1b) $f'(x) = x^3(-\sin x) + \cos x(3x^2)$
$= 3x^2 \cos x - x^3 \sin x$
$= x^2(3 \cos x - x \sin x)$

1c) $h'(t) = t^{1/2}(-2t) + (1 - t^2)\frac{1}{2}t^{-1/2}$
$= -2t^{3/2} + \frac{1}{2t^{1/2}} - \frac{1}{2}t^{3/2}$
$= -\frac{5}{2}t^{3/2} + \frac{1}{2t^{1/2}}$
$= \frac{1 - 5t^2}{2t^{1/2}} = \frac{1 - 5t^2}{2\sqrt{t}}$

1d) $y' = 2x \cos x + 2 \sin x + x^2(-\sin x) + 2x \cos x$
$= 4x \cos x + (2 - x^2) \sin x$

2b) $f(x) = \frac{3x - 1}{\sqrt{x}} = 3x^{1/2} - x^{-1/2}$
$f'(x) = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$
$= \frac{3x + 1}{2x^{3/2}}$

Alternate solution:

$f(x) = \frac{3x - 1}{\sqrt{x}} = \frac{3x - 1}{x^{1/2}}$

$f'(x) = \frac{x^{1/2}(3) - (3x - 1)\left(\frac{1}{2}\right)(x^{-1/2})}{x}$

$= \frac{\frac{1}{2}x^{-1/2}(3x + 1)}{x}$

$= \frac{3x + 1}{2x^{3/2}}$

2c)

$$f'(x) = \frac{(x^2 - 1)(-3 - 2x) - (4 - 3x - x^2)(2x)}{(x^2 - 1)^2}$$

$$= \frac{-3x^2 + 3 - 2x^3 + 2x - 8x + 6x^2 + 2x^3}{(x^2 - 1)^2}$$

$$= \frac{3x^2 - 6x + 3}{(x^2 - 1)^2}$$

$$= \frac{3(x^2 - 2x + 1)}{(x^2 - 1)^2}$$

$$= \frac{3(x - 1)^2}{(x - 1)^2(x + 1)^2} = \frac{3}{(x + 1)^2}, x \neq 1$$

2d) $f'(t) = \dfrac{-t \sin t - \cos t}{t^2} = -\dfrac{t \sin t + \cos t}{t^2}$

2e)

$$y' = \frac{(1 - \csc x)(-\csc x \cot x) - (1 + \csc x)(\csc x \cot x)}{(1 - \csc x)^2}$$

$$= \frac{-2 \csc x \cot x}{(1 - \csc x)^2}$$

3b)

$$f'(x) = \frac{(x - 3)(2x) - (x^2 - 4)(1)}{(x - 3)^2}$$

$$= \frac{2x^2 - 6x - x^2 + 4}{(x - 3)^2}$$

$$= \frac{x^2 - 6x + 4}{(x - 3)^2}$$

$$f'(1) = \frac{1 - 6 + 4}{(1 - 3)^2} = -\frac{1}{4}$$

3c) $f'(x) = (x)(-\sin x) + (\cos x)(1) = \cos x - x \sin x$

$$f'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - \frac{\pi}{4}\left(\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{8}(4 - \pi)$$

4b) $f(x) = \tan x, \quad \left(\frac{\pi}{4}, 1\right)$

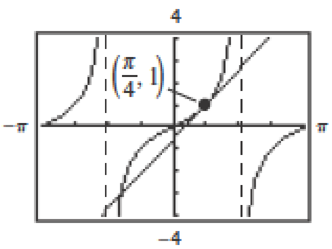
$f'(x) = \sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2; \quad \text{Slope at } \left(\frac{\pi}{4}, 1\right)$

Tangent line: $y - 1 = 2\left(x - \frac{\pi}{4}\right)$

$y - 1 = 2x - \frac{\pi}{2}$

$4x - 2y - \pi + 2 = 0$



Graphing utility confirms $\frac{dy}{dx} = 2$ at $\left(\frac{\pi}{4}, 1\right)$.

4c) $f(x) = (x^3 + 4x - 1)(x - 2), \quad (1, -4)$

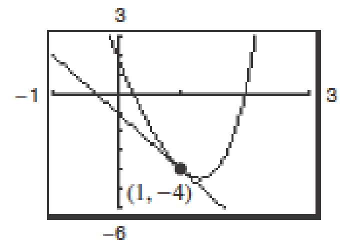
$f'(x) = (x^3 + 4x - 1)(1) + (x - 2)(3x^2 + 4)$

$= x^3 + 4x - 1 + 3x^3 - 6x^2 + 4x - 8$

$= 4x^3 - 6x^2 + 8x - 9$

$f'(1) = -3$; Slope at $(1, -4)$

Tangent line: $y + 4 = -3(x - 1) \Rightarrow y = -3x - 1$



Graphing utility confirms $\frac{dy}{dx} = -3$ at $(1, -4)$.

5b) $f(x) = \cot x(\tan x + \sec x)$

$f(x) = \cot x \bullet \tan x + \cot x \bullet \sec x$

$f(x) = 1 + \frac{1}{\sin x}$

$f(x) = 1 + \csc x$

$f'(x) = -\csc x \bullet \cot x$

6b#1) $\frac{dy}{dx}[f(x) \bullet g(x)] = f(x) \bullet g'(x) + f'(x) \bullet g(x)$

$\frac{dy}{dx}[-1] = f(-1) \bullet g'(-1) + f'(-1) \bullet g(-1)$

$\frac{dy}{dx}[-1] = 3 \bullet (-2) + 4 \bullet 1$

$= -6 + 4$

$= -2$

6b#2)

$$P'(x)=\frac{[f(x)+g(x)]\bullet\frac{dy}{dx}[f(x)\bullet g(x)]-[f(x)\bullet g(x)]\bullet\frac{dy}{dx}[f(x)+g(x)]}{[f(x)+g(x)]^2}$$

$$P'(x)=\frac{[f(x)+g(x)]\bullet[f(x)\bullet g'(x)+f'(x)\bullet g(x)]-[f(x)\bullet g(x)]\bullet[f'(x)+g'(x)]}{[f(x)+g(x)]^2}$$

$$P'(-1)=\frac{[f(-1)+g(-1)]\bullet[f(-1)\bullet g'(-1)+f'(-1)\bullet g(-1)]-[f(-1)\bullet g(-1)]\bullet[f'(-1)+g'(-1)]}{[f(-1)+g(-1)]^2}$$

$$P'(-1)=\frac{[3+1]\bullet[3\bullet(-2)+4\bullet 1]-[3\bullet 1]\bullet[4+(-2)]}{[3+1]^2}$$

$$P'(-1)=\frac{[4]\bullet[-6+4]-[3]\bullet[2]}{[4]^2}$$

$$P'(-1)=\frac{[4]\bullet[-2]-[3]\bullet[2]}{[4]^2}$$

$$P'(-1)=\frac{=-8-6}{16}$$

$$=\frac{-14}{16}\,or-\frac{7}{8}$$

7) AP MULTIPLE CHOICE EXAMPLES

1) An equation of the line tangent to the graph of $y = \frac{2x+3}{3x-2}$ at the point $(1,5)$ is

(A) $13x - y = 8$ (B) $13x + y = 18$ (C) $x - 13y = 64$

(D) $x + 13y = 66$ (E) $-2x + 3y = 13$

$y' = \frac{(3x-2)\cdot 2 - (2x+3)\cdot 3}{(3x-2)^2}$

$y'(1) = \frac{1\cdot 2 - 5\cdot 3}{1^2} = \frac{2-15}{1} = -13$

So.. $y - 5 = -13(x-1)$

$= y - 5 = -13x + 13$

$= y + 13x = 5 + 13$

$= y + 13x = 18$

2) If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$

(A) $\sec x \csc x$ (B) $\sec x - \csc x$ (C) $\sec x + \csc x$ (D) $\sec^2 x - \csc^2 x$ (E) $\sec^2 x + \csc^2 x$

$\frac{d}{dx}\tan x = \sec^2 x$

$\frac{d}{dx}\cot x = -\csc^2 x$

$\sec^2 x - (-\csc^2 x)$

$= \sec^2 x + \csc^2 x$

3) If $f(x) = \frac{x-1}{x+1}$ for all $x \neq -1$, then $f'(1) =$

(A) -1 (B) (C) (D) (E) 1

$f'(x) = \frac{(x+1)\cdot 1 - (x-1)\cdot 1}{(x+1)^2}$

$f'(1) = \frac{(1+1) - (1-1)}{(1+1)^2}$

$f'(1) = \frac{2-0}{2^2} = \frac{2}{4} = \frac{1}{2}$

4) If $y = \frac{\quad}{4+x^2}$, $\frac{dy}{dx} =$

2

(A) $\frac{-6x}{(4+x^2}$ (B) $\frac{3x}{(4+x^2)}$ (C) $\frac{\quad}{(4+x^2)^2}$ (D) $\frac{\quad}{(4+x^2)}$

$y' = \frac{(4+x^2)\cdot 0 - 3(0+2x)}{(4+x^2)^2}$

$y' = \frac{0 - 6x}{(4+x^2)^2} = \frac{-6x}{(4+x^2)^2}$