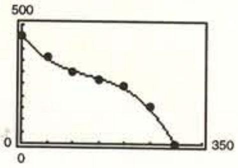
69. (a) $y = (-4.09 \times 10^{-5})x^3 + 0.016x^2 - 2.67x + 452.9$

(b)



(c) Using the integration capability of a graphing utility, you obtain

$A \approx 76{,}897.5 \text{ ft}^2$.

Section 4.3 Riemann Sums and Definite Integrals

1. $\displaystyle\int_0^5 3\,dx$

2. $\displaystyle\int_0^2 (4 - 2x)\,dx$

3. $\displaystyle\int_{-4}^4 (4 - |x|)\,dx$

4. $\displaystyle\int_0^2 x^2\,dx$

5. $\displaystyle\int_{-2}^2 (4 - x^2)\,dx$

6. $\displaystyle\int_{-1}^1 \frac{1}{x^2 + 1}\,dx$

7. $\displaystyle\int_0^\pi \sin x\,dx$

8. $\displaystyle\int_0^{\pi/4} \tan x\,dx$

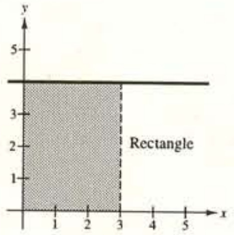
9. $\displaystyle\int_0^2 y^3\,dy$

10. $\displaystyle\int_0^2 (y - 2)^2\,dy$

11. Rectangle

$A = bh = 3(4)$

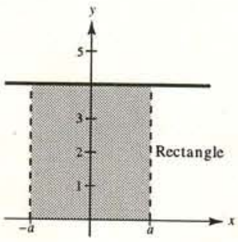
$A = \displaystyle\int_0^3 4\,dx = 12$



12. Rectangle

$A = bh = 2(4)(a)$

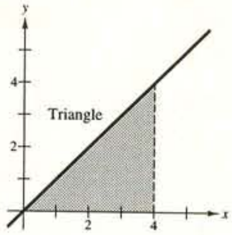
$A = \displaystyle\int_{-a}^a 4\,dx = 8a$



13. Triangle

$A = \frac{1}{2}bh = \frac{1}{2}(4)(4)$

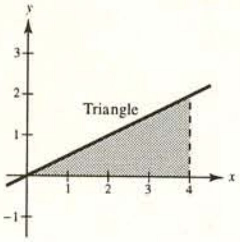
$A = \displaystyle\int_0^4 x\,dx = 8$



14. Triangle

$A = \frac{1}{2}bh = \frac{1}{2}(4)(2)$

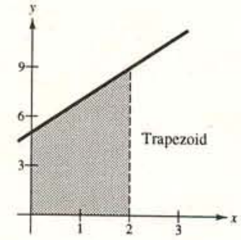
$A = \displaystyle\int_0^4 \frac{x}{2}\,dx = 4$



15. Trapezoid

$A = \dfrac{b_1 + b_2}{2}h = \left(\dfrac{5 + 9}{2}\right)2$

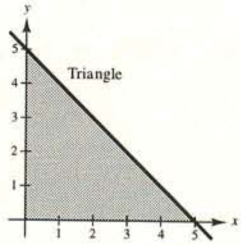
$A = \displaystyle\int_0^2 (2x + 5)\,dx = 14$



16. Triangle

$A = \frac{1}{2}bh = \frac{1}{2}(5)(5)$

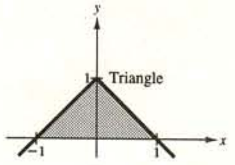
$A = \displaystyle\int_0^5 (5 - x)\,dx = \frac{25}{2}$



17. Triangle

$$A = \frac{1}{2}bh = \frac{1}{2}(2)(1)$$

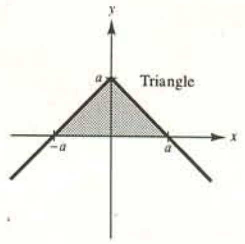
$$A = \int_{-1}^{1} (1 - |x|)\, dx = 1$$



18. Triangle

$$A = \frac{1}{2}bh = \frac{1}{2}(2a)a$$

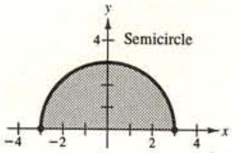
$$A = \int_{-a}^{a} (a - |x|)\, dx = a^2$$



19. Semicircle

$$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(3)^2$$

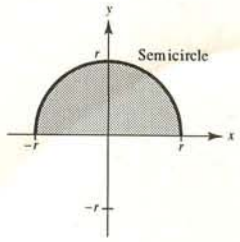
$$A = \int_{-3}^{3} \sqrt{9 - x^2}\, dx = \frac{9\pi}{2}$$



20. Semicircle

$$A = \frac{1}{2}\pi r^2$$

$$A = \int_{-r}^{r} \sqrt{r^2 - x^2}\, dx = \frac{1}{2}\pi r^2$$



21. (a) $\int_0^7 f(x)\, dx = \int_0^5 f(x)\, dx + \int_5^7 f(x)\, dx = 10 + 3 = 13$

(b) $\int_5^0 f(x)\, dx = -\int_0^5 f(x)\, dx = -10$

(c) $\int_5^5 f(x)\, dx = 0$

(d) $\int_0^5 3f(x)\, dx = 3\int_0^5 f(x)\, dx = 3(10) = 30$

22. (a) $\int_0^6 f(x)\, dx = \int_0^3 f(x)\, dx + \int_3^6 f(x)\, dx = 4 + (-1) = 3$

(b) $\int_6^3 f(x)\, dx = -\int_3^6 f(x)\, dx = -(-1) = 1$

(c) $\int_3^3 f(x)\, dx = 0$

(d) $\int_3^6 -5f(x)\, dx = -5\int_3^6 f(x)\, dx = -5(-1) = 5$

23. (a) $\int_2^6 [f(x) + g(x)]\, dx = \int_2^6 f(x)\, dx + \int_2^6 g(x)\, dx$

$= 10 + (-2) = 8$

(b) $\int_2^6 [g(x) - f(x)]\, dx = \int_2^6 g(x)\, dx - \int_2^6 f(x)\, dx$

$= -2 - 10 = -12$

(c) $\int_2^6 2g(x)\, dx = 2\int_2^6 g(x)\, dx = 2(-2) = -4$

(d) $\int_2^6 3f(x)\, dx = 3\int_2^6 f(x)\, dx = 3(10) = 30$

24. (a) $\int_{-1}^0 f(x)\, dx = \int_{-1}^1 f(x)\, dx - \int_0^1 f(x)\, dx = 0 - 5 = -5$

(b) $\int_0^1 f(x)\, dx - \int_1^0 f(x)\, dx = 5 - (-5) = 10$

(c) $\int_{-1}^1 3f(x)\, dx = 3\int_{-1}^1 f(x)\, dx = 3(0) = 0$

(d) $\int_0^1 3f(x)\, dx = 3\int_0^1 f(x)\, dx = 3(5) = 15$

25. $y = 6$ on $[4, 10]$. $\left(\textbf{Note: } \Delta x = \frac{10-4}{n} = \frac{6}{n}, \|\Delta\| \to 0 \text{ as } n \to \infty\right)$

$$\sum_{i=1}^{n} f(c_i)\,\Delta x_i = \sum_{i=1}^{n} f\left(4 + \frac{6i}{n}\right)\left(\frac{6}{n}\right) = \sum_{i=1}^{n} 6\left(\frac{6}{n}\right) = \sum_{i=1}^{n} \frac{36}{n} = 36$$

$$\int_4^{10} 6\, dx = \lim_{n\to\infty} 36 = 36$$

43. (a) Quarter circle below x-axis: $-\frac{1}{4}\pi r^2 = -\frac{1}{4}\pi(2)^2 = -\pi$

(b) Triangle: $\frac{1}{2}bh = \frac{1}{2}(4)(2) = 4$

(c) Triangle + Semicircle below x-axis: $-\frac{1}{2}(2)(1) - \frac{1}{2}\pi(2)^2 = -(1 + 2\pi)$

(d) Sum of parts (b) and (c): $4 - (1 + 2\pi) = 3 - 2\pi$

(e) Sum of absolute values of (b) and (c): $4 + (1 + 2\pi) = 5 + 2\pi$

(f) Answer to (d) plus $2(10) = 20$: $(3 - 2\pi) + 20 = 23 - 2\pi$

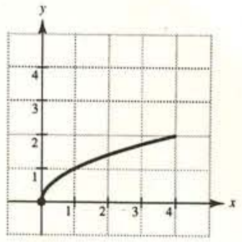
44. (a) $\int_0^5 [f(x) + 2]\,dx = \int_0^5 f(x)\,dx + \int_0^5 2\,dx = 4 + 10 = 14$

(b) $\int_{-2}^3 f(x + 2)\,dx = \int_0^5 f(x)\,dx = 4$ (Let $u = x + 2$.)

(c) $\int_{-5}^5 f(x)\,dx = 2\int_0^5 f(x)\,dx = 2(4) = 8$ (f even)

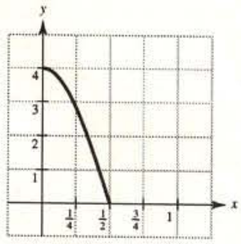
(d) $\int_{-5}^5 f(x)\,dx = 0$ (f odd)

45.



a. $A \approx 5$ square units

46.



b. $A \approx \frac{4}{3}$ square units

47. True

48. False

$$\int_0^1 x\sqrt{x}\,dx \neq \left(\int_0^1 x\,dx\right)\left(\int_0^1 \sqrt{x}\,dx\right)$$

49. True

50. True

51. False

$$\int_0^2 (-x)\,dx = -2$$

52. False

$$\int_{-2}^4 x\,dx = 6$$

53. $f(x) = x^2 + 3x$, $[0, 8]$

$x_0 = 0, x_1 = 1, x_2 = 3, x_3 = 7, x_4 = 8$

$\Delta x_1 = 1, \Delta x_2 = 2, \Delta x_3 = 4, \Delta x_4 = 1$

$c_1 = 1, c_2 = 2, c_3 = 5, c_4 = 8$

$$\sum_{i=1}^4 f(c_i)\,\Delta x = f(1)\,\Delta x_1 + f(2)\,\Delta x_2 + f(5)\,\Delta x_3 + f(8)\,\Delta x_4$$

$$= (4)(1) + (10)(2) + (40)(4) + (88)(1) = 272$$