

**Find the derivative of each function.**

1a)  $h(x) = (2x^2 + 1)(3x^3 - 4x + 10)$

1b)  $f(x) = x^3 \cos x$

$$h(x) = f(x) \cdot g(x)$$

So...let  $f(x) = (2x^2 + 1)$  and  $g(x) = (3x^3 - 4x + 10)$

Since  $h'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

$$h'(x) = (2x^2 + 1)(9x^2 - 4) + (3x^3 - 4x + 10)(4x)$$

1c)  $h(t) = \sqrt{t}(1 - t^2)$

1d)  $y = 2x \sin x + x^2 \cos x$

2a)  $h(x) = \frac{3x}{2x^2 - x + 1}$

2b)  $f(x) = \frac{3x - 1}{\sqrt{x}}$

$$h(x) = \frac{f(x)}{g(x)}$$

So...let  $f(x) = 3x$  and  $g(x) = (2x^2 - x + 1)$

Since  $h'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{(g(x))^2}$

$$h'(x) = \frac{(2x^2 - x + 1) \cdot 3 - 3x \cdot (4x - 1)}{(2x^2 - x + 1)^2}$$

$$h'(x) = \frac{(6x^2 - 3x + 3) - (12x^2 - 3x)}{(2x^2 - x + 1)^2}$$

$$h'(x) = \frac{-6x^2 + 3}{(2x^2 - x + 1)^2}$$

$$h'(x) = \frac{-3(2x^2 - 1)}{(2x^2 - x + 1)^2}$$

$$2c) f(x) = \frac{4 - 3x - x^2}{x^2 - 1}$$

$$2d) f(t) = \frac{\cos t}{t}$$

$$2e) h(x) = \frac{1 + \csc x}{1 - \csc x}$$

**Evaluate the derivative of the function at the given point.**

$$3a) f(x) = (x^3 + 4x)(3x^2 + 2x - 5) \text{ at } (0, 0)$$

$$\begin{aligned} f'(x) &= (x^3 + 4x)(6x + 2) + (3x^2 + 2x - 5)(3x^2 + 4) \\ &= 6x^4 + 24x^2 + 2x^3 + 8x + 9x^4 + 6x^3 - 15x^2 + 12x^2 + 8x - 20 \\ &= 15x^4 + 8x^3 + 21x^2 + 16x - 20 \\ f'(0) &= -20 \end{aligned}$$

$$3b) f(x) = \frac{x^2 - 4}{x - 3} \text{ at } (1, 3/2)$$

$$3c) f(x) = x \cos x \text{ at } \left( \frac{\pi}{4}, \frac{\pi\sqrt{2}}{8} \right)$$

Find an equation of the tangent line to the graph of at the given point and use the *derivative* feature of a graphing utility to confirm your results.

4a)  $f(x) = \frac{x}{x+4}, \quad (-5, 5)$

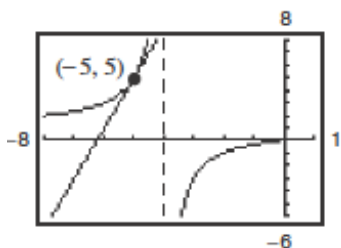
4b)  $f(x) = \tan x, \quad \left(\frac{\pi}{4}, 1\right)$

$$f(x) = \frac{x}{x+4}, \quad (-5, 5)$$

$$f'(x) = \frac{(x+4)(1) - x(1)}{(x+4)^2} = \frac{4}{(x+4)^2}$$

$$f'(-5) = \frac{4}{(-5+4)^2} = 4; \quad \text{Slope at } (-5, 5)$$

Tangent line:  $y - 5 = 4(x + 5) \Rightarrow y = 4x + 25$



Graphing utility confirms  $\frac{dy}{dx} = 4$  at  $(-5, 5)$ .

4c)  $f(x) = (x^3 + 4x - 1)(x - 2), \quad (1, -4)$

Find the derivative of each function.

5a)  $f(x) = \csc x(\tan x - \cos x)$

$$f(x) = \csc x \cdot \tan x - \csc x \cos x$$

$$f(x) = \frac{1}{\sin x} \cdot \frac{\sin x}{\cos x} - \frac{1}{\sin x} \cdot \cos x$$

$$f(x) = \frac{1}{\cos x} - \cot x$$

$$f(x) = \sec x - \cot x$$

$$f'(x) = \sec x \tan x - (-\csc^2 x)$$

$$f'(x) = \sec x \tan x + \csc^2 x$$

5b)  $f(x) = \cot x(\tan x + \sec x)$

6a) The following table lists the values of functions  $f$  and  $g$ , and of their derivatives,  $f'$  and  $g'$ , for  $x = -1$ .

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	3	1	4	-2

Evaluate  $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right]$  at  $x = -1$ .

$$\frac{f'(-1)g(-1) - f(-1)g'(-1)}{[g(-1)]^2}$$

$$= \frac{(4)(1) - (3)(-2)}{(1)^2}$$

$$= \frac{10}{1}$$

$$= 10$$

6b) Use the table above to evaluate the following:

1)  $\frac{dy}{dx} [f(x) \cdot g(x)]$  at  $x = -1$

2)  $P'(-1)$  if  $P(x) = \frac{f(x) \cdot g(x)}{f(x) + g(x)}$

## 7) AP MULTIPLE CHOICE EXAMPLES

1) An equation of the line tangent to the graph of  $y = \frac{2x+3}{3x-2}$  at the point  $(1,5)$  is

- (A)  $13x - y = 8$                       (B)  $13x + y = 18$                       (C)  $x - 13y = 64$   
(D)  $x + 13y = 66$                       (E)  $-2x + 3y = 13$

2) If  $y = \tan x - \cot x$ , then  $\frac{dy}{dx} =$

- (A)  $\sec x \csc x$     (B)  $\sec x - \csc x$     (C)  $\sec x + \csc x$     (D)  $\sec^2 x - \csc^2 x$     (E)  $\sec^2 x + \csc^2 x$

3) If  $f(x) = \frac{x-1}{x+1}$  for all  $x \neq -1$ , then  $f'(1) =$

- (A)  $-1$                       (B)  $-\frac{1}{2}$                       (C)  $0$                       (D)  $\frac{1}{2}$                       (E)  $1$

4) If  $y = \frac{3}{4+x^2}$ , then  $\frac{dy}{dx} =$

- (A)  $\frac{-6x}{(4+x^2)^2}$     (B)  $\frac{3x}{(4+x^2)^2}$     (C)  $\frac{6x}{(4+x^2)^2}$     (D)  $\frac{-3}{(4+x^2)^2}$     (E)  $\frac{3}{2x}$