100. Define $f(x) = f_2(x) - f_1(x)$. Since f_1 and f_2 are continuous on $[a, b]$, so is f.

$$f(a) = f_2(a) - f_1(a) > 0 \quad \text{and} \quad f(b) = f_2(b) - f_1(b) < 0.$$

By the Intermediate Value Theorem, there exists c in $[a, b]$ such that $f(c) = 0$.

$$f(c) = f_2(c) - f_1(c) = 0 \Rightarrow f_1(c) = f_2(c)$$

Section 1.5 Infinite Limits

1. $\lim_{x\to-2^+} \frac{1}{(x+2)^2} = \infty$

$\lim_{x\to-2^-} \frac{1}{(x+2)^2} = \infty$

2. $\lim_{x\to-2^+} \frac{1}{x+2} = \infty$

$\lim_{x\to-2^-} \frac{1}{x+2} = -\infty$

3. $\lim_{x\to-2^+} \tan\frac{\pi x}{4} = -\infty$

$\lim_{x\to-2^-} \tan\frac{\pi x}{4} = \infty$

4. $\lim_{x\to-2^+} \sec\frac{\pi x}{4} = \infty$

$\lim_{x\to-2^-} \sec\frac{\pi x}{4} = -\infty$

5. $f(x) = \frac{1}{x^2-9}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	0.308	1.639	16.64	166.6	-166.7	-16.69	-1.695	-0.364

$\lim_{x\to-3^-} f(x) = \infty$

$\lim_{x\to-3^+} f(x) = -\infty$

6. $f(x) = \frac{x}{x^2-9}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	-1.077	-5.082	-50.08	-500.1	499.9	49.92	4.915	0.9091

$\lim_{x\to-3^-} f(x) = -\infty$

$\lim_{x\to-3^+} f(x) = \infty$

7. $f(x) = \frac{x^2}{x^2-9}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	3.769	15.75	150.8	1501	-1499	-149.3	-14.25	-2.273

$\lim_{x\to-3^-} f(x) = \infty$

$\lim_{x\to-3^+} f(x) = -\infty$

8. $f(x) = \sec \frac{\pi x}{6}$

x	-3.5	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9	-2.5
$f(x)$	-3.864	-19.11	-191.0	-1910	1910	191.0	19.11	3.864

$\lim_{x \to -3^-} f(x) = -\infty$

$\lim_{x \to -3^+} f(x) = \infty$

9. $\lim_{x \to 0^+} \frac{1}{x^2} = \infty = \lim_{x \to 0^-} \frac{1}{x^2}$

Therefore, $x = 0$ is a vertical asymptote.

10. $\lim_{x \to 2^+} \frac{4}{(x-2)^3} = \infty$

$\lim_{x \to 2^-} \frac{4}{(x-2)^3} = -\infty$

Therefore, $x = 2$ is a vertical asymptote.

11. $\lim_{x \to 2^+} \frac{x^2 - 2}{(x-2)(x+1)} = \infty$

$\lim_{x \to 2^-} \frac{x^2 - 2}{(x-2)(x+1)} = -\infty$

Therefore, $x = 2$ is a vertical asymptote.

$\lim_{x \to -1^+} \frac{x^2 - 2}{(x-2)(x+1)} = \infty$

$\lim_{x \to -1^-} \frac{x^2 - 2}{(x-2)(x+1)} = -\infty$

Therefore, $x = -1$ is a vertical asymptote.

12. $\lim_{x \to 1^+} \frac{2+x}{1-x} = -\infty$

$\lim_{x \to 1^-} \frac{2+x}{1-x} = \infty$

Therefore, $x = 1$ is a vertical asymptote.

13. $\lim_{x \to -1^+} \frac{x^3}{x^2 - 1} = \infty$

$\lim_{x \to -1^-} \frac{x^3}{x^2 - 1} = -\infty$

Therefore, $x = -1$ is a vertical asymptote.

$\lim_{x \to 1^+} \frac{x^3}{x^2 - 1} = \infty$

$\lim_{x \to 1^-} \frac{x^3}{x^2 - 1} = -\infty$

Therefore, $x = 1$ is a vertical asymptote.

14. No vertical asymptote since the denominator is never zero.

15. $f(x) = \tan 2x = \frac{\sin 2x}{\cos 2x}$ has vertical asymptotes at

$$x = \frac{(2n+1)\pi}{4} = \frac{\pi}{4} + \frac{n\pi}{2}, \ n \text{ any integer.}$$

16. $f(x) = \sec \pi x = \frac{1}{\cos \pi x}$ has vertical asymptotes at

$$x = \frac{2n+1}{2}, \ n \text{ any integer.}$$

17. $\lim_{t \to 0^+} \left(1 - \frac{4}{t^2}\right) = -\infty = \lim_{t \to 0^-} \left(1 - \frac{4}{t^2}\right)$

Therefore, $t = 0$ is a vertical asymptote.

18. $\lim_{s \to 2^+} \frac{-2}{(s-2)^2} = -\infty = \lim_{s \to 2^-} \frac{-2}{(s-2)^2}$

Therefore, $s = 2$ is a vertical asymptote.