Determine whether Rolle's Theorem can be applied to on the closed interval [a, b]. If Rolle's Theorem can be applied, find all values of *c* in the open interval (a, b) such that $f'(c)=0$. If Rolle's Theorem cannot be applied, explain why not.

1a) $f(x) = -x^2 + 3x, \quad [0, 3]$

$f(0) = f(3) = 0$

f is continuous on $[0, 3]$ and differentiable on $(0, 3)$. Rolle's Theorem applies.

$f'(x) = -2x + 3$

$-2x + 3 = 0 \Rightarrow x = \frac{3}{2}$

c-value: $\frac{3}{2}$

1b) $f(x) = \dfrac{x^2 - 2x - 3}{x + 2}, \quad [-1, 3]$

1c) $f(x) = \sin x, \quad [0, 2\pi]$

1d) $f(x) = 3 - |x - 3|, \quad [0, 6]$

1e) $f(x) = (x - 3)(x + 1)^2, \quad [-1, 3]$

1f) $f(x) = \dfrac{x^2 - 1}{x}, \quad [-1, 1]$

Determine whether the Mean Value Theorem can be applied to on the closed interval [a, b]. If the MVT can be applied, find all values of *c* in the open interval (a, b) such that $f'(c)=\frac{f(b)-f(a)}{b-a}$. If the MVT cannot be applied, explain why not.

2a) $f(x) = x^2, \quad [-2, 1]$

$f(x) = x^2$ is continuous on $[-2, 1]$ and differentiable on $(-2, 1)$.

$$\frac{f(1) - f(-2)}{1 - (-2)} = \frac{1 - 4}{3} = -1$$

$$f'(x) = 2x = -1$$

$$x = -\frac{1}{2}$$

$$c = -\frac{1}{2}$$

2b) $f(x) = |2x + 1|, \quad [-1, 3]$

2c) $f(x) = x^{2/3}, \quad [0, 1]$

2d) $f(x) = \sqrt{2 - x}, \quad [-7, 2]$