Calculus **Related Rates** **Name** Key

EX 1) A pebble is dropped into a calm pond causing ripples. The radius of the outer ripple is increasing at 1 ft/sec. When the radius is 4 feet, at what rate is the area changing?

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2 \cdot \pi \cdot 4\text{feet} \cdot 1\frac{\text{ft}}{\text{sec}}$

$\frac{dA}{dt} = 8\pi \text{ ft}^2/\text{sec}$

EX 2) The radius of a sphere is increasing at a rate of 2 inches per minute. Find the rate of change of the volume when the radius is 24 inches.

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi \cdot r^2 \frac{dr}{dt}$

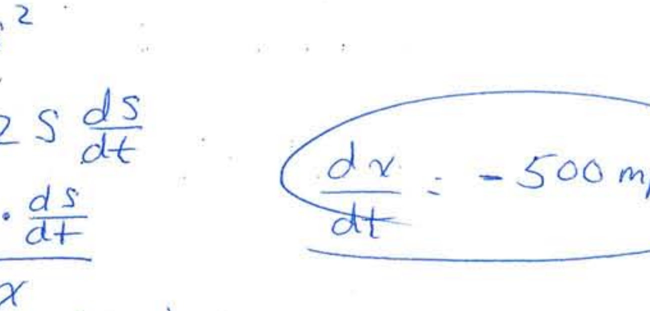
$= 4\pi (24\text{in})^2 \, 2 \frac{\text{in}}{\text{min}}$

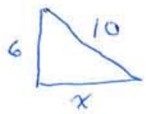
$\frac{dv}{dt} = 4608\pi \frac{\text{in}^3}{\text{min}}$

EX 3) An airplane is flying at an elevation of 6 miles above the ground, on a flight path that will take it directly over a radar station. Let "s" be the distance in miles between the radar station and the plane. If "s" is decreasing at a rate of 400 mph when "s" is 10 miles. What is the ground velocity of the plane?

$S \rightarrow$ distance from plane to radar station

$x \rightarrow$ ground distance from point below the plane to the radar station





$x^2 + 36 = 100$

$x^2 = 64$

$x = 8$

$x^2 + 36 = S^2$

$2x \frac{dx}{dt} = 2S \frac{dS}{dt}$

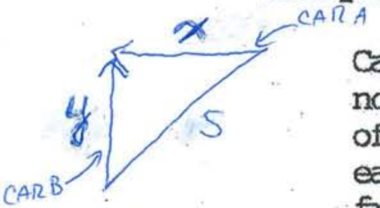
$\frac{dx}{dt} = \frac{S \cdot \frac{dS}{dt}}{x}$

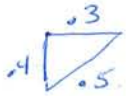
$= \frac{10\text{mi}\,(-400)\text{mph}}{8\text{mi}}$

$\frac{dx}{dt} = -500 \text{ mph}$

RELATED RATE EXAMPLES

Example #1 :





Car A is going west at 50 mi/h and Car B is traveling north at 60 mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when Car A is 0.3 mi and Car B is 0.4 mi from the intersection?

$S \rightarrow$ distance between two cars

$$x^2+y^2=S^2$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2S\frac{dS}{dt}$$

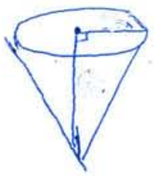
$$x\cdot\frac{dx}{dt}+y\frac{dy}{dt}=S\frac{dS}{dt}$$

$$\frac{x\frac{dx}{dt}+y\frac{dy}{dt}}{S}=\frac{dS}{dt}$$

$$\frac{(.3\,mi)(-50\,mph)+(.4\,mi)(-60\,mph)}{.5\,mi}=\frac{(-15+-24)\,mi^2/h}{.5\,mi}=-78\,mph$$

$$\boxed{-78\,mph}$$

Example #2 :



A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of 2 cubic m/min, find the rate at which the water level is rising when the water is 3m deep.

$r=\frac{1}{2}h$

$$V=\frac{1}{3}\pi r^2 h$$

$$V=\frac{1}{3}\pi\left(\frac{1}{2}h\right)^2\cdot h$$

$$V=\frac{\pi}{12}h^3$$

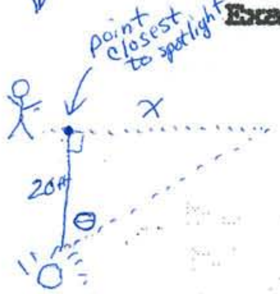
$$\frac{dV}{dt}=\frac{\pi}{4}h^2\frac{dh}{dt}$$

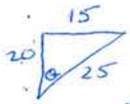
$$\frac{dh}{dt}=\frac{4\cdot\frac{dV}{dt}}{\pi h^2}=\frac{4\cdot 2\,m^3/min}{\pi\cdot(3\,m)^2}$$

$$\frac{8\,m^3/min}{(9\pi)\,m^2}$$

$$\frac{8}{9\pi}\,m/min$$

Example #3 :





A man walks along a straight path at a speed of 4ft/sec. A searchlight is located on the ground 20 feet from the path and is kept focused on the man. At what rate is the search light rotating when the man is 15ft. from the point on the path closest to the searchlight?

$\sec\theta=\frac{25}{20}=1.25$

$$\tan\theta=\frac{x}{20}$$

$x \rightarrow$ dist walked

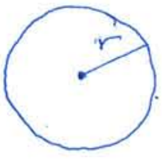
$\theta \rightarrow$ degree of rotation (in radians)

$$\sec^2\theta\frac{d\theta}{dt}=\frac{1}{20}\frac{dx}{dt}$$

$$\frac{d\theta}{dt}=\frac{.05\frac{dx}{dt}}{\sec^2\theta}=\frac{.05\,(4\,ft/sec)}{(1.25)^2}=\boxed{.128\,\frac{rad}{sec}}$$

Example #4 :

Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/sec. How fast is the area of the spill increasing when the radius of the spill is 60 ft.?



$$A=\pi r^2$$

$$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$$

$$\frac{dA}{dt}=2\cdot\pi\cdot 60\,ft\cdot 2\,ft/sec$$

$$240\,ft^2/sec$$