67. $50 - 0.5x = 0.125x$

$x = 80$

$p_1(80) = p_2(80) = 10$

Point of equilibrium: (80, 10)

$$CS = \int_0^{80} [(50 - 0.5x) - 10]\,dx = \left[-\frac{0.5x^2}{2} + 40x\right]_0^{80} = 1600$$

$$PS = \int_0^{80} [10 - 0.125x]\,dx = \left[10x - \frac{0.125x^2}{2}\right]_0^{80} = 400$$

68. $1000 - 0.4x^2 = 42x$

$x = 20$

$p_1(20) = p_2(20) = 840$

Point of equilibrium: (20, 840)

$$CS = \int_0^{20} [(1000 - 0.4x^2) - 840]\,dx = \left[160x - \frac{0.4x^3}{3}\right]_0^{20} \approx 2133.33$$

$$PS = \int_0^{20} [840 - 42x]\,dx = \left[840x - 21x^2\right]_0^{20} = 8400$$

69. True

70. True

Section 6.2 Volume: The Disc Method

1. $V = \pi\int_0^1 (-x + 1)^2\,dx = \pi\int_0^1 (x^2 - 2x + 1)\,dx = \pi\left[\frac{x^3}{3} - x^2 + x\right]_0^1 = \frac{\pi}{3}$

2. $V = \pi\int_0^2 (4 - x^2)^2\,dx = \pi\int_0^2 (x^4 - 8x^2 + 16)\,dx = \pi\left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right]_0^2 = \frac{256\pi}{15}$

3. $V = \pi\int_1^4 (\sqrt{x})^2\,dx = \pi\int_1^4 x\,dx = \pi\left[\frac{x^2}{2}\right]_1^4 = \frac{15\pi}{2}$

4. $V = \pi\int_0^2 \left(\sqrt{4 - x^2}\right)^2 dx = \pi\int_0^2 (4 - x^2)\,dx = \pi\left[4x - \frac{x^3}{3}\right]_0^2 = \frac{16\pi}{3}$

5. $V = \pi\int_0^1 [(x^2)^2 - (x^3)^2]\,dx = \pi\int_0^1 (x^4 - x^6)\,dx = \pi\left[\frac{x^5}{5} - \frac{x^7}{7}\right]_0^1 = \frac{2\pi}{35}$

6. $2 = 4 - \frac{x^2}{4}$

$8 = 16 - x^2$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

$$V = \pi\int_{-2\sqrt{2}}^{2\sqrt{2}} \left[\left(4 - \frac{x^2}{4}\right)^2 - (2)^2\right] dx$$

$$= 2\pi\int_0^{2\sqrt{2}} \left[\frac{x^4}{16} - 2x^2 + 12\right] dx$$

$$= 2\pi\left[\frac{x^5}{80} - \frac{2x^3}{3} + 12x\right]_0^{2\sqrt{2}}$$

$$= 2\pi\left[\frac{128\sqrt{2}}{80} - \frac{32\sqrt{2}}{3} + 24\sqrt{2}\right]$$

$$= \frac{448\sqrt{2}}{15}\pi \approx 132.69$$

7. $y = x^2 \Rightarrow x = \sqrt{y}$

$$V = \pi \int_0^4 (\sqrt{y})^2\,dy = \pi \int_0^4 y\,dy$$

$$= \pi \left[\frac{y^2}{2}\right]_0^4 = 8\pi$$

8. $y = \sqrt{16 - x^2} \Rightarrow x = \sqrt{16 - y^2}$

$$V = \pi \int_0^4 \left(\sqrt{16 - y^2}\right)^2 dy = \pi \int_0^4 (16 - y^2)\,dy$$

$$= \pi \left[16y - \frac{y^3}{3}\right]_0^4 = \frac{128\pi}{3}$$

9. $y = x^{2/3} \Rightarrow x = y^{3/2}$

$$V = \pi \int_0^1 (y^{3/2})^2\,dy = \pi \int_0^1 y^3\,dy = \pi\left[\frac{y^4}{4}\right]_0^1 = \frac{\pi}{4}$$

10. $V = \pi \int_1^4 (-y^2 + 4y)^2\,dy = \pi \int_1^4 (y^4 - 8y^3 + 16y^2)\,dy$

$$= \pi\left[\frac{y^5}{5} - 2y^4 + \frac{16y^3}{3}\right]_1^4$$

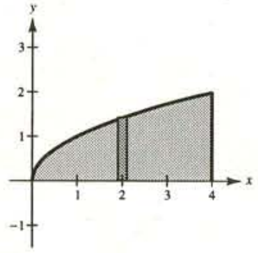
$$= \frac{459\pi}{15} = \frac{153\pi}{5}$$

11. $y = \sqrt{x},\ y = 0,\ x = 4$

(a) $R(x) = \sqrt{x},\ r(x) = 0$

$$V = \pi \int_0^4 \left(\sqrt{x}\right)^2 dx$$

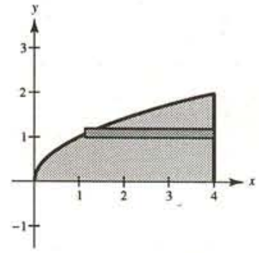
$$= \pi \int_0^4 x\,dx = \left[\frac{\pi}{2}x^2\right]_0^4 = 8\pi$$



(b) $R(y) = 4,\ r(y) = y^2$

$$V = \pi \int_0^2 (16 - y^4)\,dy$$

$$= \pi\left[16y - \frac{1}{5}y^5\right]_0^2 = \frac{128\pi}{5}$$

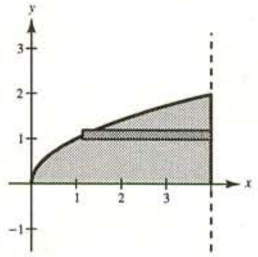


(c) $R(y) = 4 - y^2,\ r(y) = 0$

$$V = \pi \int_0^2 (4 - y^2)^2\,dy$$

$$= \pi \int_0^2 (16 - 8y^2 + y^4)\,dy$$

$$= \pi\left[16y - \frac{8}{3}y^3 + \frac{1}{5}y^5\right]_0^2 = \frac{256\pi}{15}$$

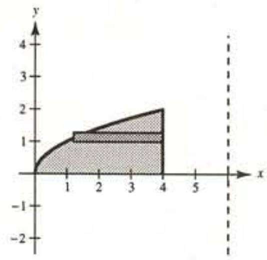


(d) $R(y) = 6 - y^2,\ r(y) = 2$

$$V = \pi \int_0^2 \left[(6 - y^2)^2 - 4\right] dy$$

$$= \pi \int_0^2 (32 - 12y^2 + y^4)\,dy$$

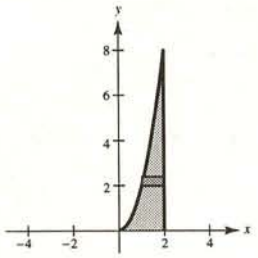
$$= \pi\left[32y - 4y^3 + \frac{1}{5}y^5\right]_0^2 = \frac{192\pi}{5}$$



12. $y = 2x^2, \ y = 0, \ x = 2$

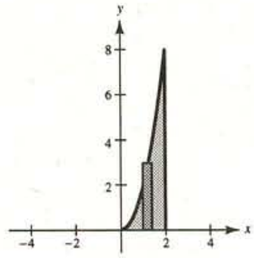
(a) $R(y) = 2, \ r(y) = \sqrt{y/2}$

$$V = \pi\int_0^8 \left(4 - \frac{y}{2}\right) dy = \pi\left[4y - \frac{y^2}{4}\right]_0^8 = 16\pi$$



(b) $R(x) = 2x^2, \ r(x) = 0$

$$V = \pi\int_0^2 4x^4\,dx = \pi\left[\frac{4x^5}{5}\right]_0^2 = \frac{128\pi}{5}$$

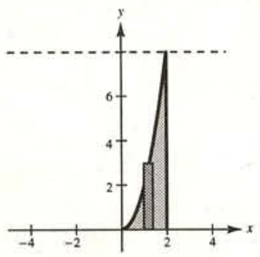


(c) $R(x) = 8, \ r(x) = 8 - 2x^2$

$$V = \pi\int_0^2 [64 - (64 - 32x^2 + 4x^4)]\,dx$$

$$= \pi\int_0^2 (32x^2 - 4x^4)\,dx = 4\pi\int_0^2 (8x^2 - x^4)\,dx$$

$$= 4\pi\left[\frac{8}{3}x^3 - \frac{1}{5}x^5\right]_0^2 = \frac{896\pi}{15}$$

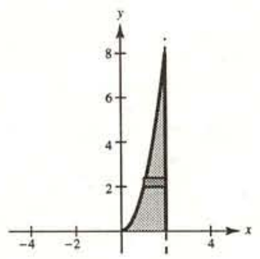


(d) $R(y) = 2 - \sqrt{y/2}, \ r(y) = 0$

$$V = \pi\int_0^8 \left(2 - \sqrt{\frac{y}{2}}\right)^2 dy$$

$$= \pi\int_0^8 \left(4 - 4\sqrt{\frac{y}{2}} + \frac{y}{2}\right) dy$$

$$= \pi\left[4y - \frac{4\sqrt{2}}{3}y^{3/2} + \frac{y^2}{4}\right]_0^8 = \frac{16\pi}{3}$$



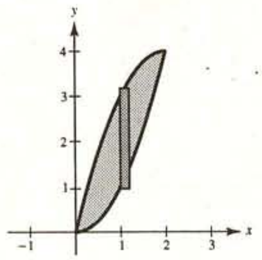
13. $y = x^2, \ y = 4x - x^2$ intersect at $(0, 0)$ and $(2, 4)$.

(a) $R(x) = 4x - x^2 \ r(x) = x^2$

$$V = \pi\int_0^2 [(4x - x^2)^2 - x^4]\,dx$$

$$= \pi\int_0^2 (16x^2 - 8x^3)\,dx$$

$$= \pi\left[\frac{16}{3}x^3 - 2x^4\right]_0^2 = \frac{32\pi}{3}$$

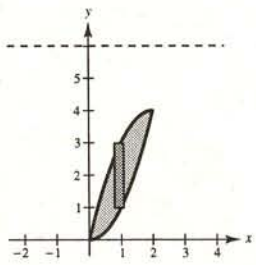


(b) $R(x) = 6 - x^2, \ r(x) = 6 - (4x - x^2)$

$$V = \pi\int_0^2 [(6 - x^2)^2 - (6 - 4x + x^2)^2]\,dx$$

$$= 8\pi\int_0^2 (x^3 - 5x^2 + 6x)\,dx$$

$$= 8\pi\left[\frac{x^4}{4} - \frac{5}{3}x^3 + 3x^2\right]_0^2 = \frac{64\pi}{3}$$



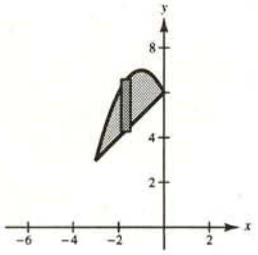
14. $y = 6 - 2x - x^2$, $y = x + 6$ intersect at $(-3, 3)$ and $(0, 6)$.

(a) $R(x) = 6 - 2x - x^2$, $r(x) = x + 6$

$$V = \pi\int_{-3}^{0} [(6 - 2x - x^2)^2 - (x + 6)^2]\, dx$$

$$= \pi\int_{-3}^{0} (x^4 + 4x^3 - 9x^2 - 36x)\, dx$$

$$= \pi\left[\frac{1}{5}x^5 + x^4 - 3x^3 - 18x^2\right]_{-3}^{0} = \frac{243\pi}{5}$$

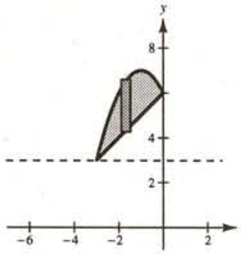


(b) $R(x) = (6 - 2x - x^2) - 3$, $r(x) = (x + 6) - 3$

$$V = \pi\int_{-3}^{0} [(3 - 2x - x^2)^2 - (x + 3)^2]\, dx$$

$$= \pi\int_{-3}^{0} (x^4 + 4x^3 - 3x^2 - 18x)\, dx$$

$$= \pi\left[\frac{1}{5}x^5 + x^4 - x^3 - 9x^2\right]_{-3}^{0} = \frac{108\pi}{5}$$

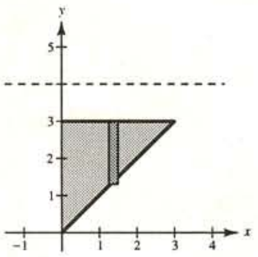


15. $R(x) = 4 - x$, $r(x) = 1$

$$V = \pi\int_{0}^{3} [(4 - x)^2 - (1)^2]\, dx$$

$$= \pi\int_{0}^{3} (x^2 - 8x + 15)\, dx$$

$$= \pi\left[\frac{x^3}{3} - 4x^2 + 15x\right]_{0}^{3} = 18\pi$$



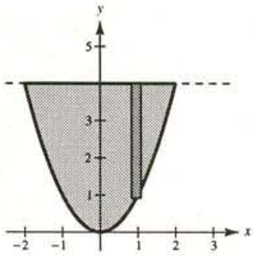
16. $R(x) = 4 - x^2$, $r(x) = 0$

$$V = 2\pi\int_{0}^{2} (4 - x^2)^2\, dx$$

$$= 2\pi\int_{0}^{2} (x^4 - 8x^2 + 16)\, dx$$

$$= 2\pi\left[\frac{x^5}{5} - \frac{8x^3}{3} + 16x\right]_{0}^{2}$$

$$= \frac{512\pi}{15}$$



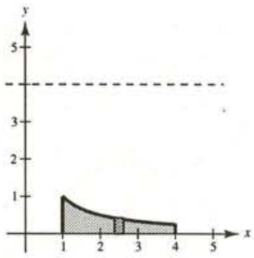
17. $R(x) = 4$, $r(x) = 4 - \frac{1}{x}$

$$V = \pi\int_{1}^{4}\left[(4)^2 - \left(4 - \frac{1}{x}\right)^2\right] dx$$

$$= \pi\int_{1}^{4}\left(\frac{8}{x} - \frac{1}{x^2}\right) dx$$

$$= \pi\left[8\ln|x| + \frac{1}{x}\right]_{1}^{4}$$

$$= \pi\left(8\ln 4 - \frac{3}{4}\right) \approx 32.49$$



18. $R(x) = 4$, $r(x) = 4 - \sec x$

$$V = \pi\int_{0}^{\pi/3} [(4)^2 - (4 - \sec x)^2]\, dx$$

$$= \pi\int_{0}^{\pi/3} (8\sec x - \sec^2 x)\, dx$$

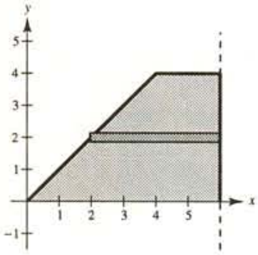
$$= \pi\left[8\ln|\sec x + \tan x| - \tan x\right]_{0}^{\pi/3}$$

$$= \pi\left[\left(8\ln|2 + \sqrt{3}| - \sqrt{3}\right) - \left(8\ln|1 + 0| - 0\right)\right]$$

$$= \pi\left[8\ln\left(2 + \sqrt{3}\right) - \sqrt{3}\right] \approx 27.66$$

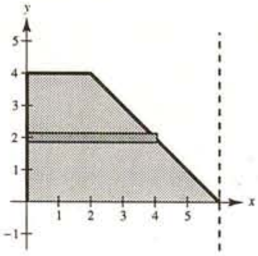
19. $R(y) = 6 - y,\ r(y) = 0$

$$V = \pi\int_0^4 (6 - y)^2\, dy$$
$$= \pi\int_0^4 (y^2 - 12y + 36)\, dy$$
$$= \pi\left[\frac{y^3}{3} - 6y^2 + 36y\right]_0^4$$
$$= \frac{208\pi}{3}$$



20. $R(y) = 6,\ r(y) = 6 - (6 - y) = y$

$$V = \pi\int_0^4 [(6)^2 - (y)^2]\, dy$$
$$= \pi\left[36y - \frac{y^3}{3}\right]_0^4 = \frac{368\pi}{3}$$

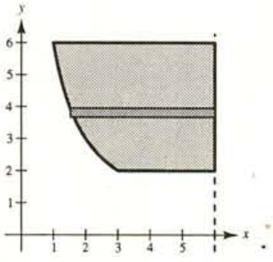


21. $R(y) = 6 - y^2,\ r(y) = 2$

$$V = \pi\int_{-2}^2 [(6 - y^2)^2 - (2)^2]\, dy$$
$$= 2\pi\int_0^2 (y^4 - 12y^2 + 32)\, dy$$
$$= 2\pi\left[\frac{y^5}{5} - 4y^3 + 32y\right]_0^2$$
$$= \frac{384\pi}{5}$$

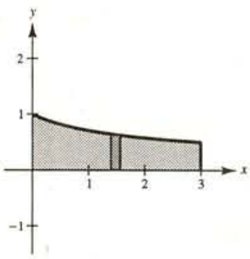
22. $R(y) = 6 - \frac{6}{y},\ r(y) = 0$

$$V = \pi\int_2^6 \left(6 - \frac{6}{y}\right)^2 dy$$
$$= 36\pi\int_2^6 \left(1 - \frac{2}{y} + \frac{1}{y^2}\right) dy$$
$$= 36\pi\left[y - 2\ln|y| - \frac{1}{y}\right]_2^6$$
$$= 36\pi\left[\left(\frac{35}{6} - 2\ln 6\right) - \left(\frac{3}{2} - 2\ln 2\right)\right]$$
$$= 36\pi\left(\frac{13}{3} + 2\ln\frac{1}{3}\right) = 12\pi(13 - 6\ln 3) \approx 241.59$$



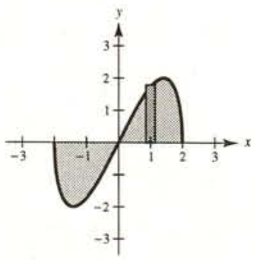
23. $R(x) = \frac{1}{\sqrt{x + 1}},\ r(x) = 0$

$$V = \pi\int_0^3 \left(\frac{1}{\sqrt{x + 1}}\right)^2 dx$$
$$= \pi\int_0^3 \frac{1}{x + 1}\, dx$$
$$= \left[\pi\ln|x + 1|\right]_0^3 = \pi\ln 4$$



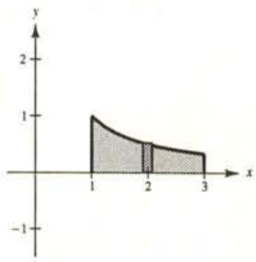
24. $R(x) = x\sqrt{4 - x^2},\quad r(x) = 0$

$$V = 2\pi\int_0^2 \left[x\sqrt{4 - x^2}\right]^2 dx$$
$$= 2\pi\int_0^2 (4x^2 - x^4)\, dx$$
$$= 2\pi\left[\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2$$
$$= \frac{128\pi}{15}$$



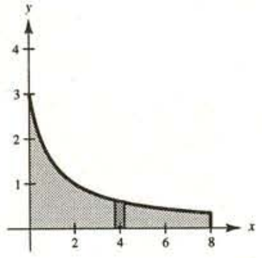
25. $R(x) = \frac{1}{x},\ r(x) = 0$

$$V = \pi\int_1^4 \left(\frac{1}{x}\right)^2 dx$$
$$= \pi\left[-\frac{1}{x}\right]_1^4$$
$$= \frac{3\pi}{4}$$



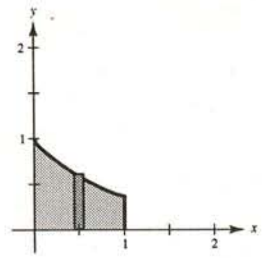
26. $R(x) = \dfrac{3}{x+1},\ r(x) = 0$

$$V = \pi\int_0^8 \left(\frac{3}{x+1}\right)^2 dx$$
$$= 9\pi\int_0^8 (x+1)^{-2}\,dx$$
$$= 9\pi\left[-\frac{1}{x+1}\right]_0^8 = 8\pi$$



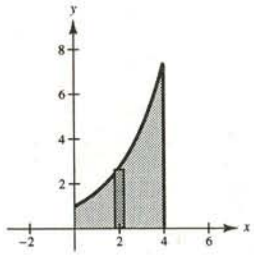
27. $R(x) = e^{-x},\ r(x) = 0$

$$V = \pi\int_0^1 (e^{-x})^2\,dx$$
$$= \pi\int_0^1 e^{-2x}\,dx$$
$$= \left[-\frac{\pi}{2}e^{-2x}\right]_0^1$$
$$= \frac{\pi}{2}(1 - e^{-2}) \approx 1.358$$



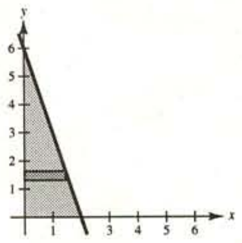
28. $R(x) = e^{x/2},\ r(x) = 0$

$$V = \pi\int_0^4 (e^{x/2})^2\,dx$$
$$= \pi\int_0^4 e^x\,dx$$
$$= \left[\pi e^x\right]_0^4$$
$$= \pi(e^4 - 1) \approx 168.38$$



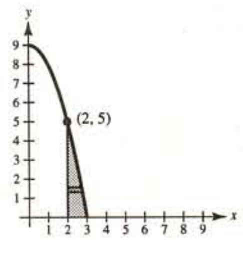
29. $y = 6 - 3x \Rightarrow x = \dfrac{1}{3}(6 - y)$

$$V = \pi\int_0^6 \left[\frac{1}{3}(6-y)\right]^2 dy$$
$$= \frac{\pi}{9}\int_0^6 [36 - 12y + y^2]\,dy$$
$$= \frac{\pi}{9}\left[36y - 6y^2 + \frac{y^3}{3}\right]_0^6$$
$$= \frac{\pi}{9}\left[216 - 216 + \frac{216}{3}\right]$$
$$= 8\pi$$

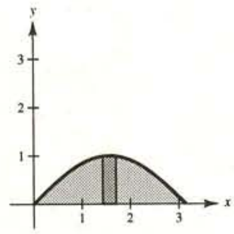


30. $y = 9 - x^2,\ y = 0,\ x = 2,\ x = 3$

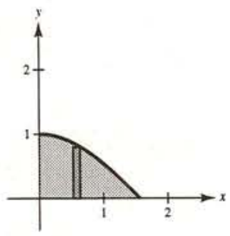
$$x = \sqrt{9-y}$$
$$V = \int_2^5 \left[\left(\sqrt{9-y}\right)^2 - 2^2\right] dy$$
$$= \int_2^5 (5 - y)\,dy$$
$$= \left[5y - \frac{y^2}{2}\right]_2^5$$
$$= \left(25 - \frac{25}{2}\right) - (10 - 2) = \frac{9}{2}$$



31. $V = \pi\displaystyle\int_0^{\pi} [\sin x]^2\,dx \approx 4.9348$



32. $V = \pi\displaystyle\int_0^{\pi/2} [\cos x]^2\,dx \approx 2.4674$



33. $V = \pi \int_0^2 [e^{-x^2}]^2\, dx \approx 1.9686$

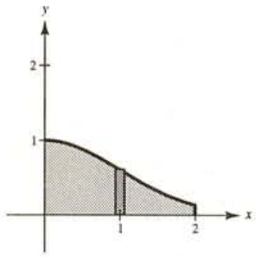
34. $V = \pi \int_1^3 [\ln x]^2\, dx \approx 3.2332$

35. $V = \pi \int_{-1}^2 [e^{x/2} + e^{-x/2}]^2\, dx \approx 49.0218$

36. $V = \pi \int_0^5 [2 \arctan (0.2x)]^2\, dx \approx 15.4115$

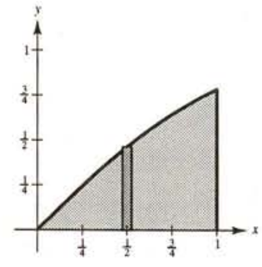
37. $A \approx 3$

Matches (a)



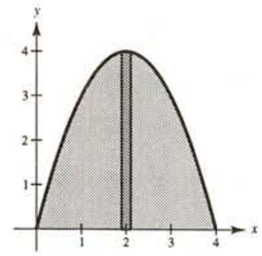
38. $A \approx \frac{3}{4}$

Matches (b)



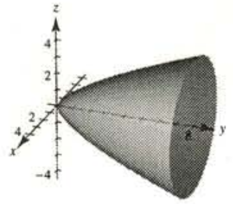
39. (a) $R(x) = 4x - x^2,\ r(x) = 0$

$$V = \pi \int_0^4 (4x - x^2)^2\, dx$$
$$= \pi \int_0^4 (16x^2 - 8x^3 + x^4)\, dx$$
$$= \pi \left[\frac{16}{3}x^3 - 2x^4 + \frac{x^5}{5}\right]_0^4 = \frac{512\pi}{15}$$

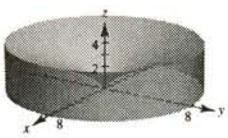


(b) Completing the square we have $4x - x^2 = 4 - (x^2 - 4x + 4) = 4 - (x - 2)^2$. Thus, $y = 4 - x^2$ has the same volume as in part (a) since the solid has been translated only horizontally.

40. (a)



(b)



(c)

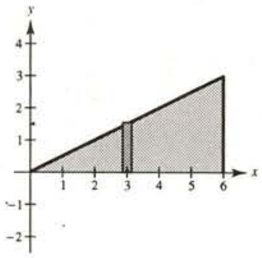
$a < c < b$.

41. $R(x) = \frac{1}{2}x,\ r(x) = 0$

$$V = \pi \int_0^6 \frac{1}{4}x^2\, dx$$
$$= \left[\frac{\pi}{12}x^3\right]_0^6 = 18\pi$$

Note: $V = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(3^2)6$$
$$= 18\pi$$



42. $R(x) = \frac{r}{h}x,\ r(x) = 0$

$$V = \pi \int_0^h \frac{r^2}{h^2}x^2\, dx$$
$$= \left[\frac{r^2\pi}{3h^2}x^3\right]_0^h$$
$$= \frac{r^2\pi}{3h^2}h^3 = \frac{1}{3}\pi r^2 h$$