24. In the same way,

$$\lim_{x\to\infty}\left(1+\frac{r}{x}\right)^x = e^r \text{ for } r > 0.$$

25. $\left(1+\frac{1}{1,000,000}\right)^{1,000,000} \approx 2.718280469$

$e \approx 2.718281828$

$e > \left(1+\frac{1}{1,000,000}\right)^{1,000,000}$

26. $1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040} = 2.71\overline{825396}$

$e \approx 2.718281828$

$e > 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040}$

27. (a) $y = e^{3x}$

$y' = 3e^{3x}$

At $(0, 1)$, $y' = 3$.

(b) $y = e^{-3x}$

$y' = -3e^{-3x}$

At $(0, 1)$, $y' = -3$.

28. (a) $y = e^{2x}$

$y' = 2e^{2x}$

At $(0, 1)$, $y' = 2$.

(b) $y = e^{-2x}$

$y' = -2e^{-2x}$

At $(0, 1)$, $y' = -2$.

29. $f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

30. $f(x) = e^{1-x}$

$f'(x) = -e^{1-x}$

31. $f(x) = e^{-2x+x^2}$

$\frac{dy}{dx} = 2(x-1)e^{-2x+x^2}$

32. $y = e^{-x^2}$

$\frac{dy}{dx} = -2xe^{-x^2}$

33. $y = e^{\sqrt{x}}$

$\frac{dy}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$

34. $y = x^2e^{-x}$

$\frac{dy}{dx} = -x^2e^{-x} + 2xe^{-x}$

$= xe^{-x}(2-x)$

35. $g(t) = (e^{-t} + e^t)^3$

$g'(t) = 3(e^{-t} + e^t)^2(e^t - e^{-t})$

36. $g(t) = e^{-1/t^2}$

$g'(t) = \frac{2e^{-1/t^2}}{t^3}$

37. $y = \ln e^{x^2} = x^2$

$\frac{dy}{dx} = 2x$

38. $y = \ln\left(\frac{1+e^x}{1-e^x}\right)$

$= \ln(1+e^x) - \ln(1-e^x)$

$\frac{dy}{dx} = \frac{e^x}{1+e^x} + \frac{e^x}{1-e^x}$

$= \frac{2e^x}{1-e^{2x}}$

39. $y = \ln(1 + e^{2x})$

$\frac{dy}{dx} = \frac{2e^{2x}}{1+e^{2x}}$

40. $y = \ln\left(\frac{e^x + e^{-x}}{2}\right)$

$= \ln(e^x + e^{-x}) - \ln 2$

$\frac{dy}{dx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

$= \frac{e^{2x} - 1}{e^{2x} + 1}$

41. $y = \frac{2}{e^x + e^{-x}} = 2(e^x + e^{-x})^{-1}$

$\frac{dy}{dx} = -2(e^x + e^{-x})^{-2}(e^x - e^{-x})$

$= \frac{-2(e^x - e^{-x})}{(e^x + e^{-x})^2}$

42. $y = \frac{e^x - e^{-x}}{2}$

$\frac{dy}{dx} = \frac{e^x + e^{-x}}{2}$

43. $y = x^2e^x - 2xe^x + 2e^x = e^x(x^2 - 2x + 2)$

$$\frac{dy}{dx} = e^x(2x - 2) + e^x(x^2 - 2x + 2) = x^2e^x$$

44. $y = xe^x - e^x = e^x(x - 1)$

$$\frac{dy}{dx} = e^x + e^x(x - 1) = xe^x$$

45. $f(x) = e^{-x} \ln x$

$$f'(x) = e^{-x}\left(\frac{1}{x}\right) - e^{-x} \ln x = e^{-x}\left(\frac{1}{x} - \ln x\right)$$

46. $f(x) = e^3 \ln x$

$$f'(x) = \frac{e^3}{x}$$

47. $y = e^x(\sin x + \cos x)$

$$\frac{dy}{dx} = e^x(\cos x - \sin x) + (\sin x + \cos x)(e^x)$$

$$= e^x(2 \cos x) = 2e^x \cos x$$

48. $y = \ln e^x = x$

$$\frac{dy}{dx} = 1$$

49. $xe^y - 10x + 3y = 0$

$$xe^y \frac{dy}{dx} + e^y - 10 + 3\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(xe^y + 3) = 10 - e^y$$

$$\frac{dy}{dx} = \frac{10 - e^y}{xe^y + 3}$$

50. $e^{xy} + x^2 - y^2 = 10$

$$\left(x\frac{dy}{dx} + y\right)e^{xy} + 2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx}(xe^{xy} - 2y) = -ye^{xy} - 2x$$

$$\frac{dy}{dx} = -\frac{ye^{xy} + 2x}{xe^{xy} - 2y}$$

51. $f(x) = (3 + 2x)e^{-3x}$

$$f'(x) = (3 + 2x)(-3e^{-3x}) + 2e^{-3x}$$

$$= (-7 - 6x)e^{-3x}$$

$$f''(x) = (-7 - 6x)(-3e^{-3x}) - 6e^{-3x}$$

$$= 3(6x + 5)e^{-3x}$$

52. $g(x) = \sqrt{x} + e^x \ln x$

$$g'(x) = \frac{1}{2\sqrt{x}} + \frac{e^x}{x} + e^x \ln x$$

$$g''(x) = -\frac{1}{4x^{3/2}} + \frac{xe^x - e^x}{x^2} + \frac{e^x}{x} + e^x \ln x$$

$$= -\frac{1}{4x\sqrt{x}} + \frac{e^x(2x - 1)}{x^2} + e^x \ln x$$

53.

$$y = e^x(\cos\sqrt{2}x + \sin\sqrt{2}x)$$

$$y' = e^x(-\sqrt{2}\sin\sqrt{2}x + \sqrt{2}\cos\sqrt{2}x) + e^x(\cos\sqrt{2}x + \sin\sqrt{2}x)$$

$$= e^x\left[(1 + \sqrt{2})\cos\sqrt{2}x + (1 - \sqrt{2})\sin\sqrt{2}x\right]$$

$$y'' = e^x\left[-(\sqrt{2} + 2)\sin\sqrt{2}x + (\sqrt{2} - 2)\cos\sqrt{2}x\right] + e^x\left[(1 + \sqrt{2})\cos\sqrt{2}x + (1 - \sqrt{2})\sin\sqrt{2}x\right]$$

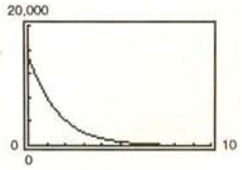
$$= e^x\left[(-1 - 2\sqrt{2})\sin\sqrt{2}x + (-1 + 2\sqrt{2})\cos\sqrt{2}x\right]$$

$$-2y' + 3y = -2e^x\left[(1 + \sqrt{2})\cos\sqrt{2}x + (1 - \sqrt{2})\sin\sqrt{2}x\right] + 3e^x\left[\cos\sqrt{2}x + \sin\sqrt{2}x\right]$$

$$= e^x\left[(1 - 2\sqrt{2})\cos\sqrt{2}x + (1 + 2\sqrt{2})\sin\sqrt{2}x\right] = -y''$$

Therefore, $-2y' + 3y = -y'' \Rightarrow y'' - 2y' + 3y = 0$.

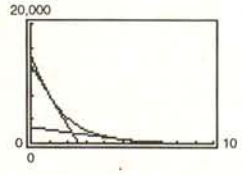
66. $V = 15{,}000e^{-0.6286t}, 0 \le t \le 10$

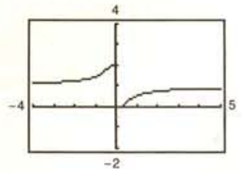
(a) 

(b) $\dfrac{dV}{dt} = -9429e^{-0.6286t}$

When $t = 1, \dfrac{dV}{dt} \approx -5028.84$.

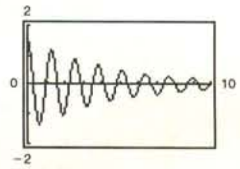
When $t = 5, \dfrac{dV}{dt} \approx -406.89$.

(c) 

67. (a) 

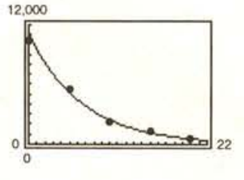
(b) When x increases without bound, $1/x$ approaches zero, and $e^{1/x}$ approaches 1. Therefore, $f(x)$ approaches $2/(1 + 1) = 1$. Thus, $f(x)$ has a horizontal asymptote at $y = 1$. As x approaches zero from the right, $1/x$ approaches ∞, $e^{1/x}$ approaches ∞ and $f(x)$ approaches zero. As x approaches zero from the left, $1/x$ approaches $-\infty$, $e^{1/x}$ approaches zero, and $f(x)$ approaches 2. The limit does not exist since the left limit does not equal the right limit. Therefore, $x = 0$ is a nonremovable discontinuity.

68. $1.56e^{-0.22t} \cos 4.9t \le 0.25$ (3 inches equals one-fourth foot.) Using a graphing utility or Newton's Method, we have $t \ge 7.79$ seconds.



69.

h	0	5	10	15	20
P	10,332	5,583	2,376	1,240	517
$\ln P$	9.243	8.627	7.773	7.123	6.248

(a) 

$y = -0.1499h + 9.3018$ is the regression line for data $(h, \ln P)$.

(b) $\ln P = ah + b$

$P = e^{ah+b} = e^b e^{ah}$

$P = Ce^{ah}, C = e^b$

For our data, $a = -0.1499$ and $C = e^{9.3018} = 10{,}957.7$

$P = 10{,}957.7e^{-0.1499h}$

(c)

(d) $\dfrac{dP}{dh} = (10{,}957.71)(-0.1499)e^{-0.1499h}$

$= -1642.56e^{-0.1499h}$

For $h = 5, \dfrac{dP}{dh} = -776.3$. For $h = 18, \dfrac{dP}{dh} \approx -110.6$.

28. $f(x) = \log_{10} x$

(a) Domain: $x > 0$

(b) $y = \log_{10} x$

$10^y = x$

$f^{-1}(x) = 10^x$

(c) $\log_{10} 1000 = \log_{10} 10^3 = 3$

$\log_{10} 10{,}000 = \log_{10} 10^4 = 4$

If $1000 \le x \le 10{,}000$, then $3 \le f(x) \le 4$.

(d) If $f(x) < 0$, then $0 < x < 1$.

(e) $f(x) + 1 = \log_{10} x + \log_{10} 10$

$= \log_{10}(10x)$

x must have been increased by a factor of 10.

(f) $\log_{10}\left(\frac{x_1}{x_2}\right) = \log_{10} x_1 - \log_{10} x_2$

$= 3n - n = 2n$

Thus, $x_1/x_2 = 10^{2n} = 100^n$.

29. $f(x) = 4^x$

$f'(x) = (\ln 4)\, 4^x$

30. $g(x) = 2^{-x}$

$g'(x) = -(\ln 2)\, 2^{-x}$

31. $y = 5^{x-2}$

$\frac{dy}{dx} = (\ln 5)\, 5^{x-2}$

32. $y = x(7^{-3x})$

$\frac{dy}{dx} = x[-3(\ln 7)\, 7^{-3x}] + 7^{-3x}$

$= 7^{-3x}[-3x(\ln 7) + 1]$

$= 7^{-3x}(1 - 3x \ln 7)$

33. $g(t) = t^2 2^t$

$g'(t) = t^2 (\ln 2)\, 2^t + (2t)\, 2^t$

$= t2^t(t \ln 2 + 2)$

$= 2^t t(2 + t \ln 2)$

34. $f(t) = \frac{3^{2t}}{t}$

$f'(t) = \frac{t(2 \ln 3)\, 3^{2t} - 3^{2t}}{t^2}$

$= \frac{3^{2t}(2t \ln 3 - 1)}{t^2}$

35. $h(\theta) = 2^{-\theta} \cos \pi\theta$

$h'(\theta) = 2^{-\theta}(-\pi \sin \pi\theta) - (\ln 2)2^{-\theta} \cos \pi\theta$

$= -2^{-\theta}[(\ln 2) \cos \pi\theta + \pi \sin \pi\theta]$

36. $g(\alpha) = 5^{-\alpha/2} \sin 2\alpha$

$g'(\alpha) = 5^{-\alpha/2}\, 2 \cos 2\alpha - \frac{1}{2}(\ln 5)\, 5^{-\alpha/2} \sin 2\alpha$

37. $y = \log_3 x$

$\frac{dy}{dx} = \frac{1}{x \ln 3}$

38. $y = \log_{10}(2x) = \log_{10} 2 + \log_{10} x$

$\frac{dy}{dx} = 0 + \frac{1}{x \ln 10} = \frac{1}{x \ln 10}$

39. $f(x) = \log_2 \frac{x^2}{x - 1}$

$= 2 \log_2 x - \log_2 (x - 1)$

$f'(x) = \frac{2}{x \ln 2} - \frac{1}{(x - 1) \ln 2}$

$= \frac{x - 2}{(\ln 2)x(x - 1)}$

40. $h(x) = \log_3 \frac{x\sqrt{x - 1}}{2}$

$= \log_3 x + \frac{1}{2} \log_3 (x - 1) - \log_3 2$

$h'(x) = \frac{1}{x \ln 3} + \frac{1}{2} \cdot \frac{1}{(x - 1) \ln 3} - 0$

$= \frac{1}{\ln 3}\left[\frac{1}{x} + \frac{1}{2(x - 1)}\right]$

$= \frac{1}{\ln 3}\left[\frac{3x - 2}{2x(x - 1)}\right]$

41. $y = \log_5 \sqrt{x^2 - 1} = \frac{1}{2} \log_5 (x^2 - 1)$

$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{2x}{(x^2 - 1)\ln 5} = \frac{x}{(x^2 - 1)\ln 5}$

50. $f(x) = a^x$

(a) $f(u + v) = a^{u+v} = a^u a^v = f(u)f(v)$

(b) $f(2x) = a^{2x} = (a^x)^2 = [f(x)]^2$

51. $C(t) = P(1.05)^t$

(a) $C(10) = 24.95(1.05)^{10}$

$\approx \$40.64$

(b) $\dfrac{dC}{dt} = P(\ln 1.05)(1.05)^t$

When $t = 1$: $\dfrac{dC}{dt} \approx 0.051P$

When $t = 8$: $\dfrac{dC}{dt} \approx 0.072P$

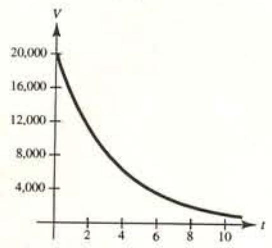
(c) $\dfrac{dC}{dt} = (\ln 1.05)[P(1.05)^t]$

$= (\ln 1.05)C(t)$

The constant of proportionality is ln 1.05.

52. $V(t) = 20{,}000\left(\dfrac{3}{4}\right)^t$

(a)



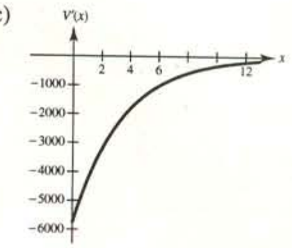
$V(2) = 20{,}000\left(\dfrac{3}{4}\right)^2 = \$11{,}250$

(b) $\dfrac{dV}{dt} = 20{,}000\left(\ln \dfrac{3}{4}\right)\left(\dfrac{3}{4}\right)^t$

When $t = 1$: $\dfrac{dV}{dt} \approx -4315.23$

When $t = 4$: $\dfrac{dV}{dt} \approx -1820.49$

(c)



Horizontal asymptote: $v' = 0$

As the car ages, it is worth less each year and depreciates less each year, but the value of the car will never reach $0.

53. $P = \$1000$, $r = 3\frac{1}{2}\% = 0.035$, $t = 10$

$A = 1000\left(1 + \dfrac{0.035}{n}\right)^{10n}$

$A = 1000e^{(0.035)(10)} = 1419.07$

n	1	2	4	12	365	Continuous
A	1410.60	1414.78	1416.91	1418.34	1419.04	1419.07

54. $P = \$2500$, $r = 6\% = 0.06$, $t = 20$

$A = 2500\left(1 + \dfrac{0.06}{n}\right)^{20n}$

$A = 2500e^{(0.06)(20)} = 8300.29$

n	1	2	4	12	365	Continuous
A	8017.84	8155.09	8226.66	8275.51	8299.47	8300.29

55. $P = \$1000$, $r = 5\% = 0.05$, $t = 30$

$A = 1000\left(1 + \dfrac{0.05}{n}\right)^{30n}$

$A = 1000e^{(0.05)30} = 4481.69$

n	1	2	4	12	365	Continuous
A	4321.94	4399.79	4440.21	4467.74	4481.23	4481.69

56. $P = \$2500$, $r = 5\% = 0.05$, $t = 40$

$A = 2500\left(1 + \dfrac{0.05}{n}\right)^{40n}$

$A = 2500e^{(0.05)(40)} = 18{,}472.64$

n	1	2	4	12	365	Continuous
A	17,599.97	18,023.92	18,245.05	18,396.04	18,470.11	18,472.64