30. $\displaystyle\int_{-\pi/2}^{\pi/2} (2t + \cos t)\, dt = \left[t^2 + \sin t\right]_{-\pi/2}^{\pi/2} = \left(\frac{\pi^2}{4} + 1\right) - \left(\frac{\pi^2}{4} - 1\right) = 2$

31. $\displaystyle\int_0^3 10{,}000(t - 6)\, dt = 10{,}000\left[\frac{t^2}{2} - 6t\right]_0^3 = -\$135{,}000$

32. $\displaystyle P = \frac{2}{\pi}\int_0^{\pi/2} \sin\theta\, d\theta = \left[-\frac{2}{\pi}\cos\theta\right]_0^{\pi/2} = -\frac{2}{\pi}(0 - 1) = \frac{2}{\pi} \approx 63.7\%$

33. $\displaystyle A = \int_0^1 (x - x^2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{6}$

34. $\displaystyle A = \int_{-1}^1 (1 - x^4)\, dx = \left[x - \frac{1}{5}x^5\right]_{-1}^1 = \frac{8}{5}$

35. $\displaystyle A = \int_0^3 (3 - x)\sqrt{x}\, dx = \int_0^3 (3x^{1/2} - x^{3/2})\, dx = \left[2x^{3/2} - \frac{2}{5}x^{5/2}\right]_0^3 = \left[\frac{x\sqrt{x}}{5}(10 - 2x)\right]_0^3 = \frac{12\sqrt{3}}{5}$

36. $\displaystyle A = \int_1^2 \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

37. $\displaystyle A = \int_0^{\pi/2} \cos x\, dx = \left[\sin x\right]_0^{\pi/2} = 1$

38. $\displaystyle A = \int_0^{\pi} (x + \sin x)\, dx = \left[\frac{x^2}{2} - \cos x\right]_0^{\pi} = \frac{\pi^2}{2} + 2 = \frac{\pi^2 + 4}{2}$

39. Since $y \geq 0$ on $[0, 2]$,

$$A = \int_0^2 (3x^2 + 1)\, dx = \left[x^3 + x\right]_0^2 = 8 + 2 = 10.$$

40. Since $y \geq 0$ on $[0, 4]$,

$$A = \int_0^4 (1 + \sqrt{x})\, dx = \left[x + \frac{2}{3}x^{3/2}\right]_0^4$$

$$= 4 + \frac{2}{3}(8) = \frac{28}{3}.$$

41. Since $y \geq 0$ on $[0, 2]$,

$$A = \int_0^2 (x^3 + x)\, dx = \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_0^2 = 4 + 2 = 6.$$

42. Since $y \geq 0$ on $[0, 3]$,

$$A = \int_0^3 (3x - x^2)\, dx = \left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3 = \frac{9}{2}.$$

43. $\displaystyle\int_0^2 (x - 2\sqrt{x})\, dx = \left[\frac{x^2}{2} - \frac{4x^{3/2}}{3}\right]_0^2 = 2 - \frac{8\sqrt{2}}{3}$

$$f(c)(2 - 0) = \frac{6 - 8\sqrt{2}}{3}$$

$$c - 2\sqrt{c} = \frac{3 - 4\sqrt{2}}{3}$$

$$c - 2\sqrt{c} + 1 = \frac{3 - 4\sqrt{2}}{3} + 1$$

$$(\sqrt{c} - 1)^2 = \frac{6 - 4\sqrt{2}}{3}$$

$$\sqrt{c} - 1 = \pm\sqrt{\frac{6 - 4\sqrt{2}}{3}}$$

$$c = \left[1 \pm \sqrt{\frac{6 - 4\sqrt{2}}{3}}\right]^2$$

$c \approx 0.4380$ or $c \approx 1.7908$

44. $\displaystyle\int_1^3 \frac{9}{x^3}\, dx = \left[-\frac{9}{2x^2}\right]_1^3 = -\frac{1}{2} + \frac{9}{2} = 4$

$$f(c)(3 - 1) = 4$$

$$\frac{9}{c^3} = 2$$

$$c^3 = \frac{9}{2}$$

$$c = \sqrt[3]{\frac{9}{2}} \approx 1.6510$$

45. $\displaystyle\int_{-\pi/4}^{\pi/4} 2\sec^2 x\,dx = \Big[2\tan x\Big]_{-\pi/4}^{\pi/4} = 2(1) - 2(-1) = 4$

$$f(c)\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = 4$$

$$2\sec^2 c = \frac{8}{\pi}$$

$$\sec^2 c = \frac{4}{\pi}$$

$$\sec c = \pm\frac{2}{\sqrt{\pi}}$$

$$c = \pm\text{arcsec}\left(\frac{2}{\sqrt{\pi}}\right)$$

$$= \pm\arccos\frac{\sqrt{\pi}}{2} \approx \pm 0.4817$$

46. $\displaystyle\int_{-\pi/3}^{\pi/3} \cos x\,dx = \Big[\sin x\Big]_{-\pi/3}^{\pi/3} = \sqrt{3}$

$$f(c)\left[\frac{\pi}{3} - \left(-\frac{\pi}{3}\right)\right] = \sqrt{3}$$

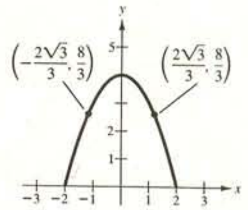
$$\cos c = \frac{3\sqrt{3}}{2\pi}$$

$$c \approx \pm 0.5971$$

47. $\displaystyle\frac{1}{2-(-2)}\int_{-2}^{2}(4 - x^2)\,dx = \frac{1}{4}\left[4x - \frac{1}{3}x^3\right]_{-2}^{2} = \frac{1}{4}\left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right] = \frac{8}{3}$

Average value $= \dfrac{8}{3}$

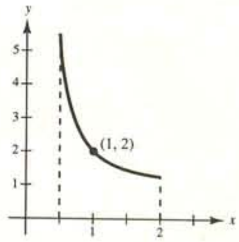
$4 - x^2 = \dfrac{8}{3}$ when $x^2 = 4 - \dfrac{8}{3}$ or $x = \pm\dfrac{2\sqrt{3}}{3} \approx \pm 1.155.$



48. $\displaystyle\frac{1}{2-(1/2)}\int_{1/2}^{2}\frac{x^2+1}{x^2}\,dx = \frac{2}{3}\int_{1/2}^{2}(1 + x^{-2})\,dx = \frac{2}{3}\left[x - \frac{1}{x}\right]_{1/2}^{2} = \frac{2}{3}\left(\frac{3}{2} + \frac{3}{2}\right) = 2$

Average value $= 2$

In the interval $\left[\dfrac{1}{2}, 2\right]$, $\dfrac{x^2+1}{x^2} = 2$ when $x^2 + 1 = 2x^2$ or $x = 1$.

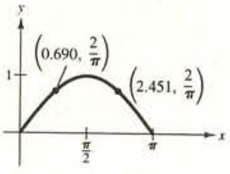


49. $\displaystyle\frac{1}{\pi - 0}\int_0^{\pi}\sin x\,dx = \left[-\frac{1}{\pi}\cos x\right]_0^{\pi} = \frac{2}{\pi}$

Average value $= \dfrac{2}{\pi}$

$$\sin x = \frac{2}{\pi}$$

$$x \approx 0.690,\ 2.451$$

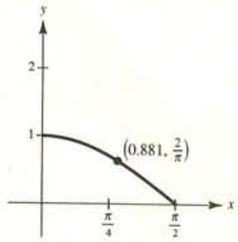


50. $\displaystyle\frac{1}{(\pi/2) - 0}\int_0^{\pi/2}\cos x\,dx = \left[\frac{2}{\pi}\sin x\right]_0^{\pi/2} = \frac{2}{\pi}$

Average value $= \dfrac{2}{\pi}$

$$\cos x = \frac{2}{\pi}$$

$$x \approx 0.881$$



51. $\displaystyle\int_0^2 f(x)\,dx = -(\text{area of region } A) = -1.5$

52. $\displaystyle\int_2^6 f(x)\,dx = (\text{area or region } B) = \int_0^6 f(x)\,dx - \int_0^2 f(x)\,dx$

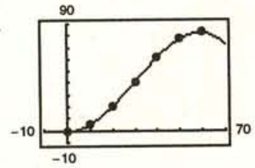
$$= 3.5 - (-1.5) = 5.0$$

53. $\displaystyle\int_0^6 |f(x)|\,dx = -\int_0^2 f(x)\,dx + \int_2^6 f(x)\,dx = 1.5 + 5.0 = 6.5$

54. $\displaystyle\int_0^2 -2f(x)\,dx = -2\int_0^2 f(x)\,dx = -2(-1.5) = 3.0$

65. (a) $v = -8.61 \times 10^{-4}t^3 + 0.0782t^2 - 0.208t + 0.0952$

(b)



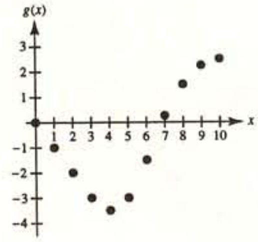
(c) $\int_0^{60} v(t)\,dt = \left[\frac{-8.61 \times 10^{-4}t^4}{4} + \frac{0.0782t^3}{3} - \frac{0.208t^2}{2} + 0.0952t\right]_0^{60} = 2472$ meters

66. (a) $g(0) = \int_0^0 f(t)\,dt = 0$

$g(1) = \int_0^1 f(t)\,dt = -1$ etc.

x	0	1	2	3	4	5	6	7	8	9	10
$g(x)$	0	-1	-2	-3	-3.5	-3	-1.5	0.25	1.5	2.25	2.5

(b)



(c) g has its minimum at $x = 4$.

(d) The first 4 points are collinear:

$(0, 0), (1, -1), (2, -2)$, and $(3, -3)$.

(e) Between $x = 5$ and $x = 6$.

67. (a) $\int_0^x (t + 2)\,dt = \left[\frac{t^2}{2} + 2t\right]_0^x = \frac{1}{2}x^2 + 2x$

(b) $\frac{d}{dx}\left[\frac{1}{2}x^2 + 2x\right] = x + 2$

68. (a) $\int_0^x t(t^2 + 1)\,dt = \int_0^x (t^3 + t)\,dt = \left[\frac{1}{4}t^4 + \frac{1}{2}t^2\right]_0^x = \frac{1}{4}x^4 + \frac{1}{2}x^2 = \frac{x^2}{4}(x^2 + 2)$

(b) $\frac{d}{dx}\left[\frac{1}{4}x^4 + \frac{1}{2}x^2\right] = x^3 + x = x(x^2 + 1)$

69. (a) $\int_8^x \sqrt[3]{t}\,dt = \left[\frac{3}{4}t^{4/3}\right]_8^x = \frac{3}{4}(x^{4/3} - 16) = \frac{3}{4}x^{4/3} - 12$

(b) $\frac{d}{dx}\left[\frac{3}{4}x^{4/3} - 12\right] = x^{1/3} = \sqrt[3]{x}$

70. (a) $\int_4^x \sqrt{t}\,dt = \left[\frac{2}{3}t^{3/2}\right]_4^x = \frac{2}{3}x^{3/2} - \frac{16}{3} = \frac{2}{3}(x^{3/2} - 8)$

(b) $\frac{d}{dx}\left[\frac{2}{3}x^{3/2} - \frac{16}{3}\right] = x^{1/2} = \sqrt{x}$

71. (a) $\int_{\pi/4}^x \sec^2 t\,dt = \left[\tan t\right]_{\pi/4}^x = \tan x - 1$

(b) $\frac{d}{dx}[\tan x - 1] = \sec^2 x$

72. (a) $\int_{\pi/3}^x \sec t \tan t\,dt = \left[\sec t\right]_{\pi/3}^x = \sec x - 2$

(b) $\frac{d}{dx}[\sec x - 2] = \sec x \tan x$

73. $F(x) = \int_{-2}^x (t^2 - 2t)\,dt$

$F'(x) = x^2 - 2x$

74. $F(x) = \int_1^x \sqrt[4]{t}\,dt$

$F'(x) = \sqrt[4]{x}$

75. $F(x) = \int_{-1}^x \sqrt{t^4 + 1}\,dt$

$F'(x) = \sqrt{x^4 + 1}$

76. $F(x) = \int_0^x \tan^4 t \, dt$

$F'(x) = \tan^4 x$

77. $F(x) = \int_0^x t \cos t \, dt$

$F'(x) = x \cos x$

78. $F(x) = \int_1^x \frac{t^2}{t^2 + 1} \, dt$

$F'(x) = \frac{x^2}{x^2 + 1}$

79. $F(x) = \int_x^{x+2} (4t + 1) \, dt$

$= \left[2t^2 + t\right]_x^{x+2}$

$= [2(x + 2)^2 + (x + 2)] - [2x^2 + x]$

$= 8x + 10$

$F'(x) = 8$

Alternate solution:

$F(x) = \int_x^{x+2} (4t + 1) \, dt$

$= \int_x^0 (4t + 1) \, dt + \int_0^{x+2} (4t + 1) \, dt$

$= -\int_0^x (4t + 1) \, dt + \int_0^{x+2} (4t + 1) \, dt$

$F'(x) = -(4x + 1) + 4(x + 2) + 1 = 8$

80. $F(x) = \int_{-x}^x t^3 \, dt = \left[\frac{t^4}{4}\right]_{-x}^x = 0$

$F'(x) = 0$

Alternate solution

$F(x) = \int_{-x}^x t^3 \, dt$

$= \int_{-x}^0 t^3 \, dt + \int_0^x t^3 \, dt$

$= -\int_0^{-x} t^3 \, dt + \int_0^x t^3 \, dt$

$F'(x) = -(-x)^3(-1) + (x^3) = 0$

81. $F(x) = \int_0^{\sin x} \sqrt{t} \, dt = \left[\frac{2}{3}t^{3/2}\right]_0^{\sin x} = \frac{2}{3}(\sin x)^{3/2}$

$F'(x) = (\sin x)^{1/2} \cos x = \cos x \sqrt{\sin x}$

Alternate solution

$F(x) = \int_0^{\sin x} \sqrt{t} \, dt$

$F'(x) = \sqrt{\sin x} \frac{d}{dx}(\sin x) = \sqrt{\sin x}(\cos x)$

82. $F(x) = \int_2^{x^2} \frac{1}{t^2} \, dt = \left[-\frac{1}{t}\right]_2^{x^2} = -\frac{1}{x^2} + \frac{1}{2}$

Alternate solution

$F(x) = \int_2^{x^2} \frac{1}{t^2} \, dt$

$F'(x) = \frac{1}{(x^2)^2}(2x) = \frac{2}{x^3}$

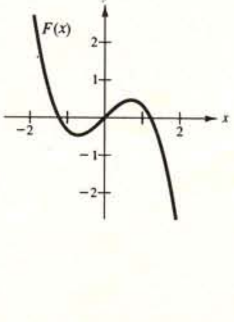
83. $F(x) = \int_0^{x^3} \sin t^2 \, dt$

$F'(x) = \sin(x^3)^2 \cdot 3x^2 = 3x^2 \sin x^6$

84. $F(x) = \int_0^{3x} \sqrt{1 + t^3} \, dt$

$F'(x) = \sqrt{1 + (3x)^3} \cdot 3 = 3\sqrt{1 + 27x^3}$

85. The extrema of F correspond to the zeros of f and the inflection point of F corresponds to the extrema of f.



86. The extremum of F corresponds to the zero of f. Since f has no extrema F has no points of inflection.