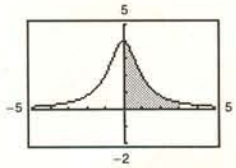
Section 4.4 The Fundamental Theorem of Calculus

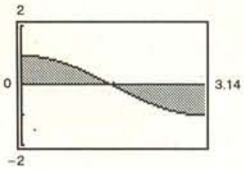
1. $f(x) = \dfrac{4}{x^2+1}$

$\displaystyle\int_0^{\pi} \frac{4}{x^2+1}\,dx$ is positive.



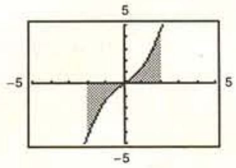
2. $f(x) = \cos x$

$\displaystyle\int_0^{\pi} \cos x\,dx = 0$



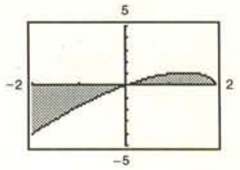
3. $f(x) = x\sqrt{x^2+1}$

$\displaystyle\int_{-2}^{2} x\sqrt{x^2+1}\,dx = 0$



4. $f(x) = x\sqrt{2-x}$

$\displaystyle\int_{-2}^{2} x\sqrt{2-x}\,dx$ is negative.



5. $\displaystyle\int_0^1 2x\,dx = \Big[x^2\Big]_0^1 = 1 - 0 = 1$

6. $\displaystyle\int_2^7 3\,dv = \Big[3v\Big]_2^7 = 3(7) - 3(2) = 15$

7. $\displaystyle\int_{-1}^{0} (x-2)\,dx = \left[\frac{x^2}{2} - 2x\right]_{-1}^{0} = 0 - \left(\frac{1}{2} + 2\right) = -\frac{5}{2}$

8. $\displaystyle\int_2^5 (-3v+4)\,dv = \left[-\frac{3}{2}v^2 + 4v\right]_2^5 = \left(-\frac{75}{2} + 20\right) - (-6+8) = -\frac{39}{2}$

9. $\displaystyle\int_{-1}^{1} (t^2-2)\,dt = \left[\frac{t^3}{3} - 2t\right]_{-1}^{1} = \left(\frac{1}{3} - 2\right) - \left(-\frac{1}{3} + 2\right) = -\frac{10}{3}$

10. $\displaystyle\int_0^3 (3x^2 + x - 2)\,dx = \left[x^3 + \frac{x^2}{2} - 2x\right]_0^3 = \left(27 + \frac{9}{2} - 6\right) - 0 = \frac{51}{2}$

11. $\displaystyle\int_0^1 (2t-1)^2\,dt = \int_0^1 (4t^2 - 4t + 1)\,dt = \left[\frac{4}{3}t^3 - 2t^2 + t\right]_0^1 = \frac{4}{3} - 2 + 1 = \frac{1}{3}$

12. $\displaystyle\int_{-1}^{1} (t^3 - 9t)\,dt = \left[\frac{1}{4}t^4 - \frac{9}{2}t^2\right]_{-1}^{1} = \left(\frac{1}{4} - \frac{9}{2}\right) - \left(\frac{1}{4} - \frac{9}{2}\right) = 0$

13. $\displaystyle\int_1^2 \left(\frac{3}{x^2} - 1\right)dx = \left[-\frac{3}{x} - x\right]_1^2 = \left(-\frac{3}{2} - 2\right) - (-3 - 1) = \frac{1}{2}$

14. $\displaystyle\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right)du = \left[\frac{u^2}{2} + \frac{1}{u}\right]_{-2}^{-1} = \left(\frac{1}{2} - 1\right) - \left(2 - \frac{1}{2}\right) = -2$

15. $\displaystyle\int_1^4 \frac{u-2}{\sqrt{u}}\,du = \int_1^4 (u^{1/2} - 2u^{-1/2})\,du = \left[\frac{2}{3}u^{3/2} - 4u^{1/2}\right]_1^4 = \left[\frac{2}{3}(\sqrt{4})^3 - 4\sqrt{4}\right] - \left[\frac{2}{3} - 4\right] = \frac{2}{3}$

16. $\displaystyle\int_{-3}^{3} v^{1/3}\,dv = \left[\frac{3}{4}v^{4/3}\right]_{-3}^{3} = \frac{3}{4}\left[(\sqrt[3]{-3})^4 - (\sqrt[3]{-3})^4\right] = 0$

17. $\int_{-1}^{1} (\sqrt[3]{t} - 2)\, dt = \left[\frac{3}{4}t^{4/3} - 2t\right]_{-1}^{1} = \left(\frac{3}{4} - 2\right) - \left(\frac{3}{4} + 2\right) = -4$

18. $\int_{1}^{8} \sqrt{\frac{2}{x}}\, dx = \sqrt{2}\int_{1}^{8} x^{-1/2}\, dx = \left[\sqrt{2}(2)x^{1/2}\right]_{1}^{8} = \left[2\sqrt{2x}\right]_{1}^{8} = 8 - 2\sqrt{2}$

19. $\int_{0}^{1} \frac{x - \sqrt{x}}{3}\, dx = \frac{1}{3}\int_{0}^{1} (x - x^{1/2})\, dx = \frac{1}{3}\left[\frac{x^2}{2} - \frac{2}{3}x^{3/2}\right]_{0}^{1} = \frac{1}{3}\left(\frac{1}{2} - \frac{2}{3}\right) = -\frac{1}{18}$

20. $\int_{0}^{2} (2 - t)\sqrt{t}\, dt = \int_{0}^{2} (2t^{1/2} - t^{3/2})\, dt = \left[\frac{4}{3}t^{3/2} - \frac{2}{5}t^{5/2}\right]_{0}^{2} = \left[\frac{t\sqrt{t}}{15}(20 - 6t)\right]_{0}^{2} = \frac{2\sqrt{2}}{15}(20 - 12) = \frac{16\sqrt{2}}{15}$

21. $\int_{-1}^{0} (t^{1/3} - t^{2/3})\, dt = \left[\frac{3}{4}t^{4/3} - \frac{3}{5}t^{5/3}\right]_{-1}^{0} = 0 - \left(\frac{3}{4} + \frac{3}{5}\right) = -\frac{27}{20}$

22. $\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\, dx = \frac{1}{2}\int_{-8}^{-1} (x^{2/3} - x^{5/3})\, dx$

$= \frac{1}{2}\left[\frac{3}{5}x^{5/3} - \frac{3}{8}x^{8/3}\right]_{-8}^{-1} = \left[\frac{x^{5/3}}{80}(24 - 15x)\right]_{-8}^{-1} = -\frac{1}{80}(39) + \frac{32}{80}(144) = \frac{4569}{80}$

23. $\int_{0}^{3} |2x - 3|\, dx = \int_{0}^{3/2} (3 - 2x)\, dx + \int_{3/2}^{3} (2x - 3)\, dx$ (split up the integral at the zero $x = \frac{3}{2}$)

$= \left[3x - x^2\right]_{0}^{3/2} + \left[x^2 - 3x\right]_{3/2}^{3} = \left(\frac{9}{2} - \frac{9}{4}\right) - 0 + (9 - 9) - \left(\frac{9}{4} - \frac{9}{2}\right) = 2\left(\frac{9}{2} - \frac{9}{4}\right) = \frac{9}{2}$

24. $\int_{0}^{4} |x^2 - 4x + 3|\, dx = \int_{0}^{1} (x^2 - 4x + 3)\, dx - \int_{1}^{3} (x^2 - 4x + 3)\, dx + \int_{3}^{4} (x^2 - 4x + 3)\, dx$ (split up the integral at the zeros $x = 1, 3$)

$= \left[\frac{x^3}{3} - 2x^2 + 3x\right]_{0}^{1} - \left[\frac{x^3}{3} - 2x^2 + 3x\right]_{1}^{3} + \left[\frac{x^3}{3} - 2x^2 + 3x\right]_{3}^{4}$

$= \left(\frac{1}{3} - 2 + 3\right) - (9 - 18 + 9) + \left(\frac{1}{3} - 2 + 3\right) + \left(\frac{64}{3} - 32 + 12\right) - (9 - 18 + 9)$

$= \frac{4}{3} - 0 + \frac{4}{3} + \frac{4}{3} - 0 = 4$

25. $\int_{0}^{\pi} (1 + \sin x)\, dx = \left[x - \cos x\right]_{0}^{\pi} = (\pi + 1) - (0 - 1) = 2 + \pi$

26. $\int_{0}^{\pi/4} \frac{1 - \sin^2\theta}{\cos^2\theta}\, d\theta = \int_{0}^{\pi/4} d\theta = \left[\theta\right]_{0}^{\pi/4} = \frac{\pi}{4}$

27. $\int_{-\pi/6}^{\pi/6} \sec^2 x\, dx = \left[\tan x\right]_{-\pi/6}^{\pi/6} = \frac{\sqrt{3}}{3} - \left(-\frac{\sqrt{3}}{3}\right) = \frac{2\sqrt{3}}{3}$

28. $\int_{\pi/4}^{\pi/2} (2 - \csc^2 x)\, dx = \left[2x + \cot x\right]_{\pi/4}^{\pi/2} = (\pi + 0) - \left(\frac{\pi}{2} + 1\right) = \frac{\pi}{2} - 1 = \frac{\pi - 2}{2}$

29. $\int_{-\pi/3}^{\pi/3} 4\sec\theta\tan\theta\, d\theta = \left[4\sec\theta\right]_{-\pi/3}^{\pi/3} = 4(2) - 4(2) = 0$

30. $\int_{-\pi/2}^{\pi/2}(2t + \cos t)\,dt = \left[t^2 + \sin t\right]_{-\pi/2}^{\pi/2} = \left(\frac{\pi^2}{4} + 1\right) - \left(\frac{\pi^2}{4} - 1\right) = 2$

31. $\int_0^3 10{,}000(t - 6)\,dt = 10{,}000\left[\frac{t^2}{2} - 6t\right]_0^3 = -\$135{,}000$

32. $P = \frac{2}{\pi}\int_0^{\pi/2}\sin\theta\,d\theta = \left[-\frac{2}{\pi}\cos\theta\right]_0^{\pi/2} = -\frac{2}{\pi}(0 - 1) = \frac{2}{\pi} \approx 63.7\%$

33. $A = \int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}$

34. $A = \int_{-1}^1 (1 - x^4)\,dx = \left[x - \frac{1}{5}x^5\right]_{-1}^1 = \frac{8}{5}$

35. $A = \int_0^3 (3 - x)\sqrt{x}\,dx = \int_0^3 (3x^{1/2} - x^{3/2})\,dx = \left[2x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^3 = \left[\frac{x\sqrt{x}}{5}(10 - 2x)\right]_0^3 = \frac{12\sqrt{3}}{5}$

36. $A = \int_1^2 \frac{1}{x^2}\,dx = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

37. $A = \int_0^{\pi/2}\cos x\,dx = \left[\sin x\right]_0^{\pi/2} = 1$

38. $A = \int_0^{\pi}(x + \sin x)\,dx = \left[\frac{x^2}{2} - \cos x\right]_0^{\pi} = \frac{\pi^2}{2} + 2 = \frac{\pi^2 + 4}{2}$

39. Since $y \geq 0$ on $[0, 2]$,

$$A = \int_0^2 (3x^2 + 1)\,dx = \left[x^3 + x\right]_0^2 = 8 + 2 = 10.$$

40. Since $y \geq 0$ on $[0, 4]$,

$$A = \int_0^4 \left(1 + \sqrt{x}\right)dx = \left[x + \frac{2}{3}x^{3/2}\right]_0^4$$
$$= 4 + \frac{2}{3}(8) = \frac{28}{3}.$$

41. Since $y \geq 0$ on $[0, 2]$,

$$A = \int_0^2 (x^3 + x)\,dx = \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^2 = 4 + 2 = 6.$$

42. Since $y \geq 0$ on $[0, 3]$,

$$A = \int_0^3 (3x - x^2)\,dx = \left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3 = \frac{9}{2}.$$

43. $\int_0^2 \left(x - 2\sqrt{x}\right)dx = \left[\frac{x^2}{2} - \frac{4x^{3/2}}{3}\right]_0^2 = 2 - \frac{8\sqrt{2}}{3}$

$$f(c)(2 - 0) = \frac{6 - 8\sqrt{2}}{3}$$
$$c - 2\sqrt{c} = \frac{3 - 4\sqrt{2}}{3}$$
$$c - 2\sqrt{c} + 1 = \frac{3 - 4\sqrt{2}}{3} + 1$$
$$\left(\sqrt{c} - 1\right)^2 = \frac{6 - 4\sqrt{2}}{3}$$
$$\sqrt{c} - 1 = \pm\sqrt{\frac{6 - 4\sqrt{2}}{3}}$$
$$c = \left[1 \pm \sqrt{\frac{6 - 4\sqrt{2}}{3}}\right]^2$$
$$c \approx 0.4380 \text{ or } c \approx 1.7908$$

44. $\int_1^3 \frac{9}{x^3}\,dx = \left[-\frac{9}{2x^2}\right]_1^3 = -\frac{1}{2} + \frac{9}{2} = 4$

$$f(c)(3 - 1) = 4$$
$$\frac{9}{c^3} = 2$$
$$c^3 = \frac{9}{2}$$
$$c = \sqrt[3]{\frac{9}{2}} \approx 1.6510$$