Section 2.3 The Product and Quotient Rules and Higher-Order Derivatives

1. $f(x) = \frac{1}{3}(2x^3 - 4)$

$f'(x) = \frac{1}{3}(6x^2) = 2x^2$

$f'(0) = 0$

2. $f(x) = (x^2 - 2x + 1)(x^3 - 1)$

$f'(x) = (x^2 - 2x + 1)(3x^2) + (x^3 - 1)(2x - 2)$

$= 3x^2(x - 1)^2 + 2(x - 1)^2(x^2 + x + 1)$

$= (x - 1)^2(5x^2 + 2x + 2)$

$f'(1) = 0$

3. $f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$

$f'(x) = (x^3 - 3x)(4x + 3) + (2x^2 + 3x + 5)(3x^2 - 3)$

$= 10x^4 + 12x^3 - 3x^2 - 18x - 15$

$f'(0) = -15$

4. $f(x) = \dfrac{x + 1}{x - 1}$

$f'(x) = \dfrac{(x - 1)(1) - (x + 1)(1)}{(x - 1)^2}$

$= \dfrac{x - 1 - x - 1}{(x - 1)^2}$

$= -\dfrac{2}{(x - 1)^2}$

$f'(2) = -\dfrac{2}{(2 - 1)^2} = -2$

5. $f(x) = x \cos x$

$f'(x) = (x)(-\sin x) + (\cos x)(1)$

$= \cos x - x \sin x$

$f'\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2} - \dfrac{\pi}{4}\left(\dfrac{\sqrt{2}}{2}\right)$

$= \dfrac{\sqrt{2}}{8}(4 - \pi)$

6. $f(x) = \dfrac{\sin x}{x}$

$f'(x) = \dfrac{(x)(\cos x) - (\sin x)(1)}{x^2}$

$= \dfrac{x \cos x - \sin x}{x^2}$

$f'\left(\dfrac{\pi}{6}\right) = \dfrac{(\pi/6)(\sqrt{3}/2) - (1/2)}{\pi^2/36}$

$= \dfrac{3\sqrt{3}\pi - 18}{\pi^2}$

$= \dfrac{3(\sqrt{3}\pi - 6)}{\pi^2}$

	Function	*Rewrite*	*Derivative*	*Simplify*
7.	$y = \dfrac{x^2 + 2x}{x}$	$y = x + 2,\ x \neq 0$	$y' = 1$	$y' = 1$
8.	$y = \dfrac{4x^{3/2}}{x}$	$y = 4\sqrt{x}, x > 0$	$y' = 2x^{-1/2}$	$y' = \dfrac{2}{\sqrt{x}}$
9.	$y = \dfrac{7}{3x^3}$	$y = \dfrac{7}{3}x^{-3}$	$y' = -7x^{-4}$	$y' = -\dfrac{7}{x^4}$
10.	$y = \dfrac{4}{5x^2}$	$y = \dfrac{4}{5}x^{-2}$	$y' = -\dfrac{8}{5}x^{-3}$	$y' = -\dfrac{8}{5x^3}$
11.	$y = \dfrac{3x^2 - 5}{7}$	$y = \dfrac{1}{7}(3x^2 - 5)$	$y' = \dfrac{1}{7}(6x)$	$y' = \dfrac{6x}{7}$
12.	$y = \dfrac{x^2 - 4}{x + 2}$	$y = x - 2,\ x \neq -2$	$y' = 1$	$y' = 1$

13. $f(x) = \dfrac{3x - 2}{2x - 3}$

$f'(x) = \dfrac{(2x - 3)(3) - (3x - 2)(2)}{(2x - 3)^2}$

$= \dfrac{-5}{(2x - 3)^2}$

14. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$

$f'(x) = \dfrac{(x^2 - 1)(3x^2 + 3) - (x^3 + 3x + 2)(2x)}{(x^2 - 1)^2}$

$= \dfrac{x^4 - 6x^2 - 4x - 3}{(x^2 - 1)^2}$

15. $f(x) = \dfrac{3 - 2x - x^2}{x^2 - 1}$

$f'(x) = \dfrac{(x^2 - 1)(-2 - 2x) - (3 - 2x - x^2)(2x)}{(x^2 - 1)^2}$

$= \dfrac{2x^2 - 4x + 2}{(x^2 - 1)^2} = \dfrac{2(x - 1)^2}{(x^2 - 1)^2}$

$= \dfrac{2}{(x + 1)^2}, x \neq 1$

16. $f(x) = x^4\left[1 - \dfrac{2}{x + 1}\right] = x^4\left[\dfrac{x - 1}{x + 1}\right]$

$f'(x) = x^4\left[\dfrac{(x + 1) - (x - 1)}{(x + 1)^2}\right] + \left[\dfrac{x - 1}{x + 1}\right](4x^3)$

$= 2x^3\left[\dfrac{2x^2 + x - 2}{(x + 1)^2}\right]$

17. $f(x) = \dfrac{x + 1}{\sqrt{x}}$

$f'(x) = \dfrac{\sqrt{x}(1) - (x + 1)\left[1/\left(2\sqrt{x}\right)\right]}{x}$

$= \dfrac{x - 1}{2x^{3/2}}$

Alternate solution:

$f(x) = \dfrac{x + 1}{\sqrt{x}} = x^{1/2} + x^{-1/2}$

$f'(x) = \dfrac{1}{2}x^{-1/2} - \dfrac{1}{2}x^{-3/2}$

$= \dfrac{1}{2x^{1/2}} - \dfrac{1}{2x^{3/2}}$

$= \dfrac{x - 1}{2x^{3/2}}$

18. $f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right) = x^{1/3}(x^{1/2} + 3)$

$f'(x) = x^{1/3}\left(\dfrac{1}{2}x^{-1/2}\right) + (x^{1/2} + 3)\left(\dfrac{1}{3}x^{-2/3}\right)$

$= \dfrac{5}{6}x^{-1/6} + x^{-2/3}$

$= \dfrac{5}{6x^{1/6}} + \dfrac{1}{x^{2/3}}$

Alternate solution:

$f(x) = \sqrt[3]{x}\left(\sqrt{x} + 3\right)$

$= x^{5/6} + 3x^{1/3}$

$f'(x) = \dfrac{5}{6}x^{-1/6} + x^{-2/3}$

$= \dfrac{5}{6x^{1/6}} + \dfrac{1}{x^{2/3}}$

19. $h(s) = (s^3 - 2)^2 = s^6 - 4s^3 + 4$

$h'(s) = 6s^5 - 12s^2 = 6s^2(s^3 - 2)$

20. $h(x) = (x^2 - 1)^2 = x^4 - 2x^2 + 1$

$h'(x) = 4x^3 - 4x = 4x(x^2 - 1)$

21. $h(t) = \dfrac{t + 1}{t^2 + 2t + 2}$

$h'(t) = \dfrac{(t^2 + 2t + 2)(1) - (t + 1)(2t + 2)}{(t^2 + 2t + 2)^2}$

$= \dfrac{-t^2 - 2t}{(t^2 + 2t + 2)^2}$

22. $f(x) = \dfrac{x(x^2 - 1)}{x + 3} = \dfrac{x^3 - x}{x + 3}$

$f'(x) = \dfrac{(x + 3)(3x^2 - 1) - (x^3 - x)}{(x + 3)^2}$

$= \dfrac{2x^3 + 9x^2 - 3}{(x + 3)^2}$

23. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

$$\begin{aligned} f'(x) &= (9x^2 + 4)(x - 5)(x + 1) + (3x^3 + 4x)(1)(x + 1) + (3x^3 + 4x)(x - 5)(1) \\ &= (9x^2 + 4)(x^2 - 4x - 5) + 3x^4 + 3x^3 + 4x^2 + 4x + 3x^4 - 15x^3 + 4x^2 - 20x \\ &= 9x^4 - 36x^3 - 41x^2 - 16x - 20 + 6x^4 - 12x^3 + 8x^2 - 16x \\ &= 15x^4 - 48x^3 - 33x^2 - 32x - 20 \end{aligned}$$

24. $f(x) = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

$$\begin{aligned} f'(x) &= (2x - 1)(x^2 + 1)(x^2 + x + 1) + (x^2 - x)(2x)(x^2 + x + 1) + (x^2 - x)(x^2 + 1)(2x + 1) \\ &= (2x - 1)(x^4 + x^3 + 2x^2 + x + 1) + (x^2 - x)(2x^3 + 2x^2 + 2x) + (x^2 - x)(2x^3 + x^2 + 2x + 1) \\ &= 2x^5 + x^4 + 3x^3 + x - 1 + 2x^5 - 2x^2 + 2x^5 - x^4 + x^3 - x^2 - x \\ &= 6x^5 + 4x^3 - 3x^2 - 1 \end{aligned}$$

25. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$

$$\begin{aligned} f'(x) &= \frac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2} \\ &= \frac{-4xc^2}{(x^2 - c^2)^2} \end{aligned}$$

26. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$

$$\begin{aligned} f'(x) &= \frac{(c^2 + x^2)(-2x) - (c^2 - x^2)(2x)}{(c^2 + x^2)^2} \\ &= \frac{-4xc^2}{(c^2 + x^2)^2} \end{aligned}$$

27. $f(t) = t^2 \sin t$

$$\begin{aligned} f'(t) &= t^2 \cos t + 2t \sin t \\ &= t(t \cos t + 2 \sin t) \end{aligned}$$

28. $f(\theta) = (\theta + 1) \cos \theta$

$$\begin{aligned} f'(\theta) &= (\theta + 1)(-\sin \theta) + (\cos \theta)(1) \\ &= \cos \theta - (\theta + 1) \sin \theta \end{aligned}$$

29. $f(t) = \dfrac{\cos t}{t}$

$$f'(t) = \frac{-t \sin t - \cos t}{t^2} = -\frac{t \sin t + \cos t}{t^2}$$

30. $f(x) = \dfrac{\sin x}{x}$

$$f'(x) = \frac{x \cos x - \sin x}{x^2}$$

31. $f(x) = -x + \tan x$

$$f'(x) = -1 + \sec^2 x = \tan^2 x$$

32. $y = x + \cot x$

$$y' = 1 - \csc^2 x = -\cot^2 x$$

33. $g(t) = \sqrt{t} + 4 \sec t$

$$\begin{aligned} g'(t) &= \frac{1}{2}t^{-1/2} + 4 \sec t \tan t \\ &= \frac{1}{2\sqrt{t}} + 4 \sec t \tan t \end{aligned}$$

34. $h(s) = \dfrac{1}{s} - 10 \csc s$

$$h'(s) = -\frac{1}{s^2} + 10 \csc s \cot s$$

35. $y = 5x \csc x$

$$\begin{aligned} y' &= -5x \csc x \cot x + 5 \csc x \\ &= 5 \csc x(-x \cot x + 1) \\ &= 5 \csc x(1 - x \cot x) \end{aligned}$$

36. $y = \dfrac{\sec x}{x}$

$$\begin{aligned} y' &= \frac{x \sec x \tan x - \sec x}{x^2} \\ &= \frac{\sec x(x \tan x - 1)}{x^2} \end{aligned}$$

37. $y = -\csc x - \sin x$

$y' = \csc x \cot x - \cos x$

$= \dfrac{\cos x}{\sin^2 x} - \cos x$

$= \cos x(\csc^2 x - 1)$

$= \cos x \cot^2 x$

38. $y = x \sin x + \cos x$

$y' = x \cos x + \sin x - \sin x = x \cos x$

39. $y = x^2 \sin x + 2x \cos x$

$y' = x^2 \cos x + 2x \sin x - 2x \sin x + 2 \cos x$

$= x^2 \cos x + 2 \cos x$

40. $f(x) = \sin x \cos x$

$f'(x) = \sin x(-\sin x) + \cos x(\cos x)$

$= \cos 2x$

41. $f(x) = x^2 \tan x$

$f'(x) = x^2 \sec^2 x + 2x \tan x$

$= x(x \sec^2 x + 2 \tan x)$

42. $h(\theta) = 5 \sec \theta + \tan \theta$

$h'(\theta) = 5 \sec \theta \tan \theta + \sec^2 \theta$

$= \sec \theta(5 \tan \theta + \sec \theta)$

43. $g(x) = \left(\dfrac{x+1}{x+2}\right)(2x - 5)$

$g'(x) = \dfrac{2x^2 + 8x - 1}{(x+2)^2}$ (form of answer may vary)

44. $f(x) = \left(\dfrac{x^2 - x - 3}{x^2 + 1}\right)(x^2 + x + 1)$

$f'(x) = 2\dfrac{x^5 + 2x^3 + 2x^2 - 2}{(x^2 + 1)^2}$ (form of answer may vary)

45. $g(\theta) = \dfrac{\theta}{1 - \sin \theta}$

$g'(\theta) = \dfrac{1 - \sin \theta + \theta \cos \theta}{(\sin \theta - 1)^2}$ (form of answer may vary)

46. $f(\theta) = \dfrac{\sin \theta}{1 - \cos \theta}$

$f'(\theta) = \dfrac{1}{\cos \theta - 1} = \dfrac{\cos \theta - 1}{(1 - \cos \theta)^2}$

(form of answer may vary)

47. $y = \dfrac{1 + \csc x}{1 - \csc x}$

$y' = \dfrac{(1 - \csc x)(-\csc x \cot x) - (1 + \csc x)(\csc x \cot x)}{(1 - \csc x)^2} = \dfrac{-2 \csc x \cot x}{(1 - \csc x)^2}$

$y'\left(\dfrac{\pi}{6}\right) = \dfrac{-2(2)(\sqrt{3})}{(1 - 2)^2} = -4\sqrt{3}$

48. $f(x) = \tan x \cot x = 1$

$f'(x) = 0$

$f'(1) = 0$

49. $h(t) = \dfrac{\sec t}{t}$

$h'(t) = \dfrac{t(\sec t \tan t) - (\sec t)(1)}{t^2}$

$= \dfrac{\sec t(t \tan t - 1)}{t^2}$

$h'(\pi) = \dfrac{\sec \pi(\pi \tan \pi - 1)}{\pi^2} = \dfrac{1}{\pi^2}$

50. $f(x) = \sin x(\sin x + \cos x)$

$f'(x) = \sin x(\cos x - \sin x) + (\sin x + \cos x)\cos x$

$= \sin x \cos x - \sin^2 x + \sin x \cos x + \cos^2 x$

$= \sin 2x + \cos 2x$

$f'\left(\dfrac{\pi}{4}\right) = \sin \dfrac{\pi}{2} + \cos \dfrac{\pi}{2} = 1$

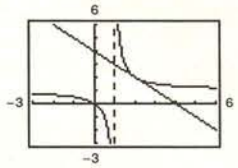
51. (a) $f(x) = \dfrac{x}{x-1}$, $(2, 2)$

$$f'(x) = \frac{(x-1)(1) - x(1)}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

$$f'(2) = \frac{-1}{(2-1)^2} = -1 = \text{slope at } (2, 2).$$

Tangent line: $y - 2 = -1(x - 2) \Rightarrow y = -x + 4$

(b)



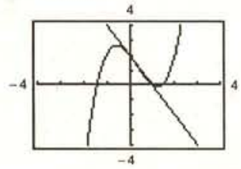
52. (a) $f(x) = (x - 1)(x^2 - 2)$, $(0, 2)$

$$f'(x) = (x-1)(2x) + (x^2 - 2)(1) = 3x^2 - 2x - 2$$

$f'(0) = -2 = $ slope at $(0, 2)$.

Tangent line: $y - 2 = -2x \Rightarrow y = -2x + 2$

(b)



53. (a) $f(x) = (x^3 - 3x + 1)(x + 2)$, $(1, -3)$

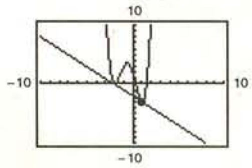
$$f'(x) = (x^3 - 3x + 1)(1) + (x + 2)(3x^2 - 3)$$

$$= 4x^3 + 6x^2 - 6x - 5$$

$f'(1) = -1 = $ slope at $(1, -3)$.

Tangent line: $y + 3 = -1(x - 1) \Rightarrow y = -x - 2$

(b)



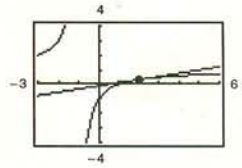
54. (a) $f(x) = \dfrac{x-1}{x+1}$, $\left(2, \dfrac{1}{3}\right)$

$$f'(x) = \frac{(x+1)(1) - (x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

$$f'(2) = \frac{2}{9} = \text{slope at } \left(2, \frac{1}{3}\right).$$

Tangent line: $y - \dfrac{1}{3} = \dfrac{2}{9}(x - 2) \Rightarrow y = \dfrac{2}{9}x - \dfrac{1}{9}$

(b)



55. (a) $f(x) = \tan x$, $\left(\dfrac{\pi}{4}, 1\right)$

$$f'(x) = \sec^2 x$$

$$f'\left(\frac{\pi}{4}\right) = 2 = \text{slope at } \left(\frac{\pi}{4}, 1\right).$$

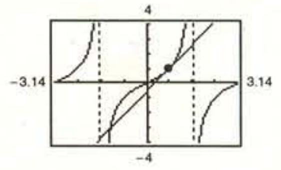
Tangent line:

$$y - 1 = 2\left(x - \frac{\pi}{4}\right)$$

$$y - 1 = 2x - \frac{\pi}{2}$$

$$4x - 2y - \pi + 2 = 0$$

(b)



56. (a) $f(x) = \sec x, \ \left(\frac{\pi}{3}, 2\right)$

$f'(x) = \sec x \tan x$

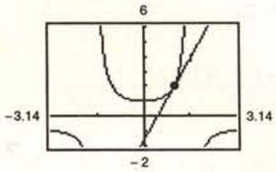
$f'\left(\frac{\pi}{3}\right) = 2\sqrt{3} = \text{slope at } \left(\frac{\pi}{3}, 2\right).$

Tangent line:

$$y - 2 = 2\sqrt{3}\left(x - \frac{\pi}{3}\right)$$

$$6\sqrt{3}x - 3y + 6 - 2\sqrt{3}\pi = 0$$

(b)



57. $f(x) = \dfrac{x^2}{x - 1}$

$$f'(x) = \frac{(x-1)(2x) - x^2(1)}{(x-1)^2}$$

$$= \frac{x^2 - 2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

$f'(x) = 0$ when $x = 0$ or $x = 2$.

Horizontal tangents are at $(0, 0)$ and $(2, 4)$.

58. $f(x) = \dfrac{x^2}{x^2 + 1}$

$$f'(x) = \frac{(x^2+1)(2x) - (x^2)(2x)}{(x^2+1)^2} = \frac{2x}{(x^2+1)^2}$$

$f'(x) = 0$ when $x = 0$.

Horizontal tangent is at $(0, 0)$.

59. $f(x) = 2g(x) + h(x)$

$$f'(x) = 2g'(x) + h'(x)$$

$$f'(2) = 2g'(2) + h'(2)$$

$$= 2(-2) + 4$$

$$= 0$$

60. $f(x) = 4 - h(x)$

$$f'(x) = -h'(x)$$

$$f'(2) = -h'(2) = -4$$

61. $f(x) = \dfrac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - g(x)h'(x)}{[h(x)]^2}$$

$$f'(2) = \frac{h(2)g'(2) - g(2)h'(2)}{[h(2)]^2}$$

$$= \frac{(-1)(-2) - (3)(4)}{(-1)^2}$$

$$= -10$$

62. $f(x) = g(x)h(x)$

$$f'(x) = g(x)h'(x) + h(x)g'(x)$$

$$f'(2) = g(2)h'(2) + h(2)g'(2)$$

$$= (3)(4) + (-1)(-2)$$

$$= 14$$

63. $f(x) = x^n \sin x$

$$f'(x) = x^n \cos x + nx^{n-1} \sin x$$

$$= x^{n-1}(x \cos x + n \sin x)$$

When $n = 1$: $f'(x) = x \cos x + \sin x$.

When $n = 2$: $f'(x) = x(x \cos x + 2 \sin x)$.

When $n = 3$: $f'(x) = x^2(x \cos x + 3 \sin x)$.

When $n = 4$: $f'(x) = x^3(x \cos x + 4 \sin x)$.

For general n, $f'(x) = x^{n-1}(x \cos x + n \sin x)$.

64. $f(x) = \dfrac{\cos x}{x^n} = x^{-n} \cos x$

$$f'(x) = -x^{-n} \sin x - nx^{-n-1} \cos x$$
$$= -x^{-n-1}(x \sin x + n \cos x)$$
$$= -\frac{x \sin x + n \cos x}{x^{n+1}}$$

When $n = 1$: $f'(x) = -\dfrac{x \sin x + \cos x}{x^2}$.

When $n = 2$: $f'(x) = -\dfrac{x \sin x + 2 \cos x}{x^3}$.

When $n = 3$: $f'(x) = -\dfrac{x \sin x + 3 \cos x}{x^4}$.

When $n = 4$: $f'(x) = -\dfrac{x \sin x + 4 \cos x}{x^5}$.

For general n, $f'(x) = -\dfrac{x \sin x + n \cos x}{x^{n+1}}$.

65. $C = 100\left(\dfrac{200}{x^2} + \dfrac{x}{x + 30}\right), \quad 1 \le x$

$$\frac{dC}{dx} = 100\left(-\frac{400}{x^3} + \frac{30}{(x + 30)^2}\right)$$

(a) When $x = 10$: $\dfrac{dC}{dx} = -\$38.13$.

(b) When $x = 15$: $\dfrac{dC}{dx} = -\$10.37$.

(c) When $x = 20$: $\dfrac{dC}{dx} = -\$3.80$.

As the order size increases, the cost per item decreases.

66. $P = \dfrac{k}{V}$

$$\frac{dP}{dV} = -\frac{k}{V^2}$$

67. $P(t) = 500\left[1 + \dfrac{4t}{50 + t^2}\right]$

$$P'(t) = 500\left[\frac{(50 + t^2)(4) - (4t)(2t)}{(50 + t^2)^2}\right] = 500\left[\frac{200 - 4t^2}{(50 + t^2)^2}\right] = 2000\left[\frac{50 - t^2}{(50 + t^2)^2}\right]$$

$P'(2) \approx 31.55$ per hour

68. $f(x) = \sec x$

$g(x) = \csc x, \quad [0, 2\pi)$

$f'(x) = g'(x)$

$$\sec x \tan x = -\csc x \cot x \Rightarrow \frac{\sec x \tan x}{\csc x \cot x} = -1 \Rightarrow \frac{\dfrac{1}{\cos x} \cdot \dfrac{\sin x}{\cos x}}{\dfrac{1}{\sin x} \cdot \dfrac{\cos x}{\sin x}} = -1 \Rightarrow$$

$$\frac{\sin^3 x}{\cos^3 x} = -1 \Rightarrow \tan^3 x = -1 \Rightarrow \tan x = -1$$

$$x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

69. (a) $\sec x = \dfrac{1}{\cos x}$

$$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right] = \frac{(\cos x)(0) - (1)(-\sin x)}{(\cos x)^2} = \frac{\sin x}{\cos x \cos x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$$

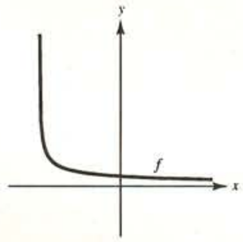
(b) $\csc x = \dfrac{1}{\sin x}$

$$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right] = \frac{(\sin x)(0) - (1)(\cos x)}{(\sin x)^2} = -\frac{\cos x}{\sin x \sin x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x$$

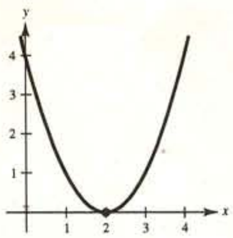
(c) $\cot x = \dfrac{\cos x}{\sin x}$

$$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right] = \frac{\sin x(-\sin x) - (\cos x)(\cos x)}{(\sin x)^2} = -\frac{\sin^2 x + \cos^2 x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$$

70. The graph of a differentiable function f such that $f > 0$ and $f' < 0$ for all real numbers x would in general look like the graph at the right.



71.



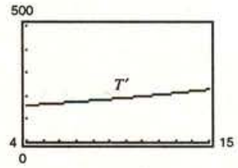
$f(2) = 0$

One such function is $f(x) = (x - 2)^2$.

72. $T = \dfrac{2{,}231{,}291 + 110{,}636t}{1000 - 14t}$

(a) Using a symbolic differentiation utility, or the quotient rule,

$$T' = \frac{70{,}937{,}037}{2(7t - 500)^2}.$$



(b) The graph of T' is getting more and more positive. That means that the tax burden per capita is increasing at a greater and greater rate.

(c) $T(20) \approx 6172$ dollars

73. $f(x) = 4x^{3/2}$

$f'(x) = 6x^{1/2}$

$f''(x) = 3x^{-1/2} = \dfrac{3}{\sqrt{x}}$

74. $f(x) = \dfrac{x^2 + 2x - 1}{x} = x + 2 - \dfrac{1}{x}$

$f'(x) = 1 + \dfrac{1}{x^2}$

$f''(x) = -\dfrac{2}{x^3}$

75. $f(x) = \dfrac{x}{x-1}$

$f'(x) = \dfrac{(x-1)(1) - x(1)}{(x-1)^2} = \dfrac{-1}{(x-1)^2}$

$f''(x) = \dfrac{2}{(x-1)^3}$

76. $f(x) = x + \dfrac{32}{x^2}$

$f'(x) = 1 - \dfrac{64}{x^3}$

$f''(x) = \dfrac{192}{x^4}$

77. $f(x) = 3 \sin x$

$f'(x) = 3 \cos x$

$f''(x) = -3 \sin x$

78. $f(x) = \sec x$

$f'(x) = \sec x \tan x$

$f''(x) = \sec x(\sec^2 x) + \tan x(\sec x \tan x)$

$= \sec x(\sec^2 x + \tan^2 x)$

79. $f'(x) = x^2$

$f''(x) = 2x$

80. $f''(x) = 2 - 2x^{-1}$

$f'''(x) = 2x^{-2} = \dfrac{2}{x^2}$

81. $f'''(x) = 2\sqrt{x}$

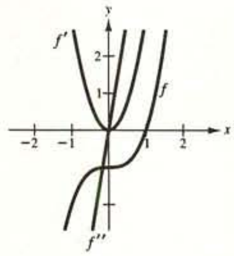
$f^{(4)}(x) = \dfrac{1}{2}(2)x^{-1/2} = \dfrac{1}{\sqrt{x}}$

82. $f^{(4)}(x) = 2x + 1$

$f^{(5)}(x) = 2$

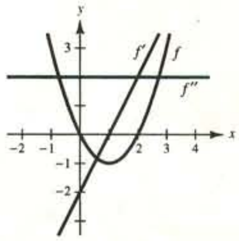
$f^{(6)}(x) = 0$

83.



It appears that f is cubic; so f' would be quadratic and f'' would be linear.

84.



It appears that f is quadratic; so f' would be linear and f'' would be constant.

85. $f(x) = g(x)h(x)$

(a) $f'(x) = g(x)h'(x) + h(x)g'(x)$

$f''(x) = g(x)h''(x) + g'(x)h'(x) + h(x)g''(x) + h'(x)g'(x)$

$= g(x)h''(x) + 2g'(x)h'(x) + h(x)g''(x)$

$f'''(x) = g(x)h'''(x) + g'(x)h''(x) + 2g'(x)h''(x) + 2g''(x)h'(x) + h(x)g'''(x) + h'(x)g''(x)$

$= g(x)h'''(x) + 3g'(x)h''(x) + 3g''(x)h'(x) + g'''(x)h(x)$

$f^{(4)}(x) = g(x)h^{(4)}(x) + g'(x)h'''(x) + 3g'(x)h'''(x) + 3g''(x)h''(x) + 3g''(x)h''(x) + 3g'''(x)h'(x)$

$+ g'''(x)h'(x) + g^{(4)}(x)h(x)$

$= g(x)h^{(4)}(x) + 4g'(x)h'''(x) + 6g''(x)h''(x) + 4g'''(x)h'(x) + g^{(4)}(x)h(x)$

—CONTINUED—

$$+\frac{n(n-1)(n-2)\cdots(2)(1)}{[(n-1)(n-2)\cdots(2)(1)](1)}g^{(n-1)}(x)h'(x)+g^{(n)}(x)h(x)$$

$$=g(x)h^{(n)}(x)+\frac{n!}{1!(n-1)!}g'(x)h^{(n-1)}(x)+\frac{n!}{2!(n-2)!}g''(x)h^{(n-2)}(x)+\cdots$$

$$+\frac{n!}{(n-1)!1!}g^{(n-1)}(x)h'(x)+g^{(n)}(x)h(x)$$

Note: $n! = n(n-1)\ldots 3 \cdot 2 \cdot 1$ (read "n factorial.")

86. (a) $f(x) = x^n$

$f^n(x) = n(n-1)(n-2)\cdots(2)(1) = n!$

(b) $f(x) = \frac{1}{x}$

$$f^{(n)}(x) = \frac{(-1)^n(n)(n-1)(n-2)\cdots(2)(1)}{x^{n+1}} = \frac{(-1)^n n!}{x^{n+1}}$$

Note: $n! = n(n-1)\cdots 3 \cdot 2 \cdot 1$ (read "n factorial.")

87. $v(t) = 36 - t^2,\ 0 \le t \le 6$

$a(t) = -2t$

$v(3) = 27$ m/sec

$a(3) = -6$ m/sec

The speed of the object is decreasing, but the rate of the decrease is increasing.

88. $v(t) = \frac{100t}{2t+15}$

$$a(t) = \frac{(2t+15)(100)-(100t)(2)}{(2t+15)^2} = \frac{1500}{(2t+15)^2}$$

(a) $a(5) = \frac{1500}{[2(5)+15]^2} = 2.4 \text{ ft/sec}^2$

(b) $a(10) = \frac{1500}{[2(10)+15]^2} \approx 1.2 \text{ ft/sec}^2$

(c) $a(20) = \frac{1500}{[2(20)+15]^2} \approx 0.5 \text{ ft/sec}^2$

89. $s(t) = -8.25t^2 + 66t$

$v(t) = -16.50t + 66$

$a(t) = -16.50$

t(sec)	0	1	2	3	4
$s(t)$ (ft)	0	57.75	99	123.75	132
$v(t) = s'(t)$ (ft/sec)	66	49.5	33	16.5	0
$a(t) = v'(t)$ (ft/sec^2)	−16.5	−16.5	−16.5	−16.5	−16.5

Average velocity on:

$[0, 1]$ is $\frac{57.75-0}{1-0} = 57.75.$

$[1, 2]$ is $\frac{99-57.75}{2-1} = 41.25.$

$[2, 3]$ is $\frac{123.75-99}{3-2} = 24.75.$

$[3, 4]$ is $\frac{132-123.75}{4-3} = 8.25.$

90. $s = -\frac{27}{10}t^2 + 27t + 6$

(a) $v(t) = -\frac{27}{5}t + 27$ ft/sec

$a(t) = -\frac{27}{5} = -5.4 \text{ ft/sec}^2$

(b) $-\frac{27}{5}t + 27 = 0$ when $t = 5$ seconds.

$s(5) = 73.5$ feet

(c) On earth, $a = -32 \text{ ft/sec}^2$.

91. $f(x) = \cos x$ $\quad f\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$f'(x) = -\sin x$ $\quad f'\left(\frac{\pi}{3}\right) = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$