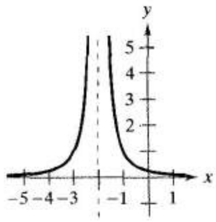
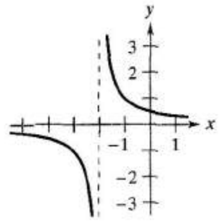
EXERCISES FOR SECTION 1.5

In Exercises 1–4, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -2 from the left and from the right.

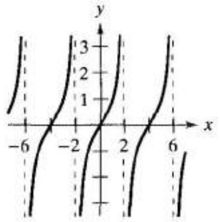
1. $f(x) = \dfrac{1}{(x+2)^2}$



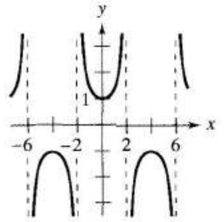
2. $f(x) = \dfrac{1}{x+2}$



3. $f(x) = \tan \dfrac{\pi x}{4}$



4. $f(x) = \sec \dfrac{\pi x}{4}$



Numerical and Graphical Analysis **In Exercises 5–8, determine whether $f(x)$ approaches ∞ or $-\infty$ as x approaches -3 from the left and from the right by completing the table. Use a graphing utility to graph the function and confirm your answer.**

x	-3.5	-3.1	-3.01	-3.001
$f(x)$				

x	-2.999	-2.99	-2.9	-2.5
$f(x)$				

5. $f(x) = \dfrac{1}{x^2-9}$

6. $f(x) = \dfrac{x}{x^2-9}$

7. $f(x) = \dfrac{x^2}{x^2-9}$

8. $f(x) = \sec \dfrac{\pi x}{6}$

In Exercises 9–24, find the vertical asymptotes (if any) of the function.

9. $f(x) = \dfrac{1}{x^2}$

10. $f(x) = \dfrac{4}{(x-2)^3}$

11. $h(x) = \dfrac{x^2-2}{x^2-x-2}$

12. $g(x) = \dfrac{2+x}{1-x}$

13. $f(x) = \dfrac{x^3}{x^2-1}$

14. $f(x) = \dfrac{-4x}{x^2+4}$

15. $f(x) = \tan 2x$

16. $f(x) = \sec \pi x$

17. $T(t) = 1 - \dfrac{4}{t^2}$

18. $V(s) = \dfrac{-2}{(s-2)^2}$

19. $f(x) = \dfrac{x}{x^2+x-2}$

20. $f(x) = \dfrac{1}{(x+3)^4}$

21. $g(x) = \dfrac{x^3+1}{x+1}$

22. $h(x) = \dfrac{x^2-4}{x^3+2x^2+x+2}$

23. $s(t) = \dfrac{t}{\sin t}$

24. $g(\theta) = \dfrac{\tan\theta}{\theta}$

In Exercises 25–28, determine whether the function has a vertical asymptote or a removable discontinuity at $x = -1$. Graph the function using a graphing utility to confirm your answer.

25. $f(x) = \dfrac{x^2-1}{x+1}$

26. $f(x) = \dfrac{x^2-6x-7}{x+1}$

27. $f(x) = \dfrac{x^2+1}{x+1}$

28. $f(x) = \dfrac{\sin(x+1)}{x+1}$

In Exercises 29–40, find the limit.

29. $\displaystyle\lim_{x\to 2^+} \frac{x-3}{x-2}$

30. $\displaystyle\lim_{x\to 1^+} \frac{2+x}{1-x}$

31. $\displaystyle\lim_{x\to 4^+} \frac{x^2}{x^2-16}$

32. $\displaystyle\lim_{x\to 4^-} \frac{x^2}{x^2+16}$

33. $\displaystyle\lim_{x\to -3^-} \frac{x^2+2x-3}{x^2+x-6}$

34. $\displaystyle\lim_{x\to(-1/2)^+} \frac{6x^2+x-1}{4x^2-4x-3}$

35. $\displaystyle\lim_{x\to 0^-} \left(1+\frac{1}{x}\right)$

36. $\displaystyle\lim_{x\to 0^-} \left(x^2-\frac{1}{x}\right)$

37. $\displaystyle\lim_{x\to 0^+} \frac{2}{\sin x}$

38. $\displaystyle\lim_{x\to(\pi/2)^+} \frac{-2}{\cos x}$

39. $\displaystyle\lim_{x\to 1} \frac{x^2-x}{(x^2+1)(x-1)}$

40. $\displaystyle\lim_{x\to 3} \frac{x-2}{x^2}$

In Exercises 41–44, use a graphing utility to graph the function and determine the one-sided limit.

41. $f(x) = \dfrac{x^2+x+1}{x^3-1}$

$\displaystyle\lim_{x\to 1^+} f(x)$

42. $f(x) = \dfrac{x^3-1}{x^2+x+1}$

$\displaystyle\lim_{x\to 1^-} f(x)$

43. $f(x) = \dfrac{1}{x^2-25}$

$\displaystyle\lim_{x\to 5^-} f(x)$

44. $f(x) = \sec \dfrac{\pi x}{6}$

$\displaystyle\lim_{x\to 3^+} f(x)$

45. A given sum S is inversely proportional to $1-r$, where $0 < |r| < 1$. Find the limit of S as $r \to 1^-$.

46. ***Boyle's Law*** For a quantity of gas at a constant temperature, the pressure P is inversely proportional to the volume V. Find the limit of P as $V \to 0^+$.

47. ***Rate of Change*** A 25-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of

$$r = \frac{2x}{\sqrt{625 - x^2}} \text{ ft/sec}$$

where x is the distance between the base of the ladder and the house.

(a) Find the rate r when x is 7 feet.

(b) Find the rate r when x is 15 feet.

(c) Find the limit of r as $x \to 25^-$.

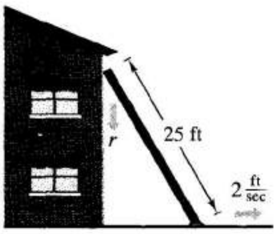


Figure for 47

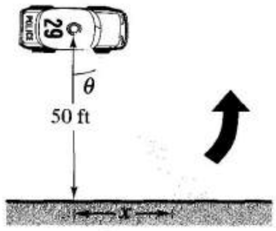


Figure for 48

48. ***Rate of Change*** A patrol car is parked 50 feet from a long warehouse (see figure). The revolving light on top of the car turns at a rate of $\frac{1}{2}$ revolution per second. The rate at which the light beam moves along the wall is

$$r = 50\pi \sec^2 \theta \text{ ft/sec.}$$

(a) Find the rate r when θ is $\pi/6$.

(b) Find the rate r when θ is $\pi/3$.

(c) Find the limit of r as $\theta \to (\pi/2)^-$.

49. ***Illegal Drugs*** The cost in millions of dollars for a governmental agency to seize $x\%$ of an illegal drug is

$$C = \frac{528x}{100 - x}, \quad 0 \le x < 100.$$

(a) Find the cost of seizing 25%.

(b) Find the cost of seizing 50%.

(c) Find the cost of seizing 75%.

(d) Find the limit of C as $x \to 100^-$.

50. ***Average Speed*** On a trip of d miles to another city, a truck driver's average speed was x miles per hour. On the return trip the average speed was y miles per hour. The average speed for the round trip was 50 miles per hour.

(a) Verify that $y = \frac{25x}{x - 25}$. What is the domain?

(b) Complete the table.

x	30	40	50	60
y				

Are the values of y different than expected? Explain.

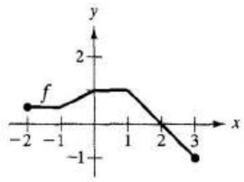
(c) Find the limit of y as $x \to 25^+$.

51. ***Relativity*** According to the theory of relativity, the mass m of a particle depends on its velocity v. That is,

$$m = \frac{m_0}{\sqrt{1 - (v^2/c^2)}}$$

where m_0 is the mass when the particle is at rest and c is the speed of light. Find the limit of the mass as v approaches c^-.

52. ***Think About It*** Use the graph of the function f (see figure) to sketch the graph of $g(x) = 1/f(x)$ on the interval $[-2, 3]$.



53. ***Numerical and Graphical Reasoning*** A crossed belt connects a 20-centimeter pulley (10-cm radius) on an electric motor with a 40-centimeter pulley (20-cm radius) on a saw arbor (see figure). The electric motor runs at 1700 revolutions per minute.

(a) Determine the number of revolutions per minute of the saw.

(b) How does crossing the belt affect the saw in relation to the motor?

(c) Let L be the total length of the belt. Write L as a function of ϕ, where ϕ is measured in radians. What is the domain of the function? [*Hint:* Add the lengths of the straight sections of the belt and the length of the belt around each pulley.]

(d) Use a graphing utility to complete the table.

ϕ	0.3	0.6	0.9	1.2	1.5
L					

(e) Use a graphing utility to graph the function over the appropriate domain.

(f) Find

$$\lim_{\phi \to (\pi/2)^-} L.$$

Use a geometric argument as the basis of a second method of finding this limit.

(g) Find $\lim_{\phi \to 0^+} L$.