44. $f(x) = x + \dfrac{1}{x}$

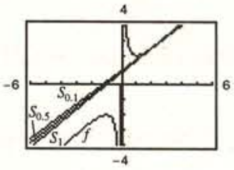
$$S_{\Delta x}(x) = \frac{f(2 + \Delta x) - f(2)}{\Delta x}(x - 2) + f(2) = \frac{(2 + \Delta x) + \dfrac{1}{2 + \Delta x} - \dfrac{5}{2}}{\Delta x}(x - 2) + \frac{5}{2}$$

$$= \frac{2(2 + \Delta x)^2 + 2 - 5(2 + \Delta x)}{2(2 + \Delta x)\,\Delta x}(x - 2) + \frac{5}{2} = \frac{(2\Delta x + 3)}{2(2 + \Delta x)}(x - 2) + \frac{5}{2}$$

(a) $\Delta x = 1$: $S_{\Delta x} = \dfrac{5}{6}(x - 2) + \dfrac{5}{2} = \dfrac{5}{6}x + \dfrac{5}{6}$

$\Delta x = 0.5$: $S_{\Delta x} = \dfrac{4}{5}(x - 2) + \dfrac{5}{2} = \dfrac{4}{5}x + \dfrac{9}{10}$

$\Delta x = 0.1$: $S_{\Delta x} = \dfrac{16}{21}(x - 2) + \dfrac{5}{2} = \dfrac{16}{21}x + \dfrac{41}{42}$



(b) As $\Delta x \to 0$, the line approaches the tangent line to f at $\left(2, \frac{5}{2}\right)$.

45. $f(x) = x^2 - 1, c = 2$

$$f'(2) = \lim_{x\to2} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to2} \frac{(x^2 - 1) - 3}{x - 2} = \lim_{x\to2} \frac{(x - 2)(x + 2)}{x - 2} = \lim_{x\to2} (x + 2) = 4$$

46. $f(x) = x^3 + 2x, c = 1$

$$f'(1) = \lim_{x\to1} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to1} \frac{x^3 + 2x - 3}{x - 1} = \lim_{x\to1} \frac{(x - 1)(x^2 + x + 3)}{x - 1} = \lim_{x\to1} (x^2 + x + 3) = 5$$

47. $f(x) = x^3 + 2x^2 + 1, c = -2$

$$f'(-2) = \lim_{x\to-2} \frac{f(x) - f(-2)}{x + 2} = \lim_{x\to-2} \frac{x^2(x + 2)}{x + 2} = \lim_{x\to-2} \frac{(x^3 + 2x^2 + 1) - 1}{x + 2} = \lim_{x\to-2} x^2 = 4$$

48. $f(x) = \dfrac{1}{x}, c = 3$

$$f'(3) = \lim_{x\to3} \frac{f(x) - f(3)}{x - 3} = \lim_{x\to3} \frac{(1/x) - (1/3)}{x - 3} = \lim_{x\to3} \frac{3 - x}{3x} \cdot \frac{1}{x - 3} = \lim_{x\to3} \left(-\frac{1}{3x}\right) = -\frac{1}{9}$$

49. $f(x) = (x - 1)^{2/3}, c = 1$

$$f'(1) = \lim_{x\to1} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to1} \frac{(x - 1)^{2/3} - 0}{x - 1} = \lim_{x\to1} \frac{1}{(x - 1)^{1/3}}$$

The limit does not exist. Thus, f is not differentiable at $x = 1$.

50. $f(x) = |x - 2|, c = 2$

$$f'(2) = \lim_{x\to2} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to2} \frac{|x - 2|}{x - 2}$$

The limit does not exist. Thus, f is not differentiable at $x = 2$.

51. $f(x)$ is differentiable everywhere except at $x = -3$. (Sharp turn in the graph.)

52. $f(x)$ is differentiable everywhere except at $x = \pm3$. (Sharp turns in the graph.)

53. $f(x)$ is differentiable everywhere except at $x = -1$. (Discontinuity)

54. $f(x)$ is differentiable everywhere except at $x = 1$. (Discontinuity)

55. $f(x)$ is differentiable everywhere except at $x = 3$. (Sharp turn in the graph)

56. $f(x)$ is differentiable everywhere except at $x = 0$. (Sharp turn in the graph)

57. $f(x)$ is differentiable on the interval $(1, \infty)$. (At $x = 1$ the tangent line is vertical.)

58. $f(x)$ is differentiable everywhere except at $x = \pm 2$. (Discontinuities)

59. $f(x)$ is differentiable everywhere except at $x = 0$. (Discontinuity)

60. $f(x)$ is differentiable everywhere except at $x = 1$. (Discontinuity)

61. $f(x) = |x - 1|$

The derivative from the left is $\lim_{x\to 1^-} \dfrac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \dfrac{|x - 1| - 0}{x - 1} = -1$.

The derivative from the right is $\lim_{x\to 1^+} \dfrac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \dfrac{|x - 1| - 0}{x - 1} = 1$.

The one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

62. $f(x) = \sqrt{1 - x^2}$

The derivative from the left does not exist because

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{\sqrt{1 - x^2} - 0}{x - 1} = \lim_{x\to 1^-} \frac{\sqrt{1 - x^2}}{x - 1} \cdot \frac{\sqrt{1 - x^2}}{\sqrt{1 - x^2}} = \lim_{x\to 1^-} -\frac{1 + x}{\sqrt{1 - x^2}} = -\infty. \text{ (Vertical tangent)}$$

The limit from the right does not exist since f is undefined for $x > 1$. Therefore, f is not differentiable at $x = 1$.

63. $f(x) = \begin{cases} (x - 1)^3, & x \le 1 \\ (x - 1)^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x\to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^-} \frac{(x - 1)^3 - 0}{x - 1} = \lim_{x\to 1^-} (x - 1)^2 = 0.$$

The derivative from the right is

$$\lim_{x\to 1^+} = \frac{f(x) - f(1)}{x - 1} = \lim_{x\to 1^+} \frac{(x - 1)^2 - 0}{x - 1} = \lim_{x\to 1^+} (x - 1) = 0.$$

These one-sided limits are equal. Therefore, f is differentiable at $x = 1$. $(f'(1) = 0)$

64. $f(x) = \begin{cases} x, & x \le 1 \\ x^2, & x > 1 \end{cases}$

The derivative from the left is

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^-} \frac{x - 1}{x - 1} = \lim_{x \to 1^-} 1 = 1.$$

The derivative from the right is

$$\lim_{x \to 1^+} = \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{x^2 - 1}{x - 1} = \lim_{x \to 1^+} (x + 1) = 2.$$

These one-sided limits are not equal. Therefore, f is not differentiable at $x = 1$.

65. Note that f is continuous at $x = 2$. $f(x) = \begin{cases} x^2 + 1, & x \le 2 \\ 4x - 3, & x > 2 \end{cases}$

The derivative from the left is

$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{(x^2 + 1) - 5}{x - 2} = \lim_{x \to 2^-} (x + 2) = 4.$$

The derivative from the right is

$$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{(4x - 3) - 5}{x - 2} = \lim_{x \to 2^+} 4 = 4.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $(f'(2) = 4)$

66. Note that f is continuous at $x = 2$. $f(x) = \begin{cases} \frac{1}{2}x + 1, & x < 2 \\ \sqrt{2x}, & x \ge 2 \end{cases}$

The derivative from the left is

$$\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{\left(\frac{1}{2}x + 1\right) - 2}{x - 2} = \lim_{x \to 2^-} \frac{\frac{1}{2}(x - 2)}{x - 2} = \frac{1}{2}.$$

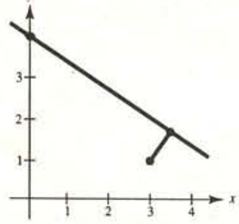
The derivative from the right is

$$\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{\sqrt{2x} - 2}{x - 2} \cdot \frac{\sqrt{2x} + 2}{\sqrt{2x} + 2}$$

$$= \lim_{x \to 2^+} \frac{2x - 4}{(x - 2)\left(\sqrt{2x} + 2\right)} = \lim_{x \to 2^+} \frac{2(x - 2)}{(x - 2)\left(\sqrt{2x} + 2\right)} = \lim_{x \to 2^+} \frac{2}{\sqrt{2x} + 2} = \frac{1}{2}.$$

The one-sided limits are equal. Therefore, f is differentiable at $x = 2$. $\left(f'(2) = \frac{1}{2}\right)$

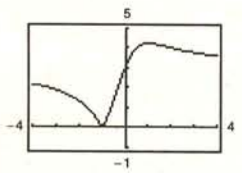
67. (a) The distance from (3, 1) to the line $mx - y + 4 = 0$ is



$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

$$= \frac{|m(3) - 1(1) + 4|}{\sqrt{m^2 + 1}} = \frac{|3m + 3|}{\sqrt{m^2 + 1}}.$$

(b)



The function d is not differentiable at $m = -1$. This corresponds to the line $y = -x + 4$, which passes through the point (3, 1).

68. (a) If f is odd and $f'(c) = 3$, then by symmetry, $f'(-c) = 3$ also.

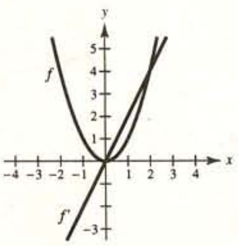
(b) If f is even and $f'(c) = 3$, then by symmetry, $f'(-c) = -3$.

69. False. $y = |x - 2|$ is continuous at $x = 2$, but is not differentiable at $x = 2$. (Sharp turn in the graph)

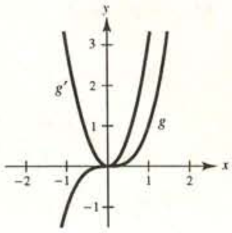
70. False. If the derivative from the left of a point does not equal the derivative from the right of a point, then the derivative does not exist at that point. For example, if $f(x) = |x|$, then the derivative from the left at $x = 0$ is -1 and the derivative from the right at $x = 0$ is 1. At $x = 0$, the derivative does not exist.

71. True—see Theorem 2.1

72. (a) $f(x) = x^2$ and $f'(x) = 2x$



(b) $g(x) = x^3$ and $g'(x) = 3x^2$



(c) The derivative is a polynomial of degree 1 less than the original function. If $h(x) = x^n$, then $h'(x) = nx^{n-1}$.

(d) If $f(x) = x^4$, then
$$\begin{aligned} f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x + \Delta x)^4 - x^4}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{x^4 + 4x^3(\Delta x) + 6x^2(\Delta x)^2 + 4x(\Delta x)^3 + (\Delta x)^4 - x^4}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x(4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3)}{\Delta x} \\ &= \lim_{\Delta x \to 0} (4x^3 + 6x^2(\Delta x) + 4x(\Delta x)^2 + (\Delta x)^3) = 4x^3 \end{aligned}$$

Hence, if $f(x) = x^4$, then $f'(x) = 4x^3$ which is consistent with the conjecture. However, this is not a proof, since you must verify the conjecture for all integer values of n, $n \geq 2$.

73. $f(x) = \begin{cases} x\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Using the Squeeze Theorem, we have $-|x| \leq x\sin(1/x) \leq |x|$, $x \neq 0$. Thus, $\lim_{x \to 0} x\sin(1/x) = 0 = f(0)$ and f is continuous at $x = 0$. Using the alternative form of the derivative we have

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x\sin(1/x) - 0}{x - 0} = \lim_{x \to 0} \left(\sin\frac{1}{x}\right).$$

Since this limit does not exist (it oscillates between -1 and 1), the function is not differentiable at $x = 0$.

$$g(x) = \begin{cases} x^2\sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

Using the Squeeze Theorem again we have $-x^2 \leq x^2\sin(1/x) \leq x^2$, $x \neq 0$. Thus, $\lim_{x \to 0} x^2\sin(1/x) = 0 = f(0)$ and f is continuous at $x = 0$. Using the alternative form of the derivative again we have

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x^2\sin(1/x) - 0}{x - 0} = \lim_{x \to 0} x\sin\frac{1}{x} = 0.$$

Therefore, g is differentiable at $x = 0$, $g'(0) = 0$.