Section 5.3 Inverse Functions

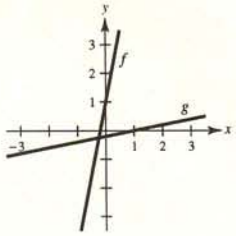
1. (a) $f(x) = 5x + 1$

$g(x) = \dfrac{x - 1}{5}$

$f(g(x)) = f\left(\dfrac{x - 1}{5}\right) = 5\left(\dfrac{x - 1}{5}\right) + 1 = x$

$g((f(x)) = g(5x + 1) = \dfrac{(5x + 1) - 1}{5} = x$

(b)



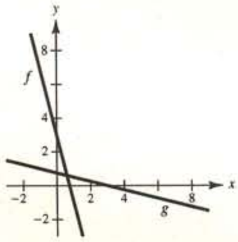
2. (a) $f(x) = 3 - 4x$

$g(x) = \dfrac{3 - x}{4}$

$f(g(x)) = f\left(\dfrac{3 - x}{4}\right) = 3 - 4\left(\dfrac{3 - x}{4}\right) = x$

$g(f(x)) = g(3 - 4x) = \dfrac{3 - (3 - 4x)}{4} = x$

(b)



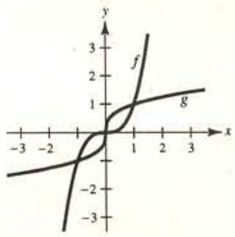
3. (a) $f(x) = x^3$

$g(x) = \sqrt[3]{x}$

$f(g(x)) = f(\sqrt[3]{x}) = (\sqrt[3]{x})^3 = x$

$g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$

(b)



4. (a) $f(x) = 1 - x^3$

$g(x) = \sqrt[3]{1 - x}$

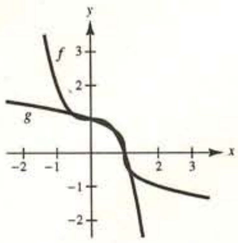
$f(g(x)) = f(\sqrt[3]{1 - x}) = 1 - (\sqrt[3]{1 - x})^3$

$= 1 - (1 - x) = x$

$g(f(x)) = g(1 - x^3)$

$= \sqrt[3]{1 - (1 - x^3)} = \sqrt[3]{x^3} = x$

(b)



5. (a) $f(x) = \sqrt{x - 4}$

$g(x) = x^2 + 4, \ x \geq 0$

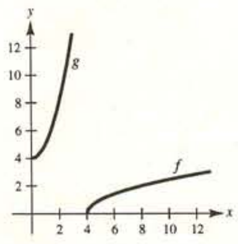
$f(g(x)) = f(x^2 + 4)$

$= \sqrt{(x^2 + 4) - 4} = \sqrt{x^2} = x$

$g(f(x)) = g(\sqrt{x - 4})$

$= (\sqrt{x - 4})^2 + 4 = x - 4 + 4 = x$

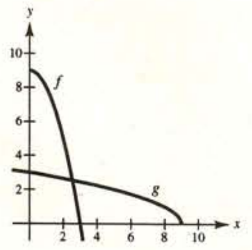
(b)



6. (a)

$$f(x) = 9 - x^2,\ x \ge 0$$
$$g(x) = \sqrt{9 - x},\ x \le 9$$
$$f(g(x)) = f\left(\sqrt{9 - x}\right)$$
$$= 9 - \left(\sqrt{9 - x}\right)^2 = 9 - (9 - x) = x$$
$$g(f(x)) = g(9 - x^2)$$
$$= \sqrt{9 - (9 - x^2)} = \sqrt{x^2} = x$$

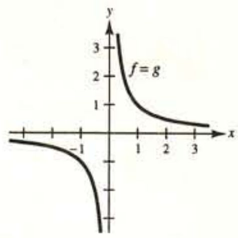
(b)



7. (a)

$$f(x) = \frac{1}{x}$$
$$g(x) = \frac{1}{x}$$
$$f(g(x)) = \frac{1}{1/x} = x$$
$$g(f(x)) = \frac{1}{1/x} = x$$

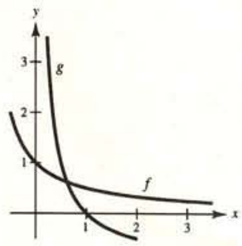
(b)



8. (a)

$$f(x) = \frac{1}{1 + x},\ x \ge 0$$
$$g(x) = \frac{1 - x}{x},\ 0 < x \le 1$$
$$f(g(x)) = f\left(\frac{1 - x}{x}\right) = \frac{1}{1 + \frac{1 - x}{x}} = \frac{1}{\frac{1}{x}} = x$$
$$g(f(x)) = g\left(\frac{1}{1 + x}\right) = \frac{1 - \frac{1}{1 + x}}{\frac{1}{1 + x}} = \frac{x}{1 + x} \cdot \frac{1 + x}{1} = x$$

(b)



9. Matches (c)

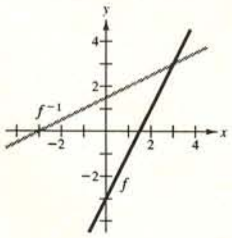
10. Matches (b)

11. Matches (a)

12. Matches (d)

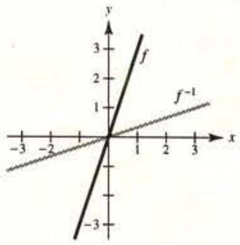
13.

$$f(x) = 2x - 3 = y$$
$$x = \frac{y + 3}{2}$$
$$y = \frac{x + 3}{2}$$
$$f^{-1}(x) = \frac{x + 3}{2}$$



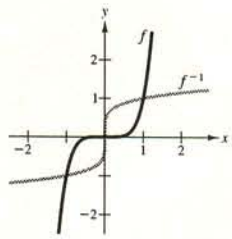
14.

$$f(x) = 3x = y$$
$$x = \frac{y}{3}$$
$$y = \frac{x}{3}$$
$$f^{-1}(x) = \frac{x}{3}$$



15.

$$f(x) = x^5 = y$$
$$x = \sqrt[5]{y}$$
$$y = \sqrt[5]{x}$$
$$f^{-1}(x) = \sqrt[5]{x} = x^{1/5}$$

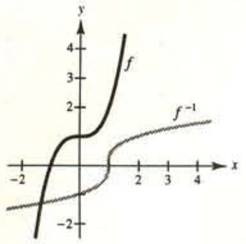


16. $f(x) = x^3 + 1 = y$

$x = \sqrt[3]{y - 1}$

$y = \sqrt[3]{x - 1}$

$f^{-1}(x) = \sqrt[3]{x - 1}$

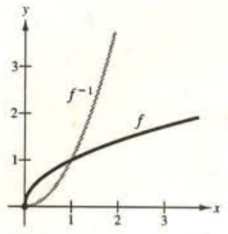


17. $f(x) = \sqrt{x} = y$

$x = y^2$

$y = x^2$

$f^{-1}(x) = x^2, \ x \geq 0$

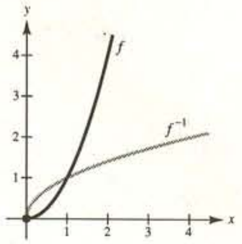


18. $f(x) = x^2 = y, \ 0 \leq x$

$x = \sqrt{y}$

$y = \sqrt{x}$

$f^{-1}(x) = \sqrt{x}$

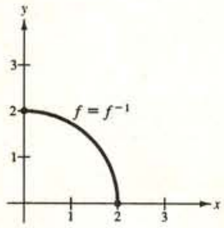


19. $f(x) = \sqrt{4 - x^2} = y, \ 0 \leq x \leq 2$

$x = \sqrt{4 - y^2}$

$y = \sqrt{4 - x^2}$

$f^{-1}(x) = \sqrt{4 - x^2}, \ 0 \leq x \leq 2$

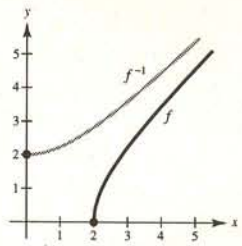


20. $f(x) = \sqrt{x^2 - 4} = y, \ x \geq 2$

$x = \sqrt{y^2 + 4}$

$y = \sqrt{x^2 + 4}$

$f^{-1}(x) = \sqrt{x^2 + 4}, \ x \geq 0$

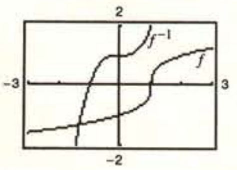


21. $f(x) = \sqrt[3]{x - 1} = y$

$x = y^3 + 1$

$y = x^3 + 1$

$f^{-1}(x) = x^3 + 1$



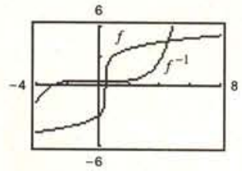
The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

22. $f(x) = 3\sqrt[5]{2x - 1} = y$

$x = \dfrac{y^5 + 243}{486}$

$y = \dfrac{x^5 + 243}{486}$

$f^{-1}(x) = \dfrac{x^5 + 243}{486}$



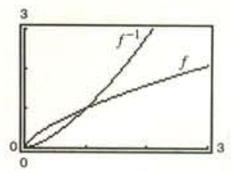
The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

23. $f(x) = x^{2/3} = y, \ x \geq 0$

$x = y^{3/2}$

$y = x^{3/2}$

$f^{-1}(x) = x^{3/2}, \ x \geq 0$



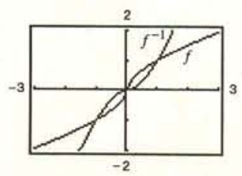
The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

24. $f(x) = x^{3/5} = y$

$x = y^{5/3}$

$y = x^{5/3}$

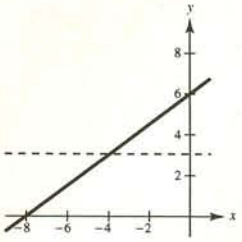
$f^{-1}(x) = x^{5/3}$



The graphs of f and f^{-1} are reflections of each other across the line $y = x$.

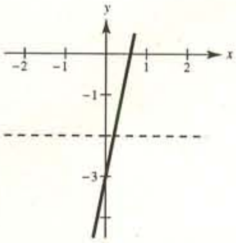
37. $f(x) = \frac{3}{4}x + 6$

One-to-one; has an inverse



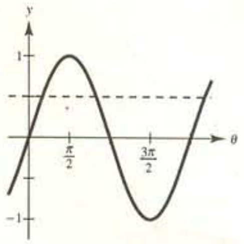
38. $f(x) = 5x - 3$

One-to-one; has an inverse



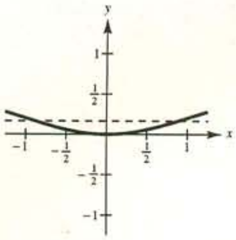
39. $f(\theta) = \sin\theta$

Not one-to-one; does not have an inverse



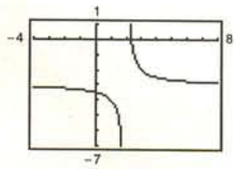
40. $F(x) = \dfrac{x^2}{x^2 + 4}$

Not one-to-one; does not have an inverse



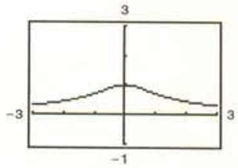
41. $h(s) = \dfrac{1}{s - 2} - 3$

One-to-one; has an inverse



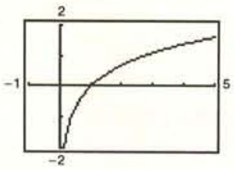
42. $g(t) = \dfrac{1}{\sqrt{t^2 + 1}}$

Not one-to-one; does not have an inverse



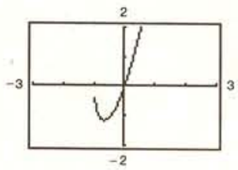
43. $f(x) = \ln x$

One-to-one; has an inverse



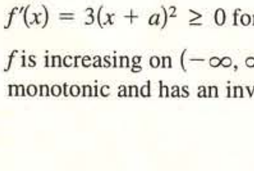
44. $f(x) = 3x\sqrt{x + 1}$

Not one-to-one; does not have an inverse



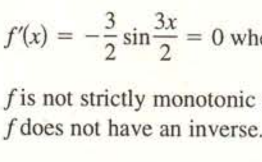
45. $g(x) = (x + 5)^3$

One-to-one; has an inverse



46. $h(x) = |x + 4| - |x - 4|$

Not one-to-one; does not have an inverse



47. $f(x) = (x + a)^3 + b$

$f'(x) = 3(x + a)^2 \geq 0$ for all x.

f is increasing on $(-\infty, \infty)$. Therefore, f is strictly monotonic and has an inverse.

48. $f(x) = \cos\dfrac{3x}{2}$

$$f'(x) = -\frac{3}{2}\sin\frac{3x}{2} = 0 \text{ when } x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \ldots$$

f is not strictly monotonic on $(-\infty, \infty)$. Therefore, f does not have an inverse.

49. $f(x) = \frac{x^4}{4} - 2x^2$

$f'(x) = x^3 - 4x = 0$ when $x = 0, 2, -2$.

f is not strictly monotonic on $(-\infty, \infty)$. Therefore, f does not have an inverse.

50. $f(x) = x^3 - 6x^2 + 12x$

$f'(x) = 3x^2 - 12x + 12 = 3(x - 2)^2 \geq 0$ for all x.

f is increasing on $(-\infty, \infty)$. Therefore, f is strictly monotonic and has an inverse.

51. $f(x) = 2 - x - x^3$

$f'(x) = -1 - 3x^2 < 0$ for all x.

f is decreasing on $(-\infty, \infty)$. Therefore, f is strictly monotonic and has an inverse.

52. $f(x) = \ln(x - 3), x > 3$

$f'(x) = \frac{1}{x - 3} > 0$ for $x > 3$.

f is increasing on $(3, \infty)$. Therefore, f is strictly monotonic and has an inverse.

53. $f(x) = (x - 4)^2$ on $[4, \infty)$

$f'(x) = 2(x - 4) > 0$ on $(4, \infty)$

f is increasing on $[4, \infty)$. Therefore, f is strictly monotonic and has an inverse.

54. $f(x) = |x + 2|$ on $[-2, \infty)$

$f'(x) = \frac{|x + 2|}{x + 2}(1) = 1 > 0$ on $(-2, \infty)$

f is increasing on $[-2, \infty)$. Therefore, f is strictly monotonic and has an inverse.

55. $f(x) = \frac{4}{x^2}$ on $(0, \infty)$

$f'(x) = -\frac{8}{x^3} < 0$ on $(0, \infty)$

f is decreasing on $(0, \infty)$. Therefore, f is strictly monotonic and has an inverse.

56. $f(x) = \tan x$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$f'(x) = \sec^2 x > 0$ on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

f is increasing on $(-\pi/2, \pi/2)$. Therefore, f is strictly monotonic and has an inverse.

57. $f(x) = \cos x$ on $[0, \pi]$

$f'(x) = -\sin x < 0$ on $(0, \pi)$

f is decreasing on $[0, \pi]$. Therefore, f is strictly monotonic and has an inverse.

58. $f(x) = \sec x$ on $\left[0, \frac{\pi}{2}\right)$

$f'(x) = \sec x \tan x > 0$ on $\left(0, \frac{\pi}{2}\right)$

f is increasing on $[0, \pi/2)$. Therefore, f is strictly monotonic and has an inverse.

59. f is not one-to-one because many different x-values yield the same y-value.

Example: $f(0) = f(\pi) = 0$

Not continuous at $\frac{(2n - 1)\pi}{2}$.

60. f is not one-to-one because different x-values yield the same y-value.

Example: $f(3) = f\left(-\frac{4}{3}\right) = \frac{3}{5}$

Not continuous at ± 2.

61. $f(x) = \sqrt{x - 2}$, Domain: $x \geq 2$

$f'(x) = \frac{1}{2\sqrt{x - 2}} > 0$ for $x > 2$.

f is one-to-one; has an inverse

$$\sqrt{x - 2} = y$$
$$x - 2 = y^2$$
$$x = y^2 + 2$$
$$y = x^2 + 2$$
$$f^{-1}(x) = x^2 + 2, x \geq 0$$

62. $f(x) = -3$

Not one-to-one; does not have an inverse

76. $f(x) = \cos 2x, f(0) = 1 = a$

$f'(x) = -2\sin 2x$

$$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))} = \frac{1}{f'(0)} = \frac{1}{-2\sin 0} = \frac{1}{0} \text{ which is undefined.}$$

77. $f(x) = x^3 - \frac{4}{x}, f(2) = 6 = a$

$f'(x) = 3x^2 + \frac{4}{x^2}$

$$(f^{-1})'(6) = \frac{1}{f'(f^{-1}(6))} = \frac{1}{f'(2)} = \frac{1}{3(2)^2 + (4/2^2)} = \frac{1}{13}$$

78. $f(x) = \sqrt{x-4}, f(8) = 2 = a$

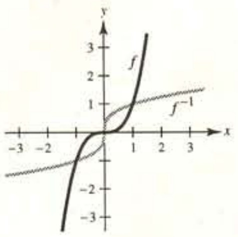
$f'(x) = \frac{1}{2\sqrt{x-4}}$

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(8)} = \frac{1}{1/(2\sqrt{8-4})} = \frac{1}{1/4} = 4$$

79. (a) Domain f = Domain $f^{-1} = (-\infty, \infty)$

(b) Range f = Range $f^{-1} = (-\infty, \infty)$

(c)



(d) $f(x) = x^3, \left(\frac{1}{2}, \frac{1}{8}\right)$

$f'(x) = 3x^2$

$f'\left(\frac{1}{2}\right) = \frac{3}{4}$

$f^{-1}(x) = \sqrt[3]{x}, \left(\frac{1}{8}, \frac{1}{2}\right)$

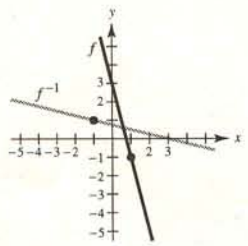
$(f^{-1})'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$(f^{-1})'\left(\frac{1}{8}\right) = \frac{4}{3}$

80. (a) Domain f = Domain $f^{-1} = (-\infty, \infty)$

(b) Range f = Range $f^{-1} = (-\infty, \infty)$

(c)



(d) $f(x) = 3 - 4x, \ (1, -1)$

$f'(x) = -4$

$f'(1) = -4$

$f^{-1}(x) = \frac{3-x}{4}, \ (-1, 1)$

$(f^{-1})'(x) = -\frac{1}{4}$

$(f^{-1})'(-1) = -\frac{1}{4}$