Name Key

Limits Practice from PSU Calculus 1 Class

Use the function $f(x)=\dfrac{|x|(x-3)}{9-x^2}$ to answer questions 1-3 that follow.

$f(x)=\dfrac{|x|(x-3)}{(3+x)(3-x)}$ $\quad f(x)=\dfrac{-|x|}{3+x}$

#1. Evaluate $\lim_{x\to 3} f(x)$. $\boxed{-\frac{1}{2}}$

#2. Determine all vertical asymptotes (if any) of $f(x)$. $\boxed{x=-3}$

#3. Find all the removable discontinuities (if any) of $f(x)$. at $x=3$ so $\boxed{\left(3,-\frac{1}{2}\right)}$
← hole in graph

#4. Determine $\lim_{\theta\to 0}\dfrac{\csc 3\theta}{\cot\theta}$. $= \lim_{\theta\to 0}\dfrac{\tan\theta}{\sin 3\theta} = \lim_{\theta\to 0}\dfrac{\tan\theta}{\sin 3\theta}\cdot\dfrac{\theta}{\theta}\cdot\dfrac{3}{3}$

$= \lim_{\theta\to 0}\dfrac{\tan\theta}{\theta}\cdot\dfrac{3\theta}{\sin 3\theta}\cdot\dfrac{1}{3}$ $= 1\cdot 1\cdot\frac{1}{3}$ $\boxed{\frac{1}{3}}$

#5. Determine the constants c and k that make the following function continuous.

$$f(x)=\begin{cases} x+2c & ,x<-2 \\ 3cx+k & ,-2\le x\le 1 \\ 3x-2k & ,x>1 \end{cases}$$

$-2+2c = -6c+k \Rightarrow 8c-k=2$

$3c+k = 3-2k \Rightarrow 3c+3k=3$

solve → $\boxed{c=\frac{1}{3},\ k=\frac{2}{3}}$

$8c-k=2$
$c+k=1$
$9c=3$
$c=1/3$

#6. What is the value of $\lim_{x\to -1}\dfrac{x^2-3x-4}{x^2-1}$? $\dfrac{(x-4)(x+1)}{(x+1)(x-1)} = \dfrac{-5}{-2}$ $\boxed{\frac{5}{2}}$

#7. Evaluate the limit, if it exists: $\lim_{x\to 9}\dfrac{\sqrt{x-5}-2}{(x-9)}\cdot\dfrac{\sqrt{x-5}+2}{\sqrt{x-5}+2}$

$\lim_{x\to 9}\dfrac{(x-9)}{(x-9)(\sqrt{x-5}+2)} = \boxed{\frac{1}{4}}$

#8. Evaluate the limit, if it exists: $\lim_{x\to 2}\dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}\cdot\dfrac{2x}{2x} = \lim_{x\to 2}\dfrac{2-x}{(x-2)\cdot 2x} = \lim_{x\to 2}\dfrac{-1}{2x}$

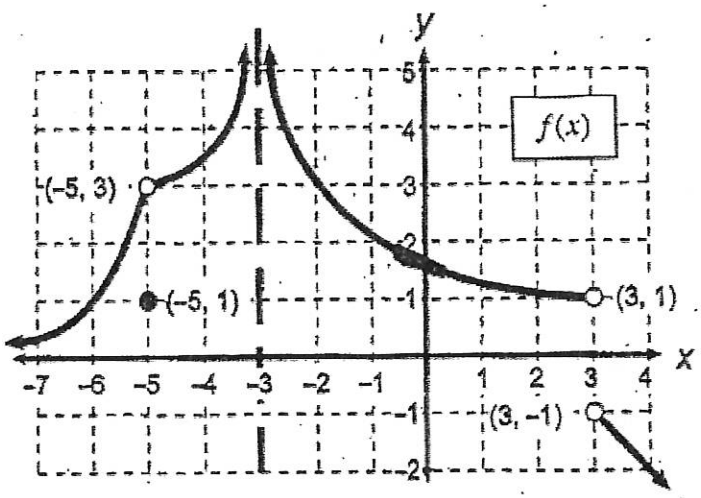
$= \boxed{-\frac{1}{4}}$

#9. Find the limit (if it exists): $\lim_{x\to\infty}\dfrac{x^4-4x^3-7x^2-31x+6}{x^3-x^2-33x+18}$

$\boxed{\text{DNE } (+\infty)}$

#10. Find the limit (if it exists): $\lim_{x\to\infty}\dfrac{3x^5+7x^3-5x^2+1}{2x^5+2x^2-8}$ = $\boxed{\frac{3}{2}}$

Use the graph of $f(x)$ below to answer questions 11 – 20 that follow.



#11. $f(-5) =$ 1

#12. $f(3) =$ DNE

#13. $\lim_{x\to -3} f(x) =$ DNE $(+\infty)$

#14. $\lim_{x\to 3^-} f(x) =$ 1

#15. $\lim_{x\to 3^+} f(x) =$ -1

#16. $\lim_{x\to 3} f(x) =$ DNE

#17. $\lim_{x\to -\infty} f(x) =$ 0

#18. $\lim_{x\to\infty} f(x) =$ DNE $(-\infty)$

#19. Is $f(x)$ continuous at $x=3$? Justify your answer using Calculus.

No $\lim_{x\to 3} f(x)$ DNE or $f(3)$ is undefined

#20. Is $f(x)$ continuous at $x=-2$? Justify your answer using Calculus.

YES $\lim_{x\to -2} f(x) = f(-2)$

$3 = 3$ ✓

#21.

Find ALL vertical asymptotes for the function $f(x)=\dfrac{1}{2\sin^2 x-\sin x-1}$

in the interval $0\le x\le 2\pi$. EXPRESS YOUR ANSWERS IN RADIANS!

$2\sin^2 x-\sin x-1=0$

$(2\sin x+1)(\sin x-1)=0$

$2\sin x+1=0$ $\quad$ $\sin x-1=0$

$\sin x=-1/2$ $\quad$ $\sin x=1$

So $\boxed{x=\frac{7\pi}{6},\ \frac{11\pi}{6},\ \frac{\pi}{2}}$