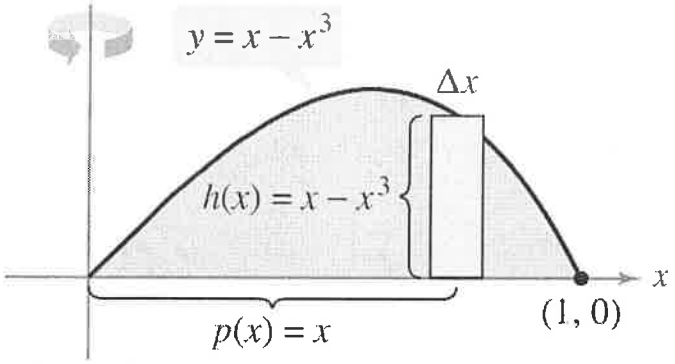
1a) Find the volume of the solid of revolution formed by revolving the region bounded by $y = x - x^3$ and the x-axis $(0 \le x \le 1)$ about the y-axis.



$$V = 2\pi \int_a^b p(x)h(x)\,dx = 2\pi \int_0^1 x(x - x^3)\,dx$$

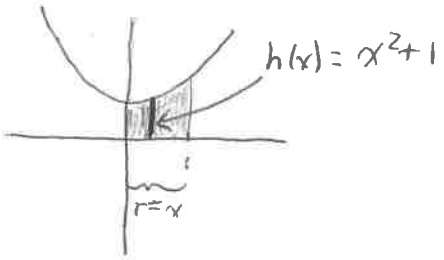
$$= 2\pi \int_0^1 (-x^4 + x^2)\,dx$$

$$= 2\pi \left[-\frac{x^5}{5} + \frac{x^3}{3} \right]_0^1$$

$$= 2\pi \left(-\frac{1}{5} + \frac{1}{3} \right)$$

$$= \frac{4\pi}{15}.$$

1b) Find the volume of the solid formed by revolving the region bounded by the graphs $y = x^2 + 1$, $y = 0$, $x = 0$, and $x = 1$ about the y-axis.



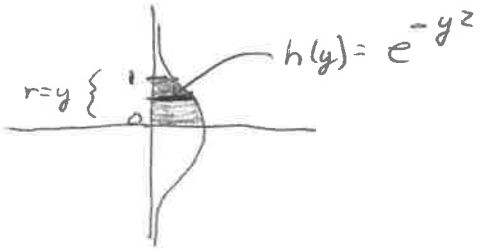
$V = 2\pi r h$

$\int_0^1 2\pi \cdot x \cdot (x^2+1)\,dx$

$2\pi \int_0^1 x^3 + x\,dx$

$= \frac{3\pi}{2}$

1c) Find the volume of the solid formed by revolving the region bounded by the graph of $x = e^{-y^2}$ and the y-axis $(0 \le y \le 1)$ about the x-axis.



$V = 2\pi r h$

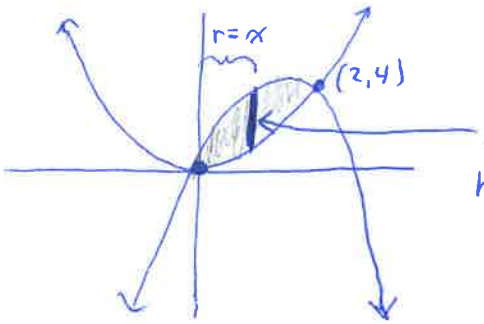
$\int_0^1 2\pi\, y \cdot e^{-y^2}\,dy$

$\approx .632\,\pi$

or

1.986

1d) Find the volume of the solid formed by revolving the region bounded by the graphs $y = x^2$, $y = 4x - x^2$ about the y-axis.



$h(x) = (4x - x^2) - x^2$

$h(x) = 4x - 2x^2$

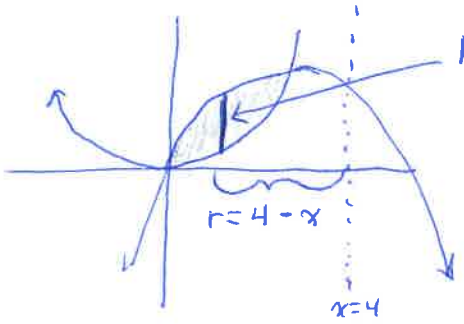
$V = 2\pi r h$

$V = \int_0^2 2\pi x (4x - 2x^2)\, dx$

$V = \int_0^2 2\pi (4x^2 - 2x^3)\, dx$

$= \boxed{\frac{16\pi}{3}}$

1e) Find the volume of the solid formed by revolving the region bounded by the graphs $y = x^2$, $y = 4x - x^2$, about the line $x = 4$.



$h(x) = 4x - 2x^2$

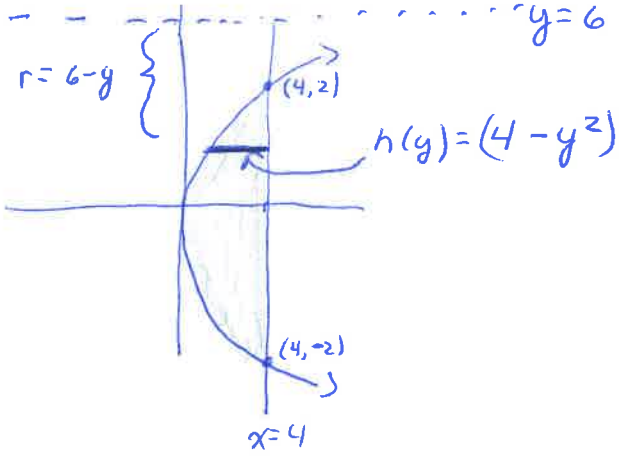
$V = 2\pi r h$

$V = \int_0^2 2\pi (4 - x)(4x - 2x^2)\, dx$

$V = \int_0^2 2\pi (2x^3 - 12x^2 - 16x)\, dx$

$= \boxed{16\pi}$

1f) Find the volume of the solid formed by revolving the region bounded by the graphs $x = y^2$, $x = 4$ about the line $y = 6$.



$h(y) = (4 - y^2)$

$V = 2\pi r h$

$V = \int_{-2}^2 2\pi (6 - y)(4 - y^2)\, dy$

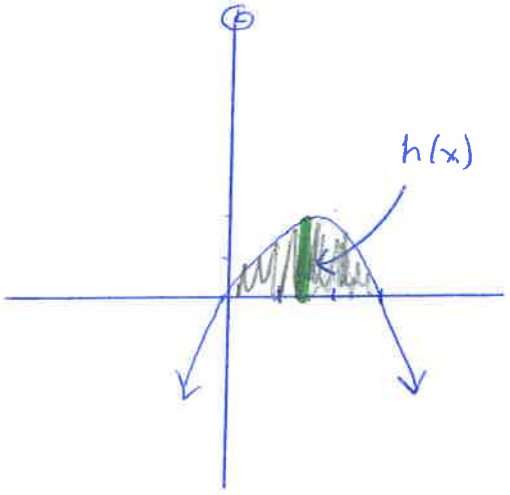
$V = \int_{-2}^2 2\pi (y^3 - 6y^2 - 4y + 24)\, dy$

$= \boxed{128\pi}$

2) AP MULTIPLE CHOICE EXAMPLES

1) The volume of the solid of revolution generated when the region in the first quadrant bounded by the graph of $y = -x^2 + 3x$ and the x-axis is revolved about the y-axis is

(A) 6.75π (B) 13.5π (C) 8.1π (D) 9π



$h(x) = -x^2+3x$

$SA(x) = 2\pi r h$

$= 2\pi x \cdot (-x^2+3x)$

$= 2\pi(-x^3+3x^2)$

$V = 2\pi \int_0^3 -x^3+3x^2\,dx$

$= 2\pi \cdot (6.5)$

2) The region in the first quadrant between the x-axis and the graph of $y = 6x - x^2$ is rotated around the y-axis. The volume of the resulting solid of revolution is given by

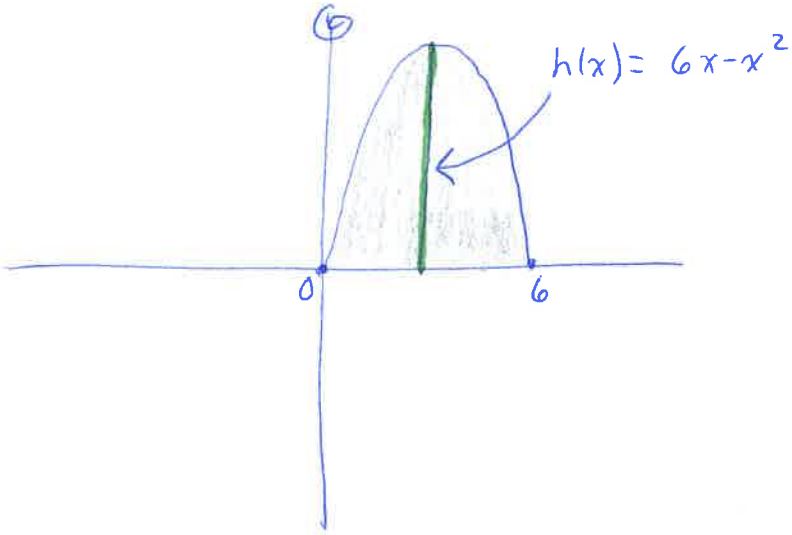
(A) $\int_0^6 \pi\left(6x-x^2\right)^2 dx$

(B) $\int_0^6 2\pi x\left(6x-x^2\right)dx$

(C) $\int_0^6 \pi x\left(6x-x^2\right)^2 dx$

(D) $\int_0^6 \pi\left(3+\sqrt{9-y}\right)^2 dy$

(E) $\int_0^9 \pi\left(3+\sqrt{9-y}\right)^2 dy$



$SA(x) = 2\pi r h$

$= 2\pi x \cdot (6x-x^2)$

$V = \int_0^6 2\pi x\,(6x-x^2)$