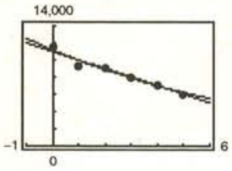
70. (a) $y = -995.57t + 11{,}018.10$

$y = 53.39t^2 - 1262.54t + 11{,}196.07$



(b) The slope represents the rate of decrease in value of the car.

(c) $\ln V = -0.1178t + 9.325$

$V = e^{-0.1178t+9.325} = 11{,}215.5e^{-0.1178t}$

(d) Horizontal asymptote: $V = 0$

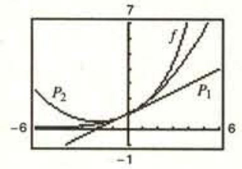
As $t \to \infty$, the value of the car approaches 0.

(e) $\dfrac{dV}{dt} = -1321.2e^{-0.1178t}$

For $t = 1$, $\dfrac{dV}{dt} \approx -1174.38$ dollars/yr.

For $t = 5$, $\dfrac{dV}{dt} \approx -733.11$ dollars/yr.

71. $f(x) = e^{x/2}, f(0) = 1$



$f'(x) = \frac{1}{2}e^{x/2}, f'(0) = \frac{1}{2}$

$f''(x) = \frac{1}{4}e^{x/2}, f''(0) = \frac{1}{4}$

$P_1(x) = 1 + \frac{1}{2}(x - 0) = \frac{x}{2} + 1, P_1(0) = 1$

$P_1'(x) = \frac{1}{2}, P_1'(0) = \frac{1}{2}$

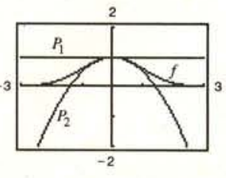
$P_2(x) = 1 + \frac{1}{2}(x - 0) + \frac{1}{8}(x - 0)^2 = \frac{x^2}{8} + \frac{x}{2} + 1, P_2(0) = 1$

$P_2'(x) = \frac{1}{4}x + \frac{1}{2}, P_2'(0) = \frac{1}{2}$

$P_2''(x) = \frac{1}{4}, P_2''(0) = \frac{1}{4}$

The values of f, P_1, P_2 and their first derivatives agree at $x = 0$. The values of the second derivatives of f and P_2 agree at $x = 0$.

72. $f(x) = e^{-x^2/2}, f(0) = 1$



$f'(x) = -xe^{-x^2/2}, f'(0) = 0$

$f''(x) = x^2e^{-x^2/2} - e^{-x^2/2} = e^{-x^2/2}(x^2 - 1), f''(0) = -1$

$P_1(x) = 1 + 0(x - 0) = 1, P_1(0) = 1$

$P_1'(x) = 0, \ P_1'(0) = 0$

$P_2(x) = 1 + 0(x - 0) - \frac{1}{2}(x - 0)^2 = 1 - \frac{x^2}{2}, P_2(0) = 1$

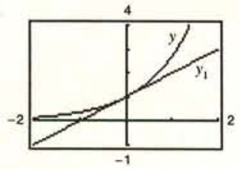
$P_2'(x) = -x, P_2'(0) = 0$

$P_2''(x) = -1, P_2''(0) = -1$

The values of f, P_1, P_2 and their first derivatives agree at $x = 0$. The values of the second derivatives of f and P_2 agree at $x = 0$.

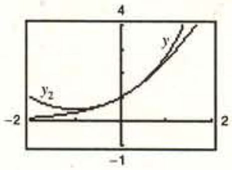
73. (a) $y = e^x$

$y_1 = 1 + x$



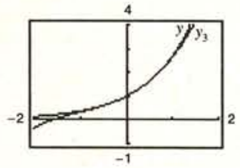
(b) $y = e^x$

$y_2 = 1 + x + \left(\frac{x^2}{2}\right)$



(c) $y = e^x$

$y_3 = 1 + x + \frac{x^2}{2} + \frac{x^3}{6}$



74. n^{th} term is $x^n/n!$ in polynomial:

$$y_4 = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

Conjecture: $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

75. Let $u = 5x$, $du = 5\,dx$.

$$\int e^{5x}\,5dx = e^{5x} + C$$

76. Let $u = -x^4$, $du = -4x^3\,dx$.

$$\int e^{-x^4}(-4x^3)\,dx = e^{-x^4} + C$$

77. Let $u = -2x$, $du = -2\,dx$.

$$\int_0^1 e^{-2x}dx = -\frac{1}{2}\int_0^1 e^{-2x}(-2)\,dx = \left[-\frac{1}{2}e^{-2x}\right]_0^1 = \frac{1}{2}(1 - e^{-2}) = \frac{e^2 - 1}{2e^2}$$

78. Let $u = 1 - x$, $du = -dx$.

$$\int_1^2 e^{1-x}dx = -\int_1^2 e^{1-x}(-1)\,dx = \left[-e^{1-x}\right]_1^2 = 1 - e^{-1} = \frac{e-1}{e}$$

79. Let $u = 1 + e^{-x}$, $du = -e^{-x}dx$.

$$\int \frac{e^{-x}}{1 + e^{-x}}\,dx = -\int \frac{-e^{-x}}{1 + e^{-x}}\,dx = -\ln(1 + e^{-x}) + C = \ln\left(\frac{e^x}{e^x + 1}\right) + C = x - \ln(e^x + 1) + C$$

80. Let $u = 1 + e^{2x}$, $du = 2e^{2x}dx$.

$$\int \frac{e^{2x}}{1 + e^{2x}}\,dx = \frac{1}{2}\int \frac{2e^{2x}}{1 + e^{2x}}\,dx = \frac{1}{2}\ln(1 + e^{2x}) + C$$

81. Let $u = \frac{3}{x}$, $du = -\frac{3}{x^2}\,dx$.

$$\int_1^3 \frac{e^{3/x}}{x^2}\,dx = -\frac{1}{3}\int_1^3 e^{3/x}\left(-\frac{3}{x^2}\right)dx = \left[-\frac{1}{3}e^{3/x}\right]_1^3 = \frac{e}{3}(e^2 - 1)$$

82. Let $u = \frac{-x^2}{2}$, $du = -x\,dx$.

$$\int_0^{\sqrt{2}} xe^{-x^2/2}\,dx = -\int_0^{\sqrt{2}} e^{-x^2/2}(-x)\,dx = \left[-e^{-x^2/2}\right]_0^{\sqrt{2}} = 1 - e^{-1} = \frac{e-1}{e}$$

83. Let $u = 1 - e^x$, $du = -e^x\,dx$.

$$\int e^x\sqrt{1-e^x}\,dx = -\int (1-e^x)^{1/2}(-e^x)\,dx = -\frac{2}{3}(1-e^x)^{3/2} + C$$

84. Let $u = e^x + e^{-x}$, $du = (e^x - e^{-x})dx$.

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \ln(e^x + e^{-x}) + C$$

85. Let $u = e^x - e^{-x}$, $du = (e^x + e^{-x})\,dx$.

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx = \ln\left|e^x - e^{-x}\right| + C$$

86. Let $u = e^x + e^{-x}$, $du = (e^x - e^{-x})\,dx$.

$$\int \frac{2e^x - 2e^{-x}}{(e^x + e^{-x})^2}\,dx = 2\int (e^x + e^{-x})^{-2}(e^x - e^{-x})\,dx$$

$$= \frac{-2}{e^x + e^{-x}} + C$$

87. $$\int \frac{5 - e^x}{e^{2x}}\,dx = \int 5e^{-2x}\,dx - \int e^{-x}\,dx$$

$$= -\frac{5}{2}e^{-2x} + e^{-x} + C$$

88. $$\int \frac{e^{2x} + 2e^x + 1}{e^x}\,dx = \int (e^x + 2 + e^{-x})\,dx$$

$$= e^x + 2x - e^{-x} + C$$

89. $$\int e^{\sin \pi x}\cos \pi x\,dx = \frac{1}{\pi}\int e^{\sin \pi x}(\pi\cos \pi x)\,dx$$

$$= \frac{1}{\pi}e^{\sin \pi x} + C$$

90. $$\int e^{\tan 2x}\sec^2 2x\,dx = \frac{1}{2}\int e^{\tan 2x}(2\sec^2 2x)\,dx$$

$$= \frac{1}{2}e^{\tan 2x} + C$$

91. $$\int e^{-x}\tan(e^{-x})\,dx = -\int [\tan(e^{-x})](-e^{-x})\,dx$$

$$= \ln\left|\cos(e^{-x})\right| + C$$

92. $$\int \ln(e^{2x-1})\,dx = \int (2x-1)\,dx$$

$$= x^2 - x + C$$

93. Let $u = ax^2$, $du = 2ax\,dx$. (Assume $a \neq 0$)

$$y = \int xe^{ax^2}\,dx$$

$$= \frac{1}{2a}\int e^{ax^2}(2ax)\,dx = \frac{1}{2a}e^{ax^2} + C$$

94. $$y = \int (e^x - e^{-x})^2\,dx$$

$$= \int (e^{2x} - 2 + e^{-2x})\,dx$$

$$= \frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + C$$

95. $$f'(x) = \int \frac{1}{2}(e^x + e^{-x})\,dx = \frac{1}{2}(e^x - e^{-x}) + C_1$$

$$f'(0) = C_1 = 0$$

$$f(x) = \int \frac{1}{2}(e^x - e^{-x})\,dx = \frac{1}{2}(e^x + e^{-x}) + C_2$$

$$f(0) = 1 + C_2 = 1 \Rightarrow C_2 = 0$$

$$f(x) = \frac{1}{2}(e^x + e^{-x})$$

96. $f'(x) = \int (\sin x + e^{2x})\, dx = -\cos x + \frac{1}{2}e^{2x} + C_1$

$f'(0) = -1 + \frac{1}{2} + C_1 = \frac{1}{2} \Rightarrow C_1 = 1$

$f'(x) = -\cos x + \frac{1}{2}e^{2x} + 1$

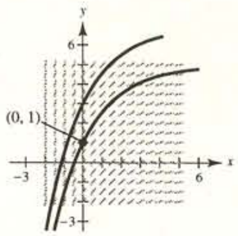
$f(x) = \int \left(-\cos x + \frac{1}{2}e^{2x} + 1\right) dx$

$= -\sin x + \frac{1}{4}e^{2x} + x + C_2$

$f(0) = \frac{1}{4} + C_2 = \frac{1}{4} \Rightarrow C_2 = 0$

$f(x) = x - \sin x + \frac{1}{4}e^{2x}$

97. (a)



(b) $\frac{dy}{dx} = 2e^{-x/2}$, $(0, 1)$

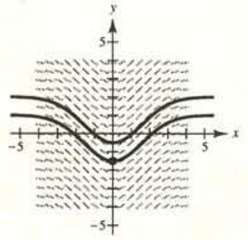
$y = \int 2e^{-x/2}\, dx = -4\int e^{-x/2}\left(-\frac{1}{2}dx\right)$

$= -4e^{-x/2} + C$

$(0, 1)$: $1 = -4e^0 + C = -4 + C \Rightarrow C = 5$

$y = -4e^{-x/2} + 5$

98. (a)



(b) $\frac{dy}{dx} = xe^{-0.2x^2}$, $\left(0, -\frac{3}{2}\right)$

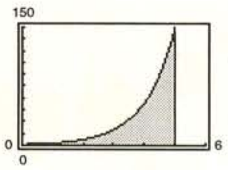
$y = \int xe^{-0.2x^2}\, dx = \frac{1}{-0.4}\int e^{-0.2x^2}(-0.4x)\, dx$

$= -\frac{1}{0.4}e^{-0.2x^2} + C = -2.5e^{-0.2x^2} + C$

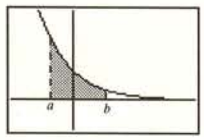
$\left(0, -\frac{3}{2}\right)$: $-\frac{3}{2} = -2.5e^0 + C = -2.5 + C \Rightarrow C = 1$

$y = -2.5e^{-0.2x^2} + 1$

99. $\int_0^5 e^x\, dx = \left[e^x\right]_0^5 = e^5 - 1 \approx 147.413$

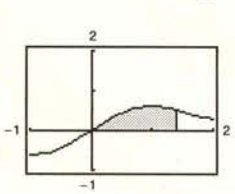


100. $\int_a^b e^{-x}\, dx = \left[-e^{-x}\right]_a^b = e^{-a} - e^{-b}$



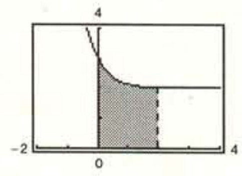
101. $\int_0^{\sqrt{2}} xe^{-(x^2/2)} dx = \left[-e^{-(x^2/2)}\right]_0^{\sqrt{2}}$

$= -e^{-1} + 1 \approx 0.632$



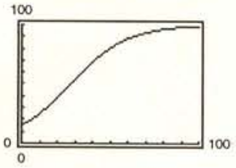
102. $\int_0^2 (e^{-2x} + 2)\, dx = \left[-\frac{1}{2}e^{-2x} + 2x\right]_0^2$

$= -\frac{1}{2}e^{-4} + 4 + \frac{1}{2} \approx 4.491$



65. $y = \dfrac{300}{3 + 17e^{-0.0625x}}$

(a)



(b) If $x = 2$ (2000 egg masses), $y \approx 16.67 \approx 16.7\%$.

(c) If $y = 66.67\%$, then $x \approx 38.8$ or 38,800 egg masses.

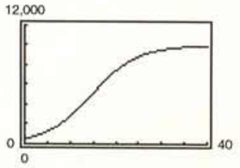
(d) $y = 300(3 + 17e^{-0.0625x})^{-1}$

$$y' = \frac{318.75e^{-0.0625x}}{(3 + 17e^{-0.0625x})^2}$$

$$y'' = \frac{19.921875e^{-0.0625x}(17e^{-0.0625x} - 3)}{(3 + 17e^{-0.0625x})^3}$$

$17e^{-0.0625x} - 3 = 0 \Rightarrow x \approx 27.8$ or 27,800 egg masses.

66. (a)



(b) Limiting size: 10,000 fish

(c) $p(t) = \dfrac{10{,}000}{1 + 19e^{-t/5}}$

$$p'(t) = \frac{e^{-t/5}}{(1 + 19e^{t/5})^2}\left(\frac{19}{5}\right)(10{,}000)$$

$$= \frac{38{,}000e^{-t/5}}{(1 + 19e^{-t/5})^2}$$

$p'(1) \approx 113.5$ fish/month

$p'(10) \approx 403.2$ fish/month

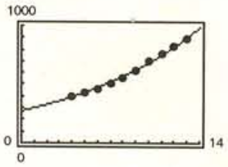
(d) $p''(t) = -\dfrac{38{,}000}{5}(e^{-t/5})\left[\dfrac{1 - 19e^{-t/5}}{(1 + 19e^{-t/5})^3}\right] = 0$

$19e^{-t/5} = 1$

$\dfrac{t}{5} = \ln 19$

$t = 5 \ln 19 \approx 14.72$

67. (a) $y = (271.92)(1.096)^x = (271.92)e^{0.0918x}$



(b) $y = -254.08 + 418.41 \ln x$

(c) The exponential model is better.

(d) Exponential model:

$$\frac{dy}{dx} = (2171.92)(0.0918)(e^{0.0918(20)})$$

$$\approx 157$$

Logarithmic model: $\dfrac{dy}{dx} = (418.41)\dfrac{1}{20} \approx 20.9$

Logarithmic model would be better.

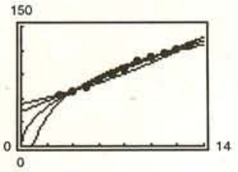
68. (a) $y_1 = 6.65x + 43.31$

$y_2 = 4.75 + 46.19 \ln x$

$y_3 = (52.97)(1.07)^x$

$y_4 = (34.31)(x^{0.51})$

(b)



The y_4 model seems best.

(c) The slope 6.65 is the annual rate of change in the amount given to philanthropy.

(d) $y_1' = 6.65$

$$y_2' = \frac{46.19}{12} = 3.85$$

$y_3' = (52.97)\ln(1.07)(1.07)^{12} = 7.54$

$y_4' = (34.31)(0.51)(12)^{0.51-1} = 5.18$

The third model is increasing most rapidly.

69. $\displaystyle\int 3^x\,dx = \frac{3^x}{\ln 3} + C$

70. $\displaystyle\int 4^{-x}\,dx = -\frac{4^{-x}}{\ln 4} + C$

71. $$\int_{-1}^{2} 2^x\,dx = \left[\frac{2^x}{\ln 2}\right]_{-1}^{2} = \frac{1}{\ln 2}\left[4 - \frac{1}{2}\right] = \frac{7}{2\ln 2} = \frac{7}{\ln 4}$$

72. $$\int_{-2}^{0} (3^3 - 5^2)\,dx = \int_{-2}^{0} (27 - 25)\,dx = \int_{-2}^{0} 2\,dx = \Big[2x\Big]_{-2}^{0} = 4$$

73. $$\int x5^{-x^2}\,dx = -\frac{1}{2}\int 5^{-x^2}(-2x)\,dx = -\left(\frac{1}{2}\right)\frac{5^{-x^2}}{\ln 5} + C = \frac{-1}{2\ln 5}(5^{-x^2}) + C$$

74. $$\int (3 - x)\,7^{(3-x)^2}\,dx = -\frac{1}{2}\int -2(3 - x)\,7^{(3-x)^2}\,dx = -\frac{1}{2\ln 7}[7^{(3-x)^2}] + C$$

75. $\displaystyle\int \frac{3^{2x}}{1 + 3^{2x}}\,dx,\ u = 1 + 3^{2x},\ du = 2(\ln 3)3^{2x}\,dx$

$$\frac{1}{2\ln 3}\int \frac{(2\ln 3)3^{2x}}{1 + 3^{2x}}\,dx = \frac{1}{2\ln 3}\ln(1 + 3^{2x}) + C$$

76. $\displaystyle\int 2^{\sin x}\cos x\,dx,\ u = \sin x,\ du = \cos x\,dx$

$$\frac{1}{\ln 2}2^{\sin x} + C$$

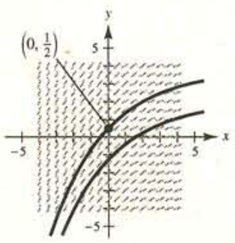
77. $\displaystyle\frac{dy}{dx} = 0.4^{x/3},\ \left(0, \frac{1}{2}\right)$

$$y = \int 0.4^{x/3}\,dx = 3\int 0.4^{x/3}\left(\frac{1}{3}\,dx\right)$$

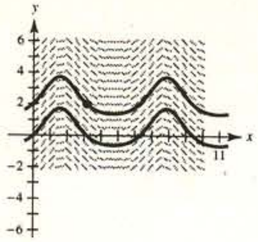
$$= \frac{3}{\ln 0.4}0.4^{x/3} + C = 3(\ln 2.5)(0.4)^{x/3} + C$$

$\left(0, \frac{1}{2}\right)$: $\frac{1}{2} = 3(\ln 2.5) + C \Rightarrow C = \frac{1}{2} - 3\ln 2.5$

$$y = 3\ln 2.5(0.4)^{x/3} + \frac{1}{2} - 3\ln 2.5 = \frac{3(1 - 0.4^{x/3})}{\ln 2.5} + \frac{1}{2}$$



78. (a)



(b) $\dfrac{dy}{dx} = e^{\sin x}\cos x \qquad (\pi, 2)$

$$y = \int e^{\sin x}\cos x\,dx = e^{\sin x} + C$$

$$(\pi, 2):\ 2 = e^{\sin \pi} + C = 1 + C \implies C = 1$$

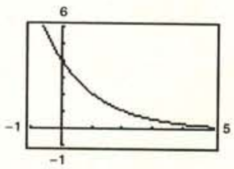
$$y = e^{\sin x} + 1$$

79. (a) $\displaystyle\int_0^4 f(t)\,dt \approx 5.67$

$\displaystyle\int_0^4 g(t)\,dt \approx 5.67$

$\displaystyle\int_0^4 h(t)\,dt \approx 5.67$

(b)



(c) The functions appear to be equal: $f(t) = g(t) = h(t)$

Analytically,

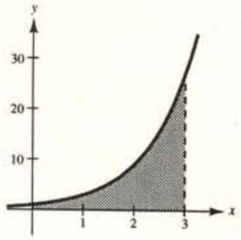
$$f(t) = 4\left(\frac{3}{8}\right)^{2t/3} = 4\left[\left(\frac{3}{8}\right)^{2/3}\right]^t = 4\left(\frac{9^{1/3}}{4}\right)^t = g(t)$$

$$h(t) = 4e^{-0.653886t} = 4[e^{-0.653886}]^t = 4(0.52002)^t$$

$$g(t) = 4\left(\frac{9^{1/3}}{4}\right)^t = 4(0.52002)^t$$

No. The definite integrals over a given interval may be equal when the functions are not equal.

80. $A = \displaystyle\int_0^3 3^x\,dx = \left[\frac{3^x}{\ln 3}\right]_0^3 = \frac{26}{\ln 3} \approx 23.666$



81. $P = \displaystyle\int_0^{10} 2000e^{-0.06t}\,dt$

$$= \left[\frac{2000}{-0.06}e^{-0.06t}\right]_0^{10}$$

$$\approx \$15{,}039.61$$

82.

x	1	10^{-1}	10^{-2}	10^{-4}	10^{-6}
$(1 + x)^{1/x}$	2	2.594	2.705	2.718	2.718

83.

t	0	1	2	3	4
y	1200	720	432	259.20	155.52

$y = C(k^t)$

When $t = 0$, $y = 1200 \implies C = 1200$.

$y = 1200(k^t)$

$$\frac{720}{1200} = 0.6,\ \frac{432}{720} = 0.6,\ \frac{259.20}{432} = 0.6,\ \frac{155.52}{259.20} = 0.6$$

Let $k = 0.6$.

$y = 1200(0.6)^t$

84.

t	0	1	2	3	4
y	600	630	661.50	694.58	729.30

$y = C(k^t)$

When $t = 0$, $y = 600 \implies C = 600$.

$y = 600(k^t)$

$$\frac{630}{600} = 1.05,\ \frac{661.50}{630} = 1.05,\ \frac{694.58}{661.50} \approx 1.05,\ \frac{729.30}{694.58} \approx 1.0$$

Let $k = 1.05$.

$y = 600(1.05)^t$

85. False. e is an irrational number.

86. True.

$$f(e^{n+1}) - f(e^n) = \ln e^{n+1} - \ln e^n$$
$$= n + 1 - n$$
$$= 1$$

87. True.

$$f(g(x)) = 2 + e^{\ln(x-2)}$$
$$= 2 + x - 2 = x$$
$$g(f(x)) = \ln(2 + e^x - 2)$$
$$= \ln e^x = x$$

88. True.

$$\frac{d^n y}{dx^n} = Ce^x$$
$$= y \text{ for } n = 1, 2, 3, \ldots$$

89. True.

$$\frac{d}{dx}[e^x] = e^x \text{ and } \frac{d}{dx}[e^{-x}] = -e^{-x}$$

$e^x = e^{-x}$ when $x = 0$.

$$(e^0)(-e^{-0}) = -1$$

90. True.

$$f(x) = g(x)e^x = 0 \Rightarrow$$

$g(x) = 0$ since $e^x > 0$ for all x.

91.
$$\frac{dy}{dt} = \frac{8}{25}y\left(\frac{5}{4} - y\right), y(0) = 1$$

$$\frac{dy}{y[(5/4) - y]} = \frac{8}{25}\,dt \Rightarrow \frac{4}{5}\int\left(\frac{1}{y} + \frac{1}{(5/4) - y}\right)dy = \int \frac{8}{25}\,dt \Rightarrow$$

$$\ln y - \ln\left(\frac{5}{4} - y\right) = \frac{2}{5}t + C$$

$$\ln\left(\frac{y}{(5/4) - y}\right) = \frac{2}{5}t + C$$

$$\frac{y}{(5/4) - y} = e^{(2/5)t + C} = C_1 e^{(2/5)t}$$

$$y(0) = 1 \Rightarrow C_1 = 4 \Rightarrow 4e^{(2/5)t} = \frac{y}{(5/4) - y}$$

$$\Rightarrow 4e^{(2/5)t}\left(\frac{5}{4} - y\right) = y \Rightarrow 5e^{(2/5)t} = 4e^{(2/5)t}y + y = (4e^{(2/5)t} + 1)y$$

$$\Rightarrow y = \frac{5e^{(2/5)t}}{4e^{(2/5)t} + 1} = \frac{5}{4 + e^{-0.4t}} = \frac{1.25}{1 + 0.25e^{-0.4t}}$$

92. $y = x^{\sin x}$

$$\ln y = \ln x^{\sin x} = \sin x \cdot \ln x$$

$$\frac{y'}{y} = \sin x\left(\frac{1}{x}\right) + \cos x \cdot \ln x$$

$$y' = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \cdot \ln x\right]$$

At $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$,

$$y' = \left(\frac{\pi}{2}\right)^{\sin(\pi/2)}\left[\frac{\sin(\pi/2)}{\pi/2} + \cos\left(\frac{\pi}{2}\right)\ln\left(\frac{\pi}{2}\right)\right]$$
$$= \frac{\pi}{2}\left[\frac{2}{\pi} + 0\right] = 1$$

Tangent line: $y - \frac{\pi}{2} = 1\left(x - \frac{\pi}{2}\right)$

$$y = x$$