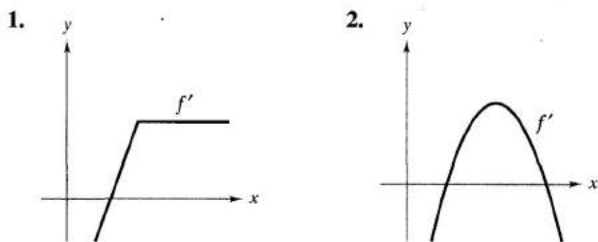


REVIEW EXERCISES FOR CHAPTER 4

In Exercises 1 and 2, use the graph of f' to sketch a graph of f .



In Exercises 3–8, find the indefinite integral.

$$3. \int (2x^2 + x - 1) dx \qquad 4. \int \frac{2}{\sqrt[3]{3x}} dx$$

$$5. \int \frac{x^3 + 1}{x^2} dx \qquad 6. \int \frac{x^3 - 2x^2 + 1}{x^2} dx$$

$$7. \int (4x - 3 \sin x) dx \qquad 8. \int (5 \cos x - 2 \sec^2 x) dx$$

9. Find the particular solution of the differential equation $f'(x) = -2x$ whose graph passes through the point $(-1, 1)$.

10. Find the particular solution of the differential equation $f''(x) = 6(x - 1)$ whose graph passes through the point $(2, 1)$ and at that point is tangent to the line $3x - y - 5 = 0$.

11. **Velocity and Acceleration** An airplane taking off from a runway travels 3600 feet before lifting off. If it starts from rest, moves with constant acceleration, and makes the run in 30 seconds, with what speed does it lift off?

12. **Velocity and Acceleration** The speed of a car traveling in a straight line is reduced from 45 to 30 miles per hour in a distance of 264 feet. Find the distance in which the car can be brought to rest from 30 miles per hour, assuming the same constant deceleration.

13. **Velocity and Acceleration** A ball is thrown vertically upward from ground level with an initial velocity of 96 feet per second.

- How long will it take the ball to rise to its maximum height?
- What is the maximum height?
- When is the velocity of the ball one-half the initial velocity?
- What is the height of the ball when its velocity is one-half the initial velocity?

14. **Velocity and Acceleration** Repeat Exercise 13 for an initial velocity of 40 meters per second.

15. Write in the sigma notation (a) the sum of the first ten positive odd integers, (b) the sum of the cubes of the first n positive integers, and (c) $6 + 10 + 14 + 18 + \cdots + 42$.

16. Evaluate each of the following sums for $x_1 = 2, x_2 = -1, x_3 = 5, x_4 = 3,$ and $x_5 = 7$.

$$(a) \frac{1}{5} \sum_{i=1}^5 x_i \qquad (b) \sum_{i=1}^5 \frac{1}{x_i}$$

$$(c) \sum_{i=1}^5 (2x_i - x_i^2) \qquad (d) \sum_{i=2}^5 (x_i - x_{i-1})$$

17. Consider the region bounded by $y = mx, y = 0, x = 0,$ and $x = b$.

- Find the upper and lower sums to approximate the area of the region when $\Delta x = b/4$.
- Find the upper and lower sums to approximate the area of the region when $\Delta x = b/n$.
- Find the area of the region by letting n approach infinity in both sums in part (b). Show that in each case you obtain the formula for the area of a triangle.
- Find the area of the region using the Fundamental Theorem of Calculus.

18. (a) Find the area of the region bounded by the graphs of $y = x^3, y = 0, x = 1,$ and $x = 3$ by the limit definition.

- Find the area of the region using the Fundamental Theorem of Calculus.

In Exercises 19 and 20, use the given values to find the value of each definite integral.

19. If $\int_2^6 f(x) dx = 10$ and $\int_2^6 g(x) dx = 3$, find

$$(a) \int_2^6 [f(x) + g(x)] dx \qquad (b) \int_2^6 [f(x) - g(x)] dx$$

$$(c) \int_2^6 [2f(x) - 3g(x)] dx \qquad (d) \int_2^6 5f(x) dx$$

20. If $\int_0^3 f(x) dx = 4$ and $\int_3^6 f(x) dx = -1$, find

$$(a) \int_0^6 f(x) dx \qquad (b) \int_6^3 f(x) dx$$

$$(c) \int_4^4 f(x) dx \qquad (d) \int_3^6 -10f(x) dx$$

In Exercises 21–34, find the indefinite integral.

$$21. \int (x^2 + 1)^3 dx \qquad 22. \int \left(x + \frac{1}{x}\right)^2 dx$$

$$23. \int \frac{x^2}{\sqrt{x^3 + 3}} dx \qquad 24. \int x^2 \sqrt{x^3 + 3} dx$$

$$25. \int x(1 - 3x^2)^4 dx \qquad 26. \int \frac{x + 3}{(x^2 + 6x - 5)^2} dx$$

27. $\int \sin^3 x \cos x \, dx$ 28. $\int x \sin 3x^2 \, dx$
29. $\int \frac{\sin \theta}{\sqrt{1 - \cos \theta}} \, d\theta$ 30. $\int \frac{\cos x}{\sqrt{\sin x}} \, dx$
31. $\int \tan^n x \sec^2 x \, dx, \quad n \neq -1$
32. $\int \sec 2x \tan 2x \, dx$
33. $\int (1 + \sec \pi x)^2 \sec \pi x \tan \pi x \, dx$
34. $\int \cot^4 \alpha \csc^2 \alpha \, d\alpha$

In Exercises 35–46, use the Fundamental Theorem of Calculus to evaluate the definite integral. Use a graphing utility to verify your result.

35. $\int_0^4 (2 + x) \, dx$ 36. $\int_{-1}^1 (t^2 + 2) \, dt$
37. $\int_{-1}^1 (4t^3 - 2t) \, dt$ 38. $\int_3^6 \frac{x}{3\sqrt{x^2 - 8}} \, dx$
39. $\int_0^3 \frac{1}{\sqrt{1 + x}} \, dx$ 40. $\int_0^1 x^2(x^3 + 1)^3 \, dx$
41. $\int_4^9 x\sqrt{x} \, dx$ 42. $\int_1^2 \left(\frac{1}{x^2} - \frac{1}{x^3}\right) \, dx$
43. $2\pi \int_0^1 (y + 1)\sqrt{1 - y} \, dy$ 44. $2\pi \int_{-1}^0 x^2\sqrt{x + 1} \, dx$
45. $\int_0^\pi \cos \frac{x}{2} \, dx$ 46. $\int_{-\pi/4}^{\pi/4} \sin 2x \, dx$

In Exercises 47–52, sketch the graph of the region whose area is given by the integral and find the area.

47. $\int_1^3 (2x - 1) \, dx$ 48. $\int_0^2 (x + 4) \, dx$
49. $\int_3^4 (x^2 - 9) \, dx$ 50. $\int_{-1}^2 (-x^2 + x + 2) \, dx$
51. $\int_0^1 (x - x^3) \, dx$ 52. $\int_0^1 \sqrt{x}(1 - x) \, dx$

In Exercises 53–56, sketch the region bounded by the graphs of the equations and determine its area.

53. $y = \frac{4}{\sqrt{x + 1}}, y = 0, x = 0, x = 8$
54. $y = x, y = x^5$
55. $y = \sec^2 x, y = 0, x = 0, x = \frac{\pi}{3}$
56. $y = \cos x, y = 0, x = -\frac{\pi}{4}, x = \frac{\pi}{4}$

In Exercises 57–60, find the average value of the function over the interval. Find the values of x where the function assumes its average value and graph the function.

Function	Interval
57. $f(x) = \frac{1}{\sqrt{x - 1}}$	[5, 10]
58. $f(x) = x^3$	[0, 2]
59. $f(x) = x$	[0, 4]
60. $f(x) = x^2 - \frac{1}{x^2}$	[1, 2]

In Exercises 61 and 62, use the Trapezoidal Rule and Simpson's Rule with $n = 4$, and use the integration capabilities of a graphing utility, to approximate the definite integral. Compare the results.

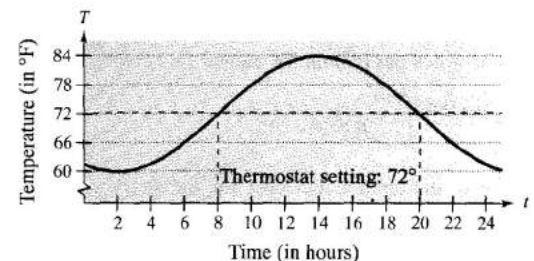
61. $\int_1^2 \frac{1}{1 + x^3} \, dx$ 62. $\int_0^1 \frac{x^{3/2}}{3 - x^2} \, dx$

63. Air Conditioning Costs The temperature in degrees Fahrenheit is $T = 72 + 12 \sin[\pi(t - 8)/12]$ where t is time in hours, with $t = 0$ representing midnight. Suppose the hourly cost of cooling a house is \$0.10 per degree.

(a) Find the cost C of cooling the house if its thermostat is set at 72°F by evaluating the integral

$$C = 0.1 \int_8^{20} \left[72 + 12 \sin \frac{\pi(t - 8)}{12} - 72 \right] dt.$$

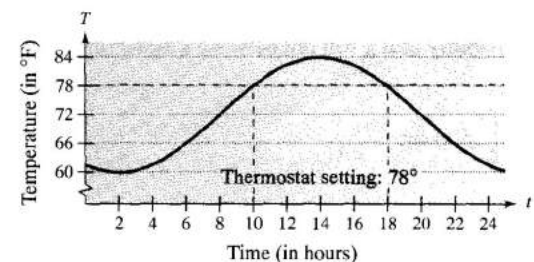
(See figure.)



(b) Find the savings from resetting the thermostat to 78°F by evaluating the integral

$$C = 0.1 \int_{10}^{18} \left[72 + 12 \sin \frac{\pi(t - 8)}{12} - 78 \right] dt.$$

(See figure.)



64. Production Scheduling A manufacturer of fertilizer finds that national sales of fertilizer follow the seasonal pattern

$$F = 100,000 \left[1 + \sin \frac{2\pi(t - 60)}{365} \right]$$

where F is measured in pounds and t is time in days, with $t = 1$ representing January 1. The manufacturer wants to set up a schedule to produce a uniform amount each day. What should this amount be?

65. Respiratory Cycle For a person at rest, the rate of air intake v , in liters per second, during a respiratory cycle is

$$v = 0.85 \sin \frac{\pi t}{3}$$

where t is time in seconds. Find the volume, in liters, of air inhaled during one cycle by integrating the function over the interval $[0, 3]$.

66. Respiratory Cycle After exercising a few minutes, a person has a respiratory cycle for which the rate of air intake is

$$v = 1.75 \sin \frac{\pi t}{2}$$

Find the volume, in liters, of air inhaled during one cycle by integrating the function over the interval $[0, 2]$. How much does the person's lung capacity increase as a result of exercising? (Compare your answer with that found in Exercise 65.)

67. Suppose that gasoline is increasing in price according to the equation

$$p = 1.20 + 0.04t$$

where p is the dollar price per gallon and $t = 0$ represents the year 1990. If an automobile is driven 15,000 miles a year and gets M miles per gallon, the annual fuel cost is

$$C = \frac{15,000}{M} \int_t^{t+1} p \, ds.$$

Estimate the annual fuel cost for the years (a) 2000 and (b) 2005.

Probability In Exercises 68 and 69, the function

$$f(x) = kx^n(1-x)^m, \quad 0 \leq x \leq 1$$

where $n > 0$, $m > 0$, and k is a constant, can be used to represent various probability distributions. If k is chosen such that

$$\int_0^1 f(x) \, dx = 1$$

the probability that x will fall between a and b ($0 \leq a \leq b \leq 1$) is

$$P_{a,b} = \int_a^b f(x) \, dx.$$

68. The probability of recall in a certain experiment is

$$P_{a,b} = \int_a^b \frac{15}{4} x \sqrt{1-x} \, dx$$

where x represents the percent of recall. (See figure.)

- (a) For a randomly chosen individual, what is the probability that he or she will recall between 50% and 75% of the material?
- (b) What is the median percent recall? That is, for what value of b is it true that the probability from 0 to b is 0.5?

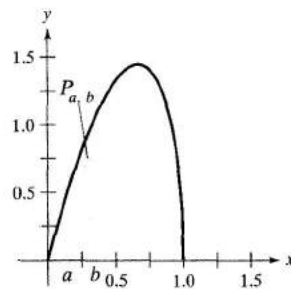


Figure for 68

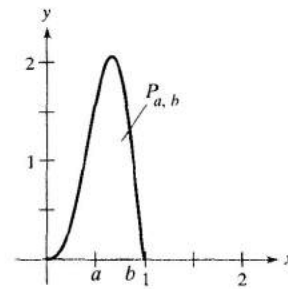


Figure for 69

69. The probability of finding iron in ore samples taken from a certain region is

$$P_{a,b} = \int_a^b \frac{1155}{32} x^3(1-x)^{3/2} \, dx.$$

(See figure.) What is the probability that a sample will contain between

- (a) 0% and 25%?
- (b) 50% and 100%?

True or False? In Exercises 70–74, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

70. $\int x f(x) \, dx = x \int f(x) \, dx$

71. $\int (1/x) \, dx = -\left(\frac{1}{x^2}\right) + C$

72. If $f(x) = -f(-x)$ on the interval $[-a, a]$, then $\int_{-a}^a f(x) \, dx = 0$.

73. The average value of the sine function over an interval of length 2π is 0.

74. $\int \tan x \, dx = \sec^2 x + C$