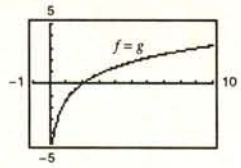
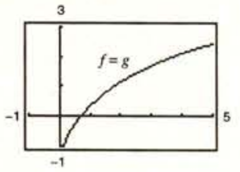
31.



32.



33. $\lim_{x\to 3^+} \ln(x-3) = -\infty$

34. $\lim_{x\to 6^-} \ln(6-x) = -\infty$

35. $\lim_{x\to 2^-} \ln[x^2(3-x)] = \ln 4 \approx 1.3863$

36. $\lim_{x\to 5^+} \ln \dfrac{x}{\sqrt{x-4}} = \ln 5 \approx 1.6094$

37. $y = \ln x^3 = 3\ln x$

$y' = \dfrac{3}{x}$

At $(1, 0)$, $y' = 3$.

38. $y = \ln x^{3/2} = \dfrac{3}{2}\ln x$

$y' = \dfrac{3}{2x}$

At $(1, 0)$, $y' = \frac{3}{2}$.

39. $y = \ln x^2 = 2\ln x$

$y' = \dfrac{2}{x}$

At $(1, 0)$, $y' = 2$.

40. $y = \ln x^{1/2} = \dfrac{1}{2}\ln x$

$y' = \dfrac{1}{2x}$

At $(1, 0)$, $y' = \frac{1}{2}$.

41. $g(x) = \ln x^2 = 2\ln x$

$g'(x) = \dfrac{2}{x}$

42. $h(x) = \ln(x^2 + 3)$

$h'(x) = \dfrac{2x}{x^2+3}$

43. $y = (\ln x)^4$

$\dfrac{dy}{dx} = 4(\ln x)^3\left(\dfrac{1}{x}\right) = \dfrac{4(\ln x)^3}{x}$

44. $y = x\ln x$

$\dfrac{dy}{dx} = x\left(\dfrac{1}{x}\right) + \ln x = 1 + \ln x$

45. $y = \ln x\sqrt{x^2-1} = \ln x + \dfrac{1}{2}\ln(x^2-1)$

$\dfrac{dy}{dx} = \dfrac{1}{x} + \dfrac{1}{2}\left(\dfrac{2x}{x^2-1}\right) = \dfrac{2x^2-1}{x(x^2-1)}$

46. $y = \ln\sqrt{x^2-4} = \dfrac{1}{2}\ln(x^2-4)$

$\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{2x}{x^2-4}\right) = \dfrac{x}{x^2-4}$

47. $f(x) = \ln\dfrac{x}{x^2+1} = \ln x - \ln(x^2+1)$

$f'(x) = \dfrac{1}{x} - \dfrac{2x}{x^2+1} = \dfrac{1-x^2}{x(x^2+1)}$

48. $f(x) = \ln\dfrac{x}{x+1} = \ln x - \ln(x+1)$

$f'(x) = \dfrac{1}{x} - \dfrac{1}{x+1} = \dfrac{1}{x^2+x}$

49. $g(t) = \dfrac{\ln t}{t^2}$

$g'(t) = \dfrac{t^2(1/t) - 2t\ln t}{t^4} = \dfrac{1-2\ln t}{t^3}$

50. $h(t) = \dfrac{\ln t}{t}$

$h'(t) = \dfrac{t(1/t) - \ln t}{t^2} = \dfrac{1-\ln t}{t^2}$

51. $y = \ln(\ln x^2)$

$\dfrac{dy}{dx} = \dfrac{1}{\ln x^2}\dfrac{d}{dx}(\ln x^2) = \dfrac{(2x/x^2)}{\ln x^2} = \dfrac{2}{x\ln x^2} = \dfrac{1}{x\ln x}$

52. $y = \ln(\ln x)$

$\dfrac{dy}{dx} = \dfrac{1/x}{\ln x} = \dfrac{1}{x\ln x}$

53. $y = \ln\sqrt{\frac{x+1}{x-1}} = \frac{1}{2}[\ln(x+1) - \ln(x-1)]$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{x-1}\right] = \frac{1}{1-x^2}$$

54. $y = \ln\sqrt{\frac{x-1}{x+1}} = \frac{1}{2}[\ln(x-1) - \ln(x+1)]$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{x-1} - \frac{1}{x+1}\right] = \frac{1}{x^2-1}$$

55. $f(x) = \ln\frac{\sqrt{4+x^2}}{x} = \frac{1}{2}\ln(4+x^2) - \ln x$

$$f'(x) = \frac{x}{4+x^2} - \frac{1}{x} = \frac{-4}{x(x^2+4)}$$

56. $f(x) = \ln\left(x + \sqrt{4+x^2}\right)$

$$f'(x) = \frac{1}{x+\sqrt{4+x^2}}\left(1 + \frac{x}{\sqrt{4+x^2}}\right)$$

$$= \frac{1}{\sqrt{4+x^2}}$$

57. $y = \frac{-\sqrt{x^2+1}}{x} + \ln\left(x + \sqrt{x^2+1}\right)$

$$\frac{dy}{dx} = \frac{-x\left(x/\sqrt{x^2+1}\right) + \sqrt{x^2+1}}{x^2} + \left(\frac{1}{x+\sqrt{x^2+1}}\right)\left(1 + \frac{x}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{x^2\sqrt{x^2+1}} + \left(\frac{1}{x+\sqrt{x^2+1}}\right)\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{x^2\sqrt{x^2+1}} + \frac{1}{\sqrt{x^2+1}} = \frac{1+x^2}{x^2\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}}{x^2}$$

58. $y = \frac{-\sqrt{x^2+4}}{2x^2} - \frac{1}{4}\ln\left(\frac{2+\sqrt{x^2+4}}{x}\right) = \frac{-\sqrt{x^2+4}}{2x^2} - \frac{1}{4}\ln\left(2+\sqrt{x^2+4}\right) + \frac{1}{4}\ln x$

$$\frac{dy}{dx} = \frac{-2x^2\left(x/\sqrt{x^2+4}\right) + 4x\sqrt{x^2+4}}{4x^4} - \frac{1}{4}\left(\frac{1}{2+\sqrt{x^2+4}}\right)\left(\frac{x}{\sqrt{x^2+4}}\right) + \frac{1}{4x}$$

Note that:

$$\frac{1}{2+\sqrt{x^2+4}} = \frac{1}{2+\sqrt{x^2+4}} \cdot \frac{2-\sqrt{x^2+4}}{2-\sqrt{x^2+4}} = \frac{2-\sqrt{x^2+4}}{-x^2}$$

Hence, $\frac{dy}{dx} = \frac{-1}{2x\sqrt{x^2+4}} + \frac{\sqrt{x^2+4}}{x^3} - \frac{1}{4}\frac{\left(2-\sqrt{x^2+4}\right)}{-x^2}\left(\frac{x}{\sqrt{x^2+4}}\right) + \frac{1}{4x}$

$$= \frac{-1 + (1/2)\left(2-\sqrt{x^2+4}\right)}{2x\sqrt{x^2+4}} + \frac{\sqrt{x^2+4}}{x^3} + \frac{1}{4x}$$

$$= \frac{-\sqrt{x^2+4}}{4x\sqrt{x^2+4}} + \frac{\sqrt{x^2+4}}{x^3} + \frac{1}{4x} = \frac{\sqrt{x^2+4}}{x^3}$$

59. $y = \ln|\sin x|$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$$

60. $y = \ln|\sec x|$

$$\frac{dy}{dx} = \frac{\sec x \tan x}{\sec x} = \tan x$$

61. $y = \ln\left|\frac{\cos x}{\cos x - 1}\right|$

$$= \ln|\cos x| - \ln|\cos x - 1|$$

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} - \frac{-\sin x}{\cos x - 1} = -\tan x + \frac{\sin x}{\cos x - 1}$$

62. $y = \ln|\sec x + \tan x|$

$$\frac{dy}{dx} = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

$$= \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x} = \sec x$$

63. $y = \ln\left|\frac{-1 + \sin x}{2 + \sin x}\right|$

$= \ln|-1 + \sin x| - \ln|2 + \sin x|$

$\frac{dy}{dx} = \frac{\cos x}{-1 + \sin x} - \frac{\cos x}{2 + \sin x}$

$= \frac{3 \cos x}{(\sin x - 1)(\sin x + 2)}$

64. $y = \ln\sqrt{1 + \sin^2 x} = \frac{1}{2}\ln(1 + \sin^2 x)$

$\frac{dy}{dx} = \left(\frac{1}{2}\right)\frac{2 \sin x \cos x}{1 + \sin^2 x} = \frac{\sin x \cos x}{1 + \sin^2 x}$

65. $f(x) = \sin 2x \ln x^2 = 2 \sin 2x \ln x$

$f'(x) = (2 \sin 2x)\left(\frac{1}{x}\right) + 4 \cos 2x \ln x$

$= \frac{2}{x}(\sin 2x + 2x \cos 2x \ln x)$

$= \frac{2}{x}(\sin 2x + x \cos 2x \ln x^2)$

66. $g(x) = \int_1^{\ln x} (t^2 + 3)dt$

$g'(x) = [(\ln x)^2 + 3]\frac{d}{dx}(\ln x) = \frac{(\ln x)^2 + 3}{x}$

(Second Fundamental Theorem of Calculus)

67. (a) $y = 3x^2 - \ln x, \quad (1, 3)$

$\frac{dy}{dx} = 6x - \frac{1}{x}$

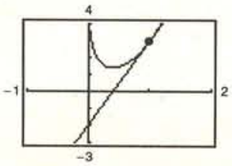
When $x = 1, \frac{dy}{dx} = 5.$

Tangent line: $y - 3 = 5(x - 1)$

$y = 5x - 2$

$0 = 5x - y - 2$

(b)



68. (a) $y = 4 - x^2 - \ln\left(\frac{1}{2}x + 1\right), \quad (0, 4)$

$\frac{dy}{dx} = -2x - \frac{1}{(1/2)x + 1}\left(\frac{1}{2}\right)$

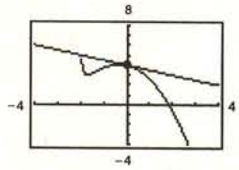
$= -2x - \frac{1}{x + 2}$

When $x = 0, \frac{dy}{dx} = -\frac{1}{2}.$

Tangent line: $y - 4 = -\frac{1}{2}(x - 0)$

$y = -\frac{1}{2}x + 4$

(b)



69. $x^2 - 3 \ln y + y^2 = 10$

$2x - \frac{3}{y}\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$2x = \frac{dy}{dx}\left(\frac{3}{y} - 2y\right)$

$\frac{dy}{dx} = \frac{2x}{(3/y) - 2y} = \frac{2xy}{3 - 2y^2}$

70. $\ln(xy) + 5x = 30$

$\ln x + \ln y + 5x = 30$

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} + 5 = 0$

$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x} - 5$

$\frac{dy}{dx} = -\frac{y}{x} - 5y = -\left(\frac{y + 5xy}{x}\right)$

71. $y = 2(\ln x) + 3$

$y' = \dfrac{2}{x}$

$y'' = -\dfrac{2}{x^2}$

$xy'' + y' = x\left(-\dfrac{2}{x^2}\right) + \dfrac{2}{x} = 0$

72. $y = x(\ln x) - 4x$

$y' = x\left(\dfrac{1}{x}\right) + \ln x - 4 = -3 + \ln x$

$(x + y) - xy' = x + x\ln x - 4x - x(-3 + \ln x)$

$= 0$

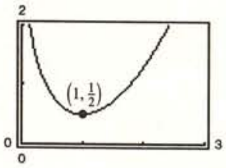
73. $y = \dfrac{x^2}{2} - \ln x$

Domain: $x > 0$

$y' = x - \dfrac{1}{x} = \dfrac{(x + 1)(x - 1)}{x} = 0$ when $x = 1$.

$y'' = 1 + \dfrac{1}{x^2} > 0$

Relative minimum: $\left(1, \frac{1}{2}\right)$



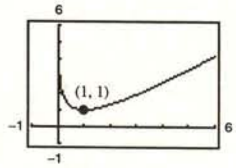
74. $y = x - \ln x$

Domain: $x > 0$

$y' = 1 - \dfrac{1}{x} = 0$ when $x = 1$.

$y'' = \dfrac{1}{x^2} > 0$

Relative minimum: $(1, 1)$



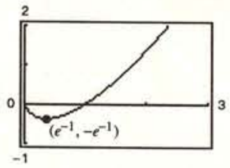
75. $y = x\ln x$

Domain: $x > 0$

$y' = x\left(\dfrac{1}{x}\right) + \ln x = 1 + \ln x = 0$ when $x = e^{-1}$.

$y'' = \dfrac{1}{x} > 0$

Relative minimum: $(e^{-1}, -e^{-1})$



76. $y = \dfrac{\ln x}{x}$

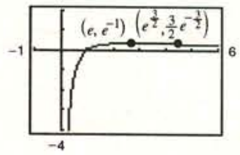
Domain: $x > 0$

$y' = \dfrac{x(1/x) - \ln x}{x^2} = \dfrac{1 - \ln x}{x^2} = 0$ when $x = e$.

$y'' = \dfrac{x^2(-1/x) - (1 - \ln x)(2x)}{x^4} = \dfrac{2(\ln x) - 3}{x^3} = 0$ when $x = e^{3/2}$.

Relative maximum: (e, e^{-1})

Point of inflection: $\left(e^{3/2}, \frac{3}{2}e^{-3/2}\right)$



81. Find x such that $\ln x = -x$.

$f(x) = (\ln x) + x = 0$

$f'(x) = \frac{1}{x} + 1$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n\left[\frac{1 - \ln x_n}{1 + x_n}\right]$

n	1	2	3
x_n	0.5	0.5644	0.5671
$f(x_n)$	−0.1931	−0.0076	−0.0001

Approximate root: $x = 0.567$

82. Find x such that $\ln x = 3 - x$.

$f(x) = x + (\ln x) - 3 = 0$

$f'(x) = 1 + \frac{1}{x}$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n\left[\frac{4 - \ln x_n}{1 + x_n}\right]$

n	1	2	3
x_n	2	2.2046	2.2079
$f(x_n)$	−0.3069	−0.0049	0.0000

Approximate root: $x = 2.208$

83. $y = x\sqrt{x^2 - 1}$

$\ln y = \ln x + \frac{1}{2}\ln(x^2 - 1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{x} + \frac{x}{x^2 - 1}$

$\frac{dy}{dx} = y\left[\frac{2x^2 - 1}{x(x^2 - 1)}\right] = \frac{2x^2 - 1}{\sqrt{x^2 - 1}}$

84. $y = \sqrt{(x - 1)(x - 2)(x - 3)}$

$\ln y = \frac{1}{2}[\ln(x - 1) + \ln(x - 2) + \ln(x - 3)]$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{2}\left[\frac{1}{x - 1} + \frac{1}{x - 2} + \frac{1}{x - 3}\right]$

$= \frac{1}{2}\left[\frac{3x^2 - 12x + 11}{(x - 1)(x - 2)(x - 3)}\right]$

$\frac{dy}{dx} = \frac{3x^2 - 12x + 11}{2y}$

$= \frac{3x^2 - 12x + 11}{2\sqrt{(x - 1)(x - 2)(x - 3)}}$

85. $y = \frac{x^2\sqrt{3x - 2}}{(x - 1)^2}$

$\ln y = 2\ln x + \frac{1}{2}\ln(3x - 2) - 2\ln(x - 1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{2}{x} + \frac{3}{2(3x - 2)} - \frac{2}{x - 1}$

$\frac{dy}{dx} = y\left[\frac{3x^2 - 15x + 8}{2x(3x - 2)(x - 1)}\right]$

$= \frac{3x^3 - 15x^2 + 8x}{2(x - 1)^3\sqrt{3x - 2}}$

86. $y = \sqrt[3]{\frac{x^2 + 1}{x^2 - 1}}$

$\ln y = \frac{1}{3}[\ln(x^2 + 1) - \ln(x + 1) - \ln(x - 1)]$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{3}\left[\frac{2x}{x^2 + 1} - \frac{1}{x + 1} - \frac{1}{x - 1}\right]$

$\frac{dy}{dx} = \frac{-4xy}{3(x^4 - 1)} = -\frac{4x}{3(x^4 - 1)}\sqrt[3]{\frac{x^2 + 1}{x^2 - 1}}$

$= \frac{-4x}{3(x^2 + 1)^{2/3}(x^2 - 1)^{4/3}}$

87. $y = \frac{x(x - 1)^{3/2}}{\sqrt{x + 1}}$

$\ln y = \ln x + \frac{3}{2}\ln(x - 1) - \frac{1}{2}\ln(x + 1)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{x} + \frac{3}{2}\left(\frac{1}{x - 1}\right) - \frac{1}{2}\left(\frac{1}{x + 1}\right)$

$\frac{dy}{dx} = \frac{y}{2}\left[\frac{2}{x} + \frac{3}{x - 1} - \frac{1}{x + 1}\right]$

$= \frac{y}{2}\left[\frac{4x^2 + 4x - 2}{x(x^2 - 1)}\right] = \frac{(2x^2 + 2x - 1)\sqrt{x - 1}}{(x + 1)^{3/2}}$

88. $y = \frac{(x + 1)(x + 2)}{(x - 1)(x - 2)}$

$\ln y = \ln(x + 1) + \ln(x + 2) - \ln(x - 1) - \ln(x - 2)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{x + 1} + \frac{1}{x + 2} - \frac{1}{x - 1} - \frac{1}{x - 2}$

$\frac{dy}{dx} = y\left[\frac{-2}{x^2 - 1} + \frac{-4}{x^2 - 4}\right] = y\left[\frac{-6x^2 + 12}{(x^2 - 1)(x^2 - 4)}\right]$

$= \frac{(x + 1)(x + 2)}{(x - 1)(x - 2)} \cdot \frac{-6(x^2 - 2)}{(x + 1)(x - 1)(x + 2)(x - 2)}$

$= -\frac{6(x^2 - 2)}{(x - 1)^2(x - 2)^2}$

24. In the same way,

$$\lim_{x\to\infty}\left(1+\frac{r}{x}\right)^x = e^r \text{ for } r > 0.$$

25. $\left(1+\dfrac{1}{1{,}000{,}000}\right)^{1{,}000{,}000} \approx 2.718280469$

$e \approx 2.718281828$

$e > \left(1+\dfrac{1}{1{,}000{,}000}\right)^{1{,}000{,}000}$

26. $1+1+\dfrac{1}{2}+\dfrac{1}{6}+\dfrac{1}{24}+\dfrac{1}{120}+\dfrac{1}{720}+\dfrac{1}{5040} = 2.71\overline{825396}$

$e \approx 2.718281828$

$e > 1+1+\dfrac{1}{2}+\dfrac{1}{6}+\dfrac{1}{24}+\dfrac{1}{120}+\dfrac{1}{720}+\dfrac{1}{5040}$

27. (a) $y = e^{3x}$

$y' = 3e^{3x}$

At $(0, 1)$, $y' = 3$.

(b) $y = e^{-3x}$

$y' = -3e^{-3x}$

At $(0, 1)$, $y' = -3$.

28. (a) $y = e^{2x}$

$y' = 2e^{2x}$

At $(0, 1)$, $y' = 2$.

(b) $y = e^{-2x}$

$y' = -2e^{-2x}$

At $(0, 1)$, $y' = -2$.

29. $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

30. $f(x) = e^{1-x}$

$f'(x) = -e^{1-x}$

31. $f(x) = e^{-2x+x^2}$

$\dfrac{dy}{dx} = 2(x-1)e^{-2x+x^2}$

32. $y = e^{-x^2}$

$\dfrac{dy}{dx} = -2xe^{-x^2}$

33. $y = e^{\sqrt{x}}$

$\dfrac{dy}{dx} = \dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

34. $y = x^2e^{-x}$

$\dfrac{dy}{dx} = -x^2e^{-x} + 2xe^{-x}$

$= xe^{-x}(2-x)$

35. $g(t) = (e^{-t} + e^t)^3$

$g'(t) = 3(e^{-t} + e^t)^2(e^t - e^{-t})$

36. $g(t) = e^{-1/t^2}$

$g'(t) = \dfrac{2e^{-1/t^2}}{t^3}$

37. $y = \ln e^{x^2} = x^2$

$\dfrac{dy}{dx} = 2x$

38. $y = \ln\left(\dfrac{1+e^x}{1-e^x}\right)$

$= \ln(1+e^x) - \ln(1-e^x)$

$\dfrac{dy}{dx} = \dfrac{e^x}{1+e^x} + \dfrac{e^x}{1-e^x}$

$= \dfrac{2e^x}{1-e^{2x}}$

39. $y = \ln(1 + e^{2x})$

$\dfrac{dy}{dx} = \dfrac{2e^{2x}}{1+e^{2x}}$

40. $y = \ln\left(\dfrac{e^x + e^{-x}}{2}\right)$

$= \ln(e^x + e^{-x}) - \ln 2$

$\dfrac{dy}{dx} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$= \dfrac{e^{2x} - 1}{e^{2x} + 1}$

41. $y = \dfrac{2}{e^x + e^{-x}} = 2(e^x + e^{-x})^{-1}$

$\dfrac{dy}{dx} = -2(e^x + e^{-x})^{-2}(e^x - e^{-x})$

$= \dfrac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}$

42. $y = \dfrac{e^x - e^{-x}}{2}$

$\dfrac{dy}{dx} = \dfrac{e^x + e^{-x}}{2}$

43. $y = x^2e^x - 2xe^x + 2e^x = e^x(x^2 - 2x + 2)$

$\frac{dy}{dx} = e^x(2x - 2) + e^x(x^2 - 2x + 2) = x^2e^x$

44. $y = xe^x - e^x = e^x(x - 1)$

$\frac{dy}{dx} = e^x + e^x(x - 1) = xe^x$

45. $f(x) = e^{-x} \ln x$

$f'(x) = e^{-x}\left(\frac{1}{x}\right) - e^{-x} \ln x = e^{-x}\left(\frac{1}{x} - \ln x\right)$

46. $f(x) = e^3 \ln x$

$f'(x) = \frac{e^3}{x}$

47. $y = e^x(\sin x + \cos x)$

$\frac{dy}{dx} = e^x(\cos x - \sin x) + (\sin x + \cos x)(e^x)$

$= e^x(2 \cos x) = 2e^x \cos x$

48. $y = \ln e^x = x$

$\frac{dy}{dx} = 1$

49. $xe^y - 10x + 3y = 0$

$xe^y \frac{dy}{dx} + e^y - 10 + 3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(xe^y + 3) = 10 - e^y$

$\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}$

50. $e^{xy} + x^2 - y^2 = 10$

$\left(x\frac{dy}{dx} + y\right)e^{xy} + 2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx}(xe^{xy} - 2y) = -ye^{xy} - 2x$

$\frac{dy}{dx} = -\frac{ye^{xy} + 2x}{xe^{xy} - 2y}$

51. $f(x) = (3 + 2x)e^{-3x}$

$f'(x) = (3 + 2x)(-3e^{-3x}) + 2e^{-3x}$

$= (-7 - 6x)e^{-3x}$

$f''(x) = (-7 - 6x)(-3e^{-3x}) - 6e^{-3x}$

$= 3(6x + 5)e^{-3x}$

52. $g(x) = \sqrt{x} + e^x \ln x$

$g'(x) = \frac{1}{2\sqrt{x}} + \frac{e^x}{x} + e^x \ln x$

$g''(x) = -\frac{1}{4x^{3/2}} + \frac{xe^x - e^x}{x^2} + \frac{e^x}{x} + e^x \ln x$

$= -\frac{1}{4x\sqrt{x}} + \frac{e^x(2x - 1)}{x^2} + e^x \ln x$

53. $y = e^x\left(\cos\sqrt{2}x + \sin\sqrt{2}x\right)$

$y' = e^x\left(-\sqrt{2}\sin\sqrt{2}x + \sqrt{2}\cos\sqrt{2}x\right) + e^x\left(\cos\sqrt{2}x + \sin\sqrt{2}x\right)$

$= e^x\left[\left(1 + \sqrt{2}\right)\cos\sqrt{2}x + \left(1 - \sqrt{2}\right)\sin\sqrt{2}x\right]$

$y'' = e^x\left[-\left(\sqrt{2} + 2\right)\sin\sqrt{2}x + \left(\sqrt{2} - 2\right)\cos\sqrt{2}x\right] + e^x\left[\left(1 + \sqrt{2}\right)\cos\sqrt{2}x + \left(1 - \sqrt{2}\right)\sin\sqrt{2}x\right]$

$= e^x\left[\left(-1 - 2\sqrt{2}\right)\sin\sqrt{2}x + \left(-1 + 2\sqrt{2}\right)\cos\sqrt{2}x\right]$

$-2y' + 3y = -2e^x\left[\left(1 + \sqrt{2}\right)\cos\sqrt{2}x + \left(1 - \sqrt{2}\right)\sin\sqrt{2}x\right] + 3e^x\left[\cos\sqrt{2}x + \sin\sqrt{2}x\right]$

$= e^x\left[\left(1 - 2\sqrt{2}\right)\cos\sqrt{2}x + \left(1 + 2\sqrt{2}\right)\sin\sqrt{2}x\right] = -y''$

Therefore, $-2y' + 3y = -y'' \Rightarrow y'' - 2y' + 3y = 0.$

28. $f(x) = \log_{10} x$

(a) Domain: $x > 0$

(b) $y = \log_{10} x$

$10^y = x$

$f^{-1}(x) = 10^x$

(c) $\log_{10} 1000 = \log_{10} 10^3 = 3$

$\log_{10} 10{,}000 = \log_{10} 10^4 = 4$

If $1000 \le x \le 10{,}000$, then $3 \le f(x) \le 4$.

(d) If $f(x) < 0$, then $0 < x < 1$.

(e) $f(x) + 1 = \log_{10} x + \log_{10} 10$

$= \log_{10}(10x)$

x must have been increased by a factor of 10.

(f) $\log_{10}\left(\dfrac{x_1}{x_2}\right) = \log_{10} x_1 - \log_{10} x_2$

$= 3n - n = 2n$

Thus, $x_1/x_2 = 10^{2n} = 100^n$.

29. $f(x) = 4^x$

$f'(x) = (\ln 4)\, 4^x$

30. $g(x) = 2^{-x}$

$g'(x) = -(\ln 2)\, 2^{-x}$

31. $y = 5^{x-2}$

$\dfrac{dy}{dx} = (\ln 5)\, 5^{x-2}$

32. $y = x(7^{-3x})$

$\dfrac{dy}{dx} = x[-3(\ln 7)\, 7^{-3x}] + 7^{-3x}$

$= 7^{-3x}[-3x(\ln 7) + 1]$

$= 7^{-3x}(1 - 3x \ln 7)$

33. $g(t) = t^2 2^t$

$g'(t) = t^2 (\ln 2)\, 2^t + (2t)\, 2^t$

$= t 2^t (t \ln 2 + 2)$

$= 2^t t(2 + t \ln 2)$

34. $f(t) = \dfrac{3^{2t}}{t}$

$f'(t) = \dfrac{t(2 \ln 3)\, 3^{2t} - 3^{2t}}{t^2}$

$= \dfrac{3^{2t}(2t \ln 3 - 1)}{t^2}$

35. $h(\theta) = 2^{-\theta} \cos \pi\theta$

$h'(\theta) = 2^{-\theta}(-\pi \sin \pi\theta) - (\ln 2) 2^{-\theta} \cos \pi\theta$

$= -2^{-\theta}[(\ln 2) \cos \pi\theta + \pi \sin \pi\theta]$

36. $g(\alpha) = 5^{-\alpha/2} \sin 2\alpha$

$g'(\alpha) = 5^{-\alpha/2}\, 2 \cos 2\alpha - \frac{1}{2}(\ln 5)\, 5^{-\alpha/2} \sin 2\alpha$

37. $y = \log_3 x$

$\dfrac{dy}{dx} = \dfrac{1}{x \ln 3}$

38. $y = \log_{10}(2x) = \log_{10} 2 + \log_{10} x$

$\dfrac{dy}{dx} = 0 + \dfrac{1}{x \ln 10} = \dfrac{1}{x \ln 10}$

39. $f(x) = \log_2 \dfrac{x^2}{x-1}$

$= 2 \log_2 x - \log_2 (x - 1)$

$f'(x) = \dfrac{2}{x \ln 2} - \dfrac{1}{(x-1) \ln 2}$

$= \dfrac{x - 2}{(\ln 2)x(x - 1)}$

40. $h(x) = \log_3 \dfrac{x\sqrt{x-1}}{2}$

$= \log_3 x + \dfrac{1}{2} \log_3 (x - 1) - \log_3 2$

$h'(x) = \dfrac{1}{x \ln 3} + \dfrac{1}{2} \cdot \dfrac{1}{(x-1) \ln 3} - 0$

$= \dfrac{1}{\ln 3}\left[\dfrac{1}{x} + \dfrac{1}{2(x-1)}\right]$

$= \dfrac{1}{\ln 3}\left[\dfrac{3x - 2}{2x(x-1)}\right]$

41. $y = \log_5 \sqrt{x^2 - 1} = \dfrac{1}{2} \log_5 (x^2 - 1)$

$\dfrac{dy}{dx} = \dfrac{1}{2} \cdot \dfrac{2x}{(x^2 - 1)\ln 5} = \dfrac{x}{(x^2 - 1)\ln 5}$

42. $y = \log_{10} \dfrac{x^2 - 1}{x}$

$= \log_{10}(x^2 - 1) - \log_{10} x$

$\dfrac{dy}{dx} = \dfrac{2x}{(x^2 - 1)\ln 10} - \dfrac{1}{x \ln 10}$

$= \dfrac{1}{\ln 10}\left[\dfrac{2x}{x^2 - 1} - \dfrac{1}{x}\right]$

$= \dfrac{1}{\ln 10}\left[\dfrac{x^2 + 1}{x(x^2 - 1)}\right]$

43. $g(t) = \dfrac{10 \log_4 t}{t} = \dfrac{10}{\ln 4}\left(\dfrac{\ln t}{t}\right)$

$g'(t) = \dfrac{10}{\ln 4}\left[\dfrac{t(1/t) - \ln t}{t^2}\right]$

$= \dfrac{10}{t^2 \ln 4}[1 - \ln t] = \dfrac{5}{t^2 \ln 2}(1 - \ln t)$

44. $f(t) = t^{3/2} \log_2 \sqrt{t + 1} = t^{3/2} \dfrac{1}{2} \dfrac{\ln(t + 1)}{\ln 2}$

$f'(t) = \dfrac{1}{2 \ln 2}\left[t^{3/2} \dfrac{1}{t + 1} + \dfrac{3}{2} t^{1/2} \ln(t + 1)\right]$

45. $y = x^{2/x}$

$\ln y = \dfrac{2}{x} \ln x$

$\dfrac{1}{y}\left(\dfrac{dy}{dx}\right) = \dfrac{2}{x}\left(\dfrac{1}{x}\right) + \ln x\left(-\dfrac{2}{x^2}\right) = \dfrac{2}{x^2}(1 - \ln x)$

$\dfrac{dy}{dx} = \dfrac{2y}{x^2}(1 - \ln x) = 2x^{(2/x) - 2}(1 - \ln x)$

46. $y = x^{x-1}$

$\ln y = (x - 1)(\ln x)$

$\dfrac{1}{y}\left(\dfrac{dy}{dx}\right) = (x - 1)\left(\dfrac{1}{x}\right) + \ln x$

$\dfrac{dy}{dx} = y\left[\dfrac{x - 1}{x} + \ln x\right]$

$= x^{x-2}(x - 1 + x \ln x)$

47. $y = (x - 2)^{x+1}$

$\ln y = (x + 1)\ln(x - 2)$

$\dfrac{1}{y}\left(\dfrac{dy}{dx}\right) = (x + 1)\left(\dfrac{1}{x - 2}\right) + \ln(x - 2)$

$\dfrac{dy}{dx} = y\left[\dfrac{x + 1}{x - 2} + \ln(x - 2)\right]$

$= (x - 2)^{x+1}\left[\dfrac{x + 1}{x - 2} + \ln(x - 2)\right]$

48. $y = (1 + x)^{1/x}$

$\ln y = \dfrac{1}{x} \ln(1 + x)$

$\dfrac{1}{y}\left(\dfrac{dy}{dx}\right) = \dfrac{1}{x}\left(\dfrac{1}{1 + x}\right) + \ln(1 + x)\left(-\dfrac{1}{x^2}\right)$

$\dfrac{dy}{dx} = \dfrac{y}{x}\left[\dfrac{1}{x + 1} - \dfrac{\ln(x + 1)}{x}\right]$

$= \dfrac{(1 + x)^{1/x}}{x}\left[\dfrac{1}{x + 1} - \dfrac{\ln(x + 1)}{x}\right]$

49. $f(x) = \log_2 x \Rightarrow f'(x) = \dfrac{1}{x \ln 2}$

$g(x) = x^x \Rightarrow g'(x) = x^x(1 + \ln x)$

[**Note:** Let $y = g(x)$. Then: $\ln y = \ln x^x = x \ln x$

$\dfrac{1}{y} y' = x \cdot \dfrac{1}{x} + \ln x$

$y' = y(1 + \ln x)$

$y' = x^x(1 + \ln x) = g'(x)$.]

$h(x) = x^2 \Rightarrow h'(x) = 2x$

$k(x) = 2^x \Rightarrow k'(x) = (\ln 2)2^x$

From greatest to smallest rate of growth:
$g(x), k(x), h(x), f(x)$