

$$1b) \quad f(-1) = f(3) = 0$$

f is continuous on $[-1, 3]$.

Note: the discontinuity, $x = -2$, is not in the interval.) f is differentiable on $(-1, 3)$. Rolle's Theorem applies.

$$f'(x) = \frac{(x+2)(2x-2) - (x^2 - 2x - 3)(1)}{(x+2)^2} = 0$$

$$\frac{x^2 + 4x - 1}{(x+2)^2} = 0$$

$$x = \frac{-4 \pm 2\sqrt{5}}{2} = -2 \pm \sqrt{5}$$

$$c\text{-value: } -2 + \sqrt{5}$$

$$1c) \quad f(0) = f(2\pi) = 0$$

f is continuous on $[0, 2\pi]$. f is differentiable on $(0, 2\pi)$. Rolle's Theorem applies.

$$f'(x) = \cos x$$

$$c\text{-values: } \frac{\pi}{2}, \frac{3\pi}{2}$$

1d) Not differentiable at $x = 3$ which is on the given interval, so Rolle's Theorem DOES NOT APPLY!

$$1e) \quad f(1) = 0$$

$$f(3) = 0$$

$$\text{so...} f(1) = f(3)$$

Continuous Polynomial so Rolle's Theorem Applies!

$$f'(x) = (x-3) \cdot 2(x+1)^1 \cdot 1 + (x+1)^2 \cdot 1$$

$$0 = 2(x-3)(x+1) + (x+1)^2$$

$$0 = (x+1)(2(x-3) + (x+1))$$

$$0 = (x+1)(2x-6+x+1)$$

$$0 = (x+1)(3x-5) \quad \text{so... } c = -1 \text{ or } \frac{5}{3}$$

1f) Not differentiable at $x = 0$ which is on the given interval, so Rolle's Theorem DOES NOT APPLY!

2b) $f(x) = |2x + 1|$ is not differentiable at $x = -1/2$. The Mean Value Theorem does not apply.

2c) $f(x) = x^{2/3}$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$.

$$\frac{f(1) - f(0)}{1 - 0} = 1$$

$$f'(x) = \frac{2}{3}x^{-1/3} = 1$$

$$x = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$$

$$c = \frac{8}{27}$$

2d) $f(x)$ is continuous on $[-7, 2]$ and Differentiable on $(-7, 2)$.

$$\frac{f(2) - f(-7)}{2 - (-7)} = \frac{0 - 3}{9} = -\frac{1}{3}$$

$$f(x) = (2 - x)^{1/2}$$

$$f'(x) = \frac{1}{2}(2 - x)^{-1/2} \cdot (-1)$$

$$f'(x) = \frac{-1}{2\sqrt{2 - x}}$$

$$-\frac{1}{3} = \frac{-1}{2\sqrt{2 - x}}$$

$$-\frac{1}{3} = \frac{-1}{2\sqrt{2 - x}}$$

$$2\sqrt{2 - x} = 3$$

$$\sqrt{2 - x} = \frac{3}{2}$$

$$2 - x = \frac{9}{4}$$

$$-x = \frac{1}{4}$$

$$x = -\frac{1}{4}$$