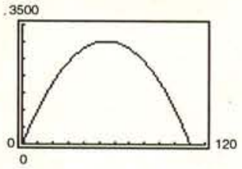
1. —CONTINUED—

(c) $P = x(110 - x) = 110x - x^2$

(d)



The solution appears to be $x = 55$.

(e) $\frac{dP}{dx} = 110 - 2x = 0$ when $x = 55$.

$\frac{d^2P}{dx^2} = -2 < 0$

P is a maximum when $x = 110 - x = 55$.

2. Let x and y be two positive numbers such that $x + y = S$.

$P = xy = x(S - x) = Sx - x^2$

$\frac{dP}{dx} = S - 2x = 0$ when $x = \frac{S}{2}$.

$\frac{d^2P}{dx^2} = -2 < 0$ when $x = \frac{S}{2}$.

P is a maximum when $x = y = S/2$.

3. Let x and y be two positive numbers such that $xy = 192$.

$S = x + y = x + \frac{192}{x}$

$\frac{dS}{dx} = 1 - \frac{192}{x^2} = 0$ when $x = \sqrt{192}$.

$\frac{d^2S}{dx^2} = \frac{384}{x^3} > 0$ when $x = \sqrt{192}$.

S is a minimum when $x = y = \sqrt{192}$.

4. Let x and y be two positive numbers such that $xy = 192$.

$S = x + 3y = \frac{192}{y} + 3y$

$\frac{dS}{dy} = 3 - \frac{192}{y^2} = 0$ when $y = 8$.

$\frac{d^2S}{dy^2} = \frac{384}{y^3} > 0$ when $y = 8$.

S is minimum when $y = 8$ and $x = 24$.

5. Let x be a positive number.

$S = x + \frac{1}{x}$

$\frac{dS}{dx} = 1 - \frac{1}{x^2} = 0$ when $x = 1$.

$\frac{d^2S}{dx^2} = \frac{2}{x^3} > 0$ when $x = 1$.

The sum is a minimum when $x = 1$ and $1/x = 1$.

6. Let x and y be two positive numbers such that $x + 2y = 100$.

$P = xy = y(100 - 2y) = 100y - 2y^2$

$\frac{dP}{dy} = 100 - 4y = 0$ when $y = 25$.

$\frac{d^2P}{dy^2} = -4 < 0$ when $y = 25$.

P is a maximum when $x = 50$ and $y = 25$.

7. Let x be the length and y the width of the rectangle.

$2x + 2y = 100$

$y = 50 - x$

$A = xy = x(50 - x)$

$\frac{dA}{dx} = 50 - 2x = 0$ when $x = 25$.

$\frac{d^2A}{dx^2} = -2 < 0$ when $x = 25$.

A is maximum when $x = y = 25$ meters.

8. Let x be the length and y the width of the rectangle.

$$2x + 2y = P$$

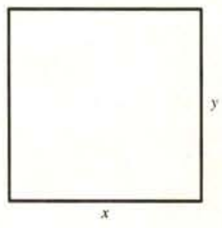
$$y = \frac{P - 2x}{2} = \frac{P}{2} - x$$

$$A = xy = x\left(\frac{P}{2} - x\right) = \frac{P}{2}x - x^2$$

$$\frac{dA}{dx} = \frac{P}{2} - 2x = 0 \text{ when } x = \frac{P}{4}.$$

$$\frac{d^2A}{dx^2} = -2 < 0 \text{ when } x = \frac{P}{4}.$$

A is maximum when $x = y = P/4$ units. (A square!)



9. Let x be the length and y the width of the rectangle.

$$xy = 64$$

$$y = \frac{64}{x}$$

$$P = 2x + 2y = 2x + 2\left(\frac{64}{x}\right) = 2x + \frac{128}{x}$$

$$\frac{dP}{dx} = 2 - \frac{128}{x^2} = 0 \text{ when } x = 8.$$

$$\frac{d^2P}{dx^2} = \frac{256}{x^3} > 0 \text{ when } x = 8.$$

P is minimum when $x = y = 8$ feet.

10. Let x be the length and y the width of the rectangle.

$$xy = A$$

$$y = \frac{A}{x}$$

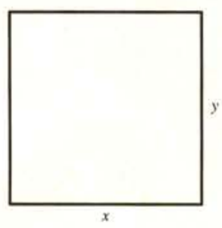
$$P = 2x + 2y = 2x + 2\left(\frac{A}{x}\right) = 2x + \frac{2A}{x}$$

$$\frac{dP}{dx} = 2 - \frac{2A}{x^2} = 0 \text{ when } x = \sqrt{A}.$$

$$\frac{d^2P}{dx^2} = \frac{4A}{x^3} > 0 \text{ when } x = \sqrt{A}.$$

P is minimum when $x = y = \sqrt{A}$ centimeters.

(A square!)



11. $d = \sqrt{(x - 4)^2 + (\sqrt{x} - 0)^2}$

$= \sqrt{x^2 - 7x + 16}$

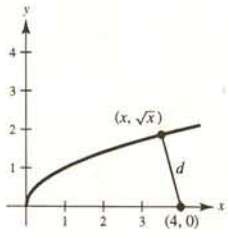
Since d is smallest when the expression inside the radical is smallest, you need only find the critical numbers of

$$f(x) = x^2 - 7x + 16.$$

$$f'(x) = 2x - 7 = 0$$

$$x = \tfrac{7}{2}$$

By the First Derivative Test, the point nearest to $(4, 0)$ is $(7/2, \sqrt{7/2})$.



12. $d = \sqrt{(x - 2)^2 + [x^2 - (1/2)]^2}$

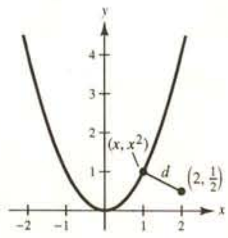
$= \sqrt{x^4 - 4x + (17/4)}$

Since d is smallest when the expression inside the radical is smallest, you need only find the critical numbers of $f(x) = x^4 - 4x + \frac{17}{4}$.

$$f'(x) = 4x^3 - 4 = 0$$

$$x = 1$$

By the First Derivative Test, the point nearest to $\left(2, \frac{1}{2}\right)$ is $(1, 1)$.



13. $\frac{dQ}{dx} = kx(Q_0 - x) = kQ_0x - kx^2$

$\frac{d^2Q}{dx^2} = kQ_0 - 2kx$

$= k(Q_0 - 2x) = 0$ when $x = \frac{Q_0}{2}$.

$\frac{d^3Q}{dx^3} = -2k < 0$ when $x = \frac{Q_0}{2}$.

dQ/dx is maximum when $x = Q_0/2$.

14. $F = \frac{v}{22 + 0.02v^2}$

$\frac{dF}{dv} = \frac{22 - 0.02v^2}{(22 + 0.02v^2)^2}$

$= 0$ when $v = \sqrt{1100} \approx 33.166$.

By the First Derivative Test, the flow rate on the road is maximized when $v \approx 33$ mph.

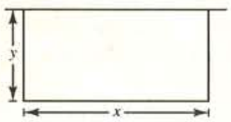
15. $xy = 180{,}000$ (see figure)

$S = x + 2y = \left(x + \frac{360{,}000}{x}\right)$ where S is the length of fence needed.

$\frac{dS}{dx} = 1 - \frac{360{,}000}{x^2} = 0$ when $x = 600$.

$\frac{d^2S}{dx^2} = \frac{720{,}000}{x^3} > 0$ when $x = 600$.

S is a minimum when $x = 600$ meters and $y = 300$ meters.



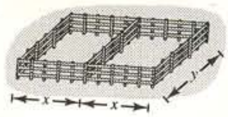
16. $4x + 3y = 200$ is the perimeter. (see figure)

$A = 2xy = 2x\left(\frac{200 - 4x}{3}\right) = \frac{8}{3}(50x - x^2)$

$\frac{dA}{dx} = \frac{8}{3}(50 - 2x) = 0$ when $x = 25$.

$\frac{d^2A}{dx^2} = -\frac{16}{3} < 0$ when $x = 25$.

A is a maximum when $x = 25$ feet and $y = \frac{100}{3}$ feet.



17. (a) $A = 4$(area of side) $+ 2$(area of Top)

(a) $A = 4(3)(11) + 2(3)(3) = 150$ square inches

(b) $A = 4(5)(5) + 2(5)(5) = 150$ square inches

(c) $A = 4(3.25)(6) + 2(6)(6) = 150$ square inches

(b) $V =$ (length)(width)(height)

(a) $V = (3)(3)(11) = 99$ cubic inches

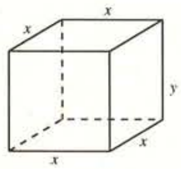
(b) $V = (5)(5)(5) = 125$ cubic inches

(c) $V = (6)(6)(3.25) = 117$ cubic inches

(c) $S = 4xy + 2x^2 = 150 \Rightarrow y = \frac{150 - 2x^2}{4x}$

$V = x^2y = x^2\left(\frac{150 - 2x^2}{4x}\right) = \frac{75}{2}x - \frac{1}{2}x^3$

$V' = \frac{75}{2} - \frac{3}{2}x^2 = 0 \Rightarrow x = \pm 5$



By the First Derivative Test, $x = 5$ yields the maximum volume. Dimensions: $5 \times 5 \times 5$. (A cube!)

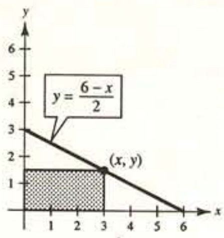
22. You can see from the figure that $A = xy$ and $y = \dfrac{6 - x}{2}$.

$$A = x\left(\frac{6 - x}{2}\right) = \frac{1}{2}(6x - x^2).$$

$$\frac{dA}{dx} = \frac{1}{2}(6 - 2x) = 0 \text{ when } x = 3.$$

$$\frac{d^2A}{dx^2} = -1 < 0 \text{ when } x = 3.$$

A is a maximum when $x = 3$ and $y = 3/2$.



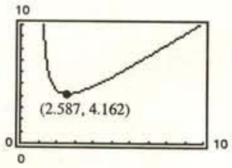
23. (a) $\dfrac{y - 2}{0 - 1} = \dfrac{0 - 2}{x - 1}$

$$y = 2 + \frac{2}{x - 1}$$

$$L = \sqrt{x^2 + y^2} = \sqrt{x^2 + \left(2 + \frac{2}{x - 1}\right)^2}$$

$$= \sqrt{x^2 + 4 + \frac{8}{x - 1} + \frac{4}{(x - 1)^2}}, \quad x > 1$$

(b)



L is minimum when $x \approx 2.587$ and $L \approx 4.162$.

(c) $\text{Area} = \dfrac{1}{2}(\text{base})(\text{height})$

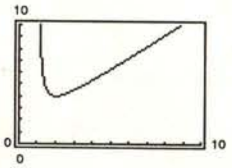
$$= \frac{1}{2}(x)(y)$$

$$= \frac{1}{2}(x)\left(2 + \frac{2}{x + 1}\right)$$

$$= x + \frac{x}{x - 1}$$

Use a graphing utility to approximate x:

$$y = x + \frac{x}{x - 1}.$$



Area is minimum when $x = 2$ and $y = 4$.
Vertices: $(0, 0)$, $(2, 0)$, and $(0, 4)$

24. $A = \dfrac{1}{2}\text{base} \times \text{height}$

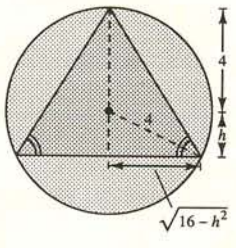
$$= \frac{1}{2}\left(2\sqrt{16 - h^2}\right)(4 + h) = \sqrt{16 - h^2}(4 + h)$$

$$\frac{dA}{dh} = \frac{1}{2}(16 - h^2)^{-1/2}(-2h)(4 + h) + (16 - h^2)^{1/2}$$

$$= (16 - h^2)^{-1/2}[-h(4 + h) + (16 - h^2)]$$

$$= \frac{-2[h^2 + 2h - 8]}{\sqrt{16 - h^2}} = \frac{-2(h + 4)(h - 2)}{\sqrt{16 - h^2}}$$

$dA/dh = 0$ when $h = 2$, which is a maximum by the First Derivative Test. Hence, the sides of the equilateral triangle are $2\sqrt{16 - h^2} = 2\sqrt{16 - 4} = 4\sqrt{3}$.



25. $A = 2xy = 2x\sqrt{25 - x^2}$ (see figure)

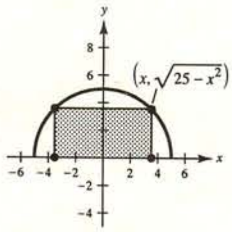
$$\frac{dA}{dx} = 2x\left(\frac{1}{2}\right)\left(\frac{-2x}{\sqrt{25 - x^2}}\right) + 2\sqrt{25 - x^2}$$

$$= 2\left(\frac{25 - 2x^2}{\sqrt{25 - x^2}}\right) = 0 \text{ when } x = y = \frac{5\sqrt{2}}{2} \approx 3.54.$$

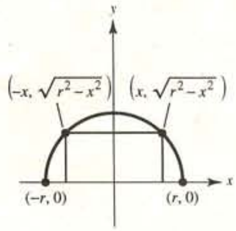
By the First Derivative Test, the inscribed rectangle of maximum area has vertices

$$\left(\pm\frac{5\sqrt{2}}{2}, 0\right), \left(\pm\frac{5\sqrt{2}}{2}, \frac{5\sqrt{2}}{2}\right).$$

Width: $\dfrac{5\sqrt{2}}{2}$; Length: $5\sqrt{2}$



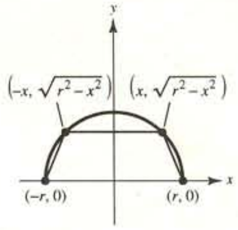
26. $A = 2xy = 2x\sqrt{r^2 - x^2}$ (see figure)



$$\frac{dA}{dx} = \frac{2(r^2 - 2x^2)}{\sqrt{r^2 - x^2}} = 0 \text{ when } x = \frac{\sqrt{2}r}{2}.$$

By the First Derivative Test, A is maximum when the rectangle has dimensions $\sqrt{2}r$ by $(\sqrt{2}r)/2$.

27. $A = \frac{1}{2}(2r + 2x)\sqrt{r^2 - x^2} = (r + x)\sqrt{r^2 - x^2}$ (see figure)



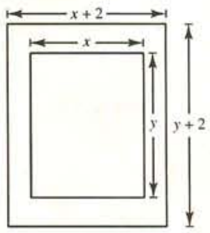
$$\frac{dA}{dx} = (r + x)\left(\frac{1}{2}\right)(r^2 - x^2)^{-1/2}(-2x) + \sqrt{r^2 - x^2}$$

$$= \frac{-x(r + x)}{\sqrt{r^2 - x^2}} + \sqrt{r^2 - x^2} = \frac{r^2 - rx - 2x^2}{\sqrt{r^2 - x^2}}$$

$$= \frac{(r - 2x)(r + x)}{\sqrt{r^2 - x^2}} = 0 \text{ when } x = \frac{r}{2}.$$

By the First Derivative Test, A will be a maximum when the trapezoid bases are r and $2r$, and the altitude is $(\sqrt{3}r)/2$.

28. $xy = 30 \Rightarrow y = \frac{30}{x}$



$$A = (x + 2)\left(\frac{30}{x} + 2\right) \quad \text{(see figure)}$$

$$\frac{dA}{dx} = (x + 2)\left(\frac{-30}{x^2}\right) + \left(\frac{30}{x} + 2\right) = \frac{2(x^2 - 30)}{x^2} = 0 \text{ when } x = \sqrt{30}.$$

$$y = \frac{30}{\sqrt{30}} = \sqrt{30}$$

By the First Derivative Test, the dimensions $(x + 2)$ by $(y + 2)$ are $(2 + \sqrt{30})$ by $(2 + \sqrt{30})$ (approximately 7.477 by 7.477). These dimensions yield a minimum area.

30. $V = \pi r^2 h = V_0$ cubic units or $h = \dfrac{V_0}{\pi r^2}$

$$S = 2\pi r^2 + 2\pi rh = 2\left(\pi r^2 + \frac{V_0}{r}\right)$$

$$\frac{dS}{dr} = 2\left(2\pi r - \frac{V_0}{r^2}\right) = 0 \text{ when } r = \sqrt[3]{\frac{V_0}{2\pi}} \text{ units.}$$

$$h = \frac{V_0}{\pi\left(\sqrt[3]{V_0/2\pi}\right)^2} = \frac{V_0(2\pi)^{2/3}}{\pi V_0^{2/3}} = \frac{2V_0^{1/3}}{(2\pi)^{1/3}} = 2r$$

By the First Derivative Test, this will yield the minimum surface area.

31. Let x be the sides of the square ends and y the length of the package.

$$P = 4x + y = 108 \implies y = 108 - 4x$$

$$V = x^2y = x^2(108 - 4x) = 108x^2 - 4x^3$$

$$\frac{dV}{dx} = 216x - 12x^2$$

$$= 12x(18 - x) = 0 \text{ when } x = 18.$$

$$\frac{d^2V}{dx^2} = 216 - 24x = -216 < 0 \text{ when } x = 18.$$

The volume is maximum when $x = 18$ inches and $y = 108 - 4(18) = 36$ inches.

32. $V = \pi r^2 x$

$$x + 2\pi r = 108 \implies x = 108 - 2\pi r \quad \text{(see figure)}$$

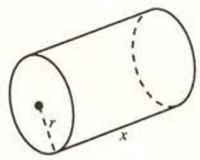
$$V = \pi r^2(108 - 2\pi r) = \pi(108r^2 - 2\pi r^3)$$

$$\frac{dV}{dr} = \pi(216r - 6\pi r^2) = 6\pi r(36 - \pi r)$$

$$= 0 \text{ when } r = \frac{36}{\pi} \text{ and } x = 36.$$

$$\frac{d^2V}{dr^2} = \pi(216 - 12\pi r) < 0 \text{ when } r = \frac{36}{\pi}.$$

Volume is maximum when $x = 36$ inches and $r = 36/\pi \approx 11.459$ inches.



33. $V = \dfrac{1}{3}\pi x^2 h = \dfrac{1}{3}\pi x^2\left(r + \sqrt{r^2 - x^2}\right)$ (see figure)

$$\frac{dV}{dx} = \frac{1}{3}\pi\left[\frac{-x^3}{\sqrt{r^2 - x^2}} + 2x\left(r + \sqrt{r^2 - x^2}\right)\right] = \frac{\pi x}{3\sqrt{r^2 - x^2}}\left(2r^2 + 2r\sqrt{r^2 - x^2} - 3x^2\right) = 0$$

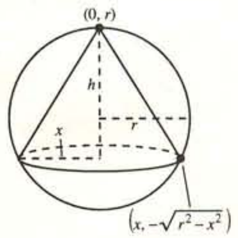
$$2r^2 + 2r\sqrt{r^2 - x^2} - 3x^2 = 0$$

$$2r\sqrt{r^2 - x^2} = 3x^2 - 2r^2$$

$$4r^2(r^2 - x^2) = 9x^4 - 12x^2r^2 + 4r^4$$

$$0 = 9x^4 - 8x^2r^2 = x^2(9x^2 - 8r^2)$$

$$x = 0, \frac{2\sqrt{2}r}{3}$$



By the First Derivative Test, the volume is a maximum when

$$x = \frac{2\sqrt{2}r}{3} \text{ and } h = r + \sqrt{r^2 - x^2} = \frac{4r}{3}.$$

Thus, the maximum volume is

$$V = \frac{1}{3}\pi\left(\frac{8r^2}{9}\right)\left(\frac{4r}{3}\right) = \frac{32\pi r^3}{81} \text{ cubic units.}$$