

Therefore, $y = 2x^2 - 3x + 1$.