

6. $\int \ln 3x \, dx$

$u = \ln 3x, \, dv = dx$

7. $\int x \sec^2 x \, dx$

$u = x, \, dv = \sec^2 x \, dx$

8. $\int x^2 \cos x \, dx$

$u = x^2, \, dv = \cos x \, dx$

9. $dv = e^{-2x} \, dx \Rightarrow v = \int e^{-2x} \, dx = -\frac{1}{2}e^{-2x}$

$u = x \Rightarrow du = dx$

$$\begin{aligned} \int x e^{-2x} \, dx &= -\frac{1}{2} x e^{-2x} - \int -\frac{1}{2} e^{-2x} \, dx \\ &= -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C = \frac{-1}{4e^{2x}}(2x + 1) + C \end{aligned}$$

10. $dv = e^{-x} \, dx \Rightarrow v = \int e^{-x} \, dx = -e^{-x}$

$u = x \Rightarrow du = dx$

$$\begin{aligned} \int \frac{x}{e^x} \, dx &= \int x e^{-x} \, dx \\ &= -x e^{-x} - \int -e^{-x} \, dx = -x e^{-x} - e^{-x} + C \end{aligned}$$

11. Use integration by parts three times.

(1) $dv = e^x \, dx \Rightarrow v = \int e^x \, dx = e^x$

$u = x^3 \Rightarrow du = 3x^2 \, dx$

(2) $dv = e^x \, dx \Rightarrow v = \int e^x \, dx = e^x$

$u = x^2 \Rightarrow du = 2x \, dx$

(3) $dv = e^x \, dx \Rightarrow v = \int e^x \, dx = e^x$

$u = x \Rightarrow du = dx$

$$\begin{aligned} \int x^3 e^x \, dx &= x^3 e^x - 3 \int x^2 e^x \, dx = x^3 e^x - 3x^2 e^x + 6 \int x e^x \, dx \\ &= x^3 e^x - 3x^2 e^x + 6x e^x - 6e^x + C = e^x(x^3 - 3x^2 + 6x - 6) + C \end{aligned}$$

12. $\int \frac{e^{1/t}}{t^2} \, dt = -\int e^{1/t} \left(\frac{-1}{t^2} \right) dt = -e^{1/t} + C$

13. $\int x^2 e^{x^3} \, dx = \frac{1}{3} \int e^{x^3} (3x^2) \, dx = \frac{1}{3} e^{x^3} + C$

14. $dv = x^3 \, dx \Rightarrow v = \int x^3 \, dx = \frac{x^4}{4}$

$u = \ln x \Rightarrow du = \frac{1}{x} \, dx$

$$\begin{aligned} \int x^3 \ln x \, dx &= \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 \, dx = \frac{x^4}{4} \ln x - \frac{x^4}{16} + C \\ &= \frac{x^4}{16} (4 \ln x - 1) + C \end{aligned}$$

15. $dv = t \, dt \Rightarrow v = \int t \, dt = \frac{t^2}{2}$

$u = \ln(t+1) \Rightarrow du = \frac{1}{t+1} \, dt$

$$\begin{aligned} \int t \ln(t+1) \, dt &= \frac{t^2}{2} \ln(t+1) - \frac{1}{2} \int \frac{t^2}{t+1} \, dt \\ &= \frac{t^2}{2} \ln(t+1) - \frac{1}{2} \int \left(t - 1 + \frac{1}{t+1} \right) dt \\ &= \frac{t^2}{2} \ln(t+1) - \frac{1}{2} \left[\frac{t^2}{2} - t + \ln(t+1) \right] + C \\ &= \frac{1}{4} [2(t^2 - 1) \ln|t+1| - t^2 + 2t] + C \end{aligned}$$

16. Let $u = \ln x, \, du = \frac{1}{x} \, dx$.

$$\int \frac{1}{x(\ln x)^3} \, dx = \int (\ln x)^{-3} \left(\frac{1}{x} \right) dx = \frac{-1}{2(\ln x)^2} + C$$

17. Let $u = \ln x, \, du = \frac{1}{x} \, dx$.

$$\int \frac{(\ln x)^2}{x} \, dx = \int (\ln x)^2 \left(\frac{1}{x} \right) dx = \frac{(\ln x)^3}{3} + C$$

$$18. dv = \frac{1}{x^2} dx \Rightarrow v = \int \frac{1}{x^2} dx = -\frac{1}{x}$$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$\int \frac{\ln x}{x^2} dx = -\frac{\ln x}{x} + \int \frac{1}{x^2} dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

$$19. dv = \frac{1}{(2x+1)^2} dx \Rightarrow v = \int (2x+1)^{-2} dx = -\frac{1}{2(2x+1)}$$

$$u = xe^{2x} \Rightarrow du = (2xe^{2x} + e^{2x}) dx = e^{2x}(2x+1) dx$$

$$\begin{aligned} \int \frac{xe^{2x}}{(2x+1)^2} dx &= -\frac{xe^{2x}}{2(2x+1)} + \int \frac{e^{2x}}{2} dx \\ &= -\frac{xe^{2x}}{2(2x+1)} + \frac{e^{2x}}{4} + C \\ &= \frac{e^{2x}}{4(2x+1)} + C \end{aligned}$$

$$20. dv = \frac{x}{(x^2+1)^2} dx \Rightarrow v = \int (x^2+1)^{-2} x dx = -\frac{1}{2(x^2+1)}$$

$$u = x^2 e^{x^2} \Rightarrow du = (2x^3 e^{x^2} + 2x e^{x^2}) dx = 2x e^{x^2} (x^2+1) dx$$

$$\int \frac{x^3 e^{x^2}}{(x^2+1)^2} dx = -\frac{x^2 e^{x^2}}{2(x^2+1)} + \int x e^{x^2} dx = -\frac{x^2 e^{x^2}}{2(x^2+1)} + \frac{e^{x^2}}{2} + C = \frac{e^{x^2}}{2(x^2+1)} + C$$

21. Use integration by parts twice.

$$(1) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = x^2 \Rightarrow du = 2x dx$$

$$\int (x^2-1)e^x dx = \int x^2 e^x dx - \int e^x dx = x^2 e^x - 2 \int x e^x dx - e^x$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right] - e^x = x^2 e^x - 2x e^x + e^x + C = (x-1)^2 e^x + C$$

$$(2) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = x \Rightarrow du = dx$$

$$22. dv = \frac{1}{x^2} dx \Rightarrow v = \int \frac{1}{x^2} dx = -\frac{1}{x}$$

$$u = \ln 2x \Rightarrow du = \frac{1}{x} dx$$

$$\begin{aligned} \int \frac{\ln(2x)}{x^2} dx &= -\frac{\ln(2x)}{x} + \int \frac{1}{x^2} dx = -\frac{\ln(2x)}{x} - \frac{1}{x} + C \\ &= -\frac{\ln(2x)+1}{x} + C \end{aligned}$$

$$23. dv = \sqrt{x-1} dx \Rightarrow v = \int (x-1)^{1/2} dx = \frac{2}{3}(x-1)^{3/2}$$

$$u = x \Rightarrow du = dx$$

$$\begin{aligned} \int x \sqrt{x-1} dx &= \frac{2}{3} x (x-1)^{3/2} - \frac{2}{3} \int (x-1)^{3/2} dx \\ &= \frac{2}{3} x (x-1)^{3/2} - \frac{4}{15} (x-1)^{5/2} + C \\ &= \frac{2(x-1)^{3/2}}{15} (3x+2) + C \end{aligned}$$

$$24. dv = \frac{1}{\sqrt{2+3x}} dx \Rightarrow v = \int (2+3x)^{-1/2} dx = \frac{2}{3} \sqrt{2+3x}$$

$$u = x \Rightarrow du = dx$$

$$\int \frac{x}{\sqrt{2+3x}} dx = \frac{2x\sqrt{2+3x}}{3} - \frac{2}{3} \int \sqrt{2+3x} dx$$

$$= \frac{2x\sqrt{2+3x}}{3} - \frac{4}{27} (2+3x)^{3/2} + C = \frac{2\sqrt{2+3x}}{27} [9x - 2(2+3x)] + C = \frac{2\sqrt{2+3x}}{27} (3x-4) + C$$

$$25. dv = \cos x dx \Rightarrow v = \int \cos x dx = \sin x$$

$$u = x \Rightarrow du = dx$$

$$\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$$26. dv = \sec \theta \tan \theta d\theta \Rightarrow v = \int \sec \theta \tan \theta d\theta = \sec \theta$$

$$u = \theta \Rightarrow du = d\theta$$

$$\begin{aligned} \int \theta \sec \theta \tan \theta d\theta &= \theta \sec \theta - \int \sec \theta d\theta \\ &= \theta \sec \theta - \ln|\sec \theta + \tan \theta| + C \end{aligned}$$

$$27. dv = dx \Rightarrow v = \int dx = x$$

$$u = \arctan x \Rightarrow du = \frac{1}{1+x^2} dx$$

$$\begin{aligned} \int \arctan x dx &= x \arctan x - \int \frac{x}{1+x^2} dx \\ &= x \arctan x - \frac{1}{2} \ln(1+x^2) + C \end{aligned}$$

$$28. dv = dx \Rightarrow v = \int dx = x$$

$$u = \arccos x \Rightarrow du = -\frac{1}{\sqrt{1-x^2}} dx$$

$$\begin{aligned} \int \arccos x dx &= x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx \\ &= x \arccos x - \sqrt{1-x^2} + C \end{aligned}$$

29. Use integration by parts twice.

$$(1) dv = e^{2x} dx \Rightarrow v = \int e^{2x} dx = \frac{1}{2} e^{2x}$$

$$u = \sin x \Rightarrow du = \cos x dx$$

$$\int e^{2x} \sin x dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{2} \int e^{2x} \cos x dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{2} \left(\frac{1}{2} e^{2x} \cos x + \frac{1}{2} \int e^{2x} \sin x dx \right)$$

$$\frac{5}{4} \int e^{2x} \sin x dx = \frac{1}{2} e^{2x} \sin x - \frac{1}{4} e^{2x} \cos x$$

$$\int e^{2x} \sin x dx = \frac{1}{5} e^{2x} (2 \sin x - \cos x) + C$$

$$(2) dv = e^{2x} dx \Rightarrow v = \int e^{2x} dx = \frac{1}{2} e^{2x}$$

$$u = \cos x \Rightarrow du = -\sin x dx$$

30. Use integration by parts twice.

$$(1) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = \cos 2x \Rightarrow du = -2 \sin 2x dx$$

$$\int e^x \cos 2x dx = e^x \cos 2x + 2 \int e^x \sin 2x dx = e^x \cos 2x + 2 \left(e^x \sin 2x - 2 \int e^x \cos 2x dx \right)$$

$$5 \int e^x \cos 2x dx = e^x \cos 2x + 2e^x \sin 2x$$

$$\int e^x \cos 2x dx = \frac{e^x}{5} (\cos 2x + 2 \sin 2x) + C$$

$$(2) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = \sin 2x \Rightarrow du = 2 \cos 2x dx$$

$$31. y' = xe^{x^2}$$

$$y = \int xe^{x^2} dx = \frac{1}{2} e^{x^2} + C$$

$$32. dv = dx \Rightarrow v = x$$

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$y' = \ln x$$

$$y = \int \ln x dx = x \ln x - \int x \left(\frac{1}{x} \right) dx$$

$$= x \ln x - x + C = x(-1 + \ln x) + C$$

33. Use integration by parts twice.

$$(1) dv = \frac{1}{\sqrt{2+3t}} dt \Rightarrow v = \int (2+3t)^{-1/2} dt = \frac{2}{3} \sqrt{2+3t}$$

$$u = t^2 \quad \Rightarrow du = 2t dt$$

$$(2) dv = \sqrt{2+3t} dt \Rightarrow v = \int (2+3t)^{1/2} dt = \frac{2}{9} (2+3t)^{3/2}$$

$$u = t \quad \Rightarrow du = dt$$

$$\begin{aligned} y &= \int \frac{t^2}{\sqrt{2+3t}} dt = \frac{2t^2\sqrt{2+3t}}{3} - \frac{4}{3} \int t\sqrt{2+3t} dt \\ &= \frac{2t^2\sqrt{2+3t}}{3} - \frac{4}{3} \left[\frac{2t}{9} (2+3t)^{3/2} - \frac{2}{9} \int (2+3t)^{3/2} dt \right] \\ &= \frac{2t^2\sqrt{2+3t}}{3} - \frac{8t}{27} (2+3t)^{3/2} + \frac{16}{405} (2+3t)^{5/2} + C \\ &= \frac{2\sqrt{2+3t}}{405} (27t^2 - 24t + 32) + C \end{aligned}$$

34. Use integration by parts twice.

$$(1) dv = \sqrt{x-1} dx \Rightarrow v = \int (x-1)^{1/2} dx = \frac{2}{3} (x-1)^{3/2}$$

$$u = x^2 \quad \Rightarrow du = 2x dx$$

$$(2) dv = (x-1)^{3/2} dx \Rightarrow v = \int (x-1)^{3/2} dx = \frac{2}{5} (x-1)^{5/2}$$

$$u = x \quad \Rightarrow du = dx$$

$$\begin{aligned} y &= \int x^2 \sqrt{x-1} dx \\ &= \frac{2}{3} x^2 (x-1)^{3/2} - \frac{4}{3} \int x(x-1)^{3/2} dx \\ &= \frac{2}{3} x^2 (x-1)^{3/2} - \frac{4}{3} \left[\frac{2}{5} x(x-1)^{5/2} - \frac{2}{5} \int (x-1)^{5/2} dx \right] \\ &= \frac{2}{3} x^2 (x-1)^{3/2} - \frac{8}{15} x(x-1)^{5/2} + \frac{16}{105} (x-1)^{7/2} + C \\ &= \frac{2(x-1)^{3/2}}{105} (15x^2 + 12x + 8) + C \end{aligned}$$

35. $(\cos y)y' = 2x$

$$\int \cos y dy = \int 2x dx$$

$$\sin y = x^2 + C$$

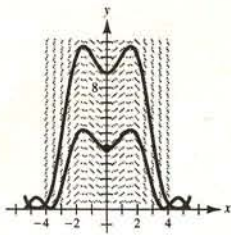
36. $dv = dx \Rightarrow v = \int dx = x$

$$u = \arctan \frac{x}{2} \Rightarrow du = \frac{1}{1+(x/2)^2} \left(\frac{1}{2} \right) dx = \frac{2}{4+x^2} dx$$

$$y = \int \arctan \frac{x}{2} dx = x \arctan \frac{x}{2} - \int \frac{2x}{4+x^2} dx$$

$$= x \arctan \frac{x}{2} - \ln(4+x^2) + C$$

37. (a)



$$(b) \quad \frac{dy}{dx} = x\sqrt{y} \cos x, (0, 4)$$

$$\int \frac{dy}{\sqrt{y}} = \int x \cos x \, dx$$

$$\int y^{-1/2} dy = \int x \cos x \, dx \quad (u = x, du = dx, dv = \cos x \, dx, v = \sin x)$$

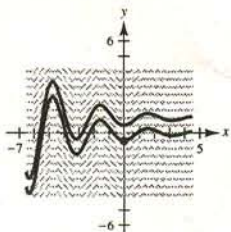
$$2y^{1/2} = x \sin x - \int \sin x \, dx$$

$$= x \sin x + \cos x + C$$

$$(0, 4): 2(4)^{1/2} = 0 + 1 + C \Rightarrow C = 3$$

$$2\sqrt{y} = x \sin x + \cos x + 3$$

38. (a)



$$(b) \quad \frac{dy}{dx} = e^{-x/3} \sin 2x, \left(0, -\frac{18}{37}\right)$$

$$y = \int e^{-x/3} \sin 2x \, dx$$

Use integration by parts twice.

$$(1) \quad u = \sin 2x, du = 2 \cos 2x$$

$$dv = e^{-x/3} \, dx, v = -3e^{-x/3}$$

$$\int e^{-x/3} \sin 2x \, dx = -3e^{-x/3} \sin 2x + \int 6e^{-x/3} \cos 2x \, dx$$

$$(2) \quad u = \cos 2x, du = -2 \sin 2x$$

$$dv = e^{-x/3} \, dx, v = -3e^{-x/3}$$

$$\int e^{-x/3} \sin 2x \, dx = -3e^{-x/3} \sin 2x + 6 \left[-3e^{-x/3} \cos 2x - \int 6e^{-x/3} \sin 2x \, dx \right] + C$$

$$37 \int e^{-x/3} \sin 2x \, dx = -3e^{-x/3} \sin 2x - 18e^{-x/3} \cos 2x + C$$

$$y = \int e^{-x/3} \sin 2x \, dx = \frac{1}{37} \left[-3e^{-x/3} \sin 2x - 18e^{-x/3} \cos 2x \right] + C$$

$$\left(0, -\frac{18}{37}\right): \frac{-18}{37} = \frac{1}{37} [0 - 18] + C \Rightarrow C = 0$$

$$y = \frac{-1}{37} [3e^{-x/3} \sin 2x + 18e^{-x/3} \cos 2x]$$

$$39. dv = \sin 2x dx \Rightarrow v = \int \sin 2x dx = -\frac{1}{2} \cos 2x$$

$$u = x \Rightarrow du = dx$$

$$\begin{aligned} \int x \sin 2x dx &= \frac{-1}{2} x \cos 2x + \frac{1}{2} \int \cos 2x dx \\ &= \frac{-1}{2} x \cos 2x + \frac{1}{4} \sin 2x + C \\ &= \frac{1}{4} (\sin 2x - 2x \cos 2x) + C \end{aligned}$$

$$\text{Thus, } \int_0^{\pi} x \sin 2x dx = \left[\frac{1}{4} (\sin 2x - 2x \cos 2x) \right]_0^{\pi} = -\frac{\pi}{2}.$$

$$40. dv = x dx \Rightarrow v = \int x dx = \frac{x^2}{2}$$

$$u = \arcsin x^2 \Rightarrow du = \frac{2x}{\sqrt{1-x^4}} dx$$

$$\begin{aligned} \int x \arcsin x^2 dx &= \frac{x^2}{2} \arcsin x^2 - \int \frac{x^3}{\sqrt{1-x^4}} dx \\ &= \frac{x^2}{2} \arcsin x^2 + \frac{1}{4} (2)(1-x^4)^{1/2} + C \\ &= \frac{1}{2} [x^2 \arcsin x^2 + \sqrt{1-x^4}] + C \end{aligned}$$

$$\begin{aligned} \text{Thus, } \int_0^1 x \arcsin x^2 dx &= \frac{1}{2} [x^2 \arcsin x^2 + \sqrt{1-x^4}]_0^1 \\ &= \frac{1}{4} (\pi - 2). \end{aligned}$$

41. Use integration by parts twice.

$$(1) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = \sin x \Rightarrow du = \cos x dx$$

$$\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx = e^x \sin x - e^x \cos x - \int e^x \sin x dx$$

$$2 \int e^x \sin x dx = e^x (\sin x - \cos x)$$

$$\int e^x \sin x dx = \frac{e^x}{2} (\sin x - \cos x) + C$$

$$\text{Thus, } \int_0^1 e^x \sin x dx = \left[\frac{e^x}{2} (\sin x - \cos x) \right]_0^1 = \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2} = \frac{e(\sin 1 - \cos 1) + 1}{2} \approx 0.909.$$

$$(2) dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$u = \cos x \Rightarrow du = -\sin x dx$$

42. See Exercise 1.

$$\int_0^1 x^2 e^x dx = [x^2 e^x - 2x e^x + 2e^x]_0^1 = e - 2 \approx 0.718$$

43. See Exercise 25.

$$\int_0^{\pi/2} x \cos x dx = [x \sin x + \cos x]_0^{\pi/2} = \frac{\pi}{2} - 1$$

$$44. dv = dx \Rightarrow v = \int dx = x$$

$$u = \ln(1+x^2) \Rightarrow du = \frac{2x}{1+x^2} dx$$

$$\int \ln(1+x^2) dx = x \ln(1+x^2) - \int \frac{2x^2}{1+x^2} dx$$

$$= x \ln(1+x^2) - 2 \int \left[1 - \frac{1}{1+x^2} \right] dx = x \ln(1+x^2) - 2x + 2 \arctan x + C$$

$$\text{Thus, } \int_0^1 \ln(1+x^2) dx = [x \ln(1+x^2) - 2x + 2 \arctan x]_0^1 = \ln 2 - 2 + \frac{\pi}{2}.$$