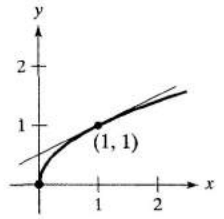
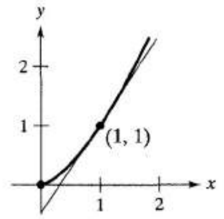
EXERCISES FOR SECTION 2.2

In Exercises 1 and 2, find the slope of the tangent line to $y = x^n$ at the point $(1, 1)$.

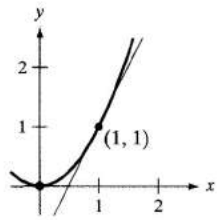
1. (a) $y = x^{1/2}$



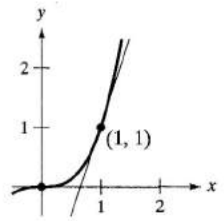
(b) $y = x^{3/2}$



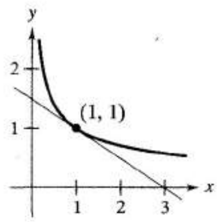
(c) $y = x^2$



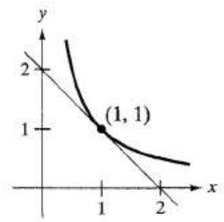
(d) $y = x^3$



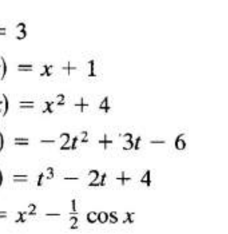
2. (a) $y = x^{-1/2}$



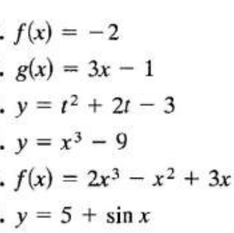
(b) $y = x^{-1}$



(c) $y = x^{-3/2}$



(d) $y = x^{-2}$



In Exercises 3–16, find the derivative of the function.

3. $y = 3$
4. $f(x) = -2$
5. $f(x) = x + 1$
6. $g(x) = 3x - 1$
7. $g(x) = x^2 + 4$
8. $y = t^2 + 2t - 3$
9. $f(t) = -2t^2 + 3t - 6$
10. $y = x^3 - 9$
11. $s(t) = t^3 - 2t + 4$
12. $f(x) = 2x^3 - x^2 + 3x$
13. $y = x^2 - \frac{1}{2}\cos x$
14. $y = 5 + \sin x$
15. $y = \dfrac{1}{x} - 3\sin x$
16. $g(t) = \pi \cos t$

In Exercises 17–22, complete the table, using Example 6 as a model.

	Original Function	*Rewrite*	*Differentiate*	*Simplify*
17.	$y = \dfrac{1}{3x^3}$			
18.	$y = \dfrac{2}{3x^2}$			
19.	$y = \dfrac{1}{(3x)^3}$			
20.	$y = \dfrac{\pi}{(3x)^2}$			
21.	$y = \dfrac{\sqrt{x}}{x}$			
22.	$y = \dfrac{4}{x^{-3}}$			

In Exercises 23–30, find the value of the derivative of the function at the indicated point. Use the derivative feature of a graphing utility to confirm your results.

	Function	*Point*
23.	$f(x) = \dfrac{1}{x}$	$(1, 1)$
24.	$f(t) = 3 - \dfrac{3}{5t}$	$(\frac{3}{5}, 2)$
25.	$f(x) = -\frac{1}{2} + \frac{7}{5}x^3$	$(0, -\frac{1}{2})$
26.	$y = 3x\left(x^2 - \dfrac{2}{x}\right)$	$(2, 18)$
27.	$y = (2x + 1)^2$	$(0, 1)$
28.	$f(x) = 3(5 - x)^2$	$(5, 0)$
29.	$f(\theta) = 4\sin\theta - \theta$	$(0, 0)$
30.	$g(t) = 2 + 3\cos t$	$(\pi, -1)$

In Exercises 31–42, find the derivative of the function.

31. $f(x) = x^3 - 3x - 2x^{-4}$
32. $f(x) = x^2 - 3x - 3x^{-2}$
33. $g(t) = t^2 - \dfrac{4}{t}$
34. $f(x) = x + \dfrac{1}{x^2}$
35. $f(x) = \dfrac{x^3 - 3x^2 + 4}{x^2}$
36. $h(x) = \dfrac{2x^2 - 3x + 1}{x}$
37. $y = x(x^2 + 1)$
38. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}$
39. $h(s) = s^{4/5}$
40. $f(t) = t^{1/3} - 1$
41. $f(x) = 4\sqrt{x} + 3\cos x$
42. $f(x) = 2\sin x + 3\cos x$

In Exercises 43–46, (a) find an equation of the tangent line to the graph of f at the indicated point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the derivative feature of a graphing utility to confirm your results.

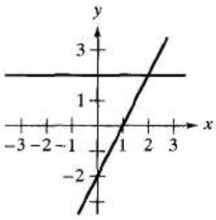
Function	*Point*
43. $y = x^4 - 3x^2 + 2$	$(1, 0)$
44. $y = x^3 + x$	$(-1, -2)$
45. $f(x) = \dfrac{1}{\sqrt[3]{x^2}}$	$\left(8, \frac{1}{4}\right)$
46. $y = (x^2 + 2x)(x + 1)$	$(1, 6)$

In Exercises 47–52, determine the point(s) (if any) at which the function has a horizontal tangent line.

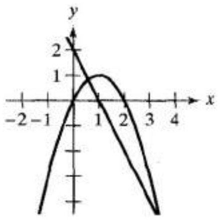
47. $y = x^4 - 8x^2 + 2$
48. $y = x^3 + x$
49. $y = \dfrac{1}{x^2}$
50. $y = x^2 + 1$
51. $y = x + \sin x, \quad 0 \le x < 2\pi$
52. $y = \sqrt{3}x + 2\cos x, \quad 0 \le x < 2\pi$

Writing **In Exercises 53 and 54, the graphs of a function f and its derivative f' are given on the same set of coordinate axes. Label the graphs as f or f' and write a short paragraph stating the criteria used in making the selection.**

53.



54.



55. Sketch the graphs of the two equations $y = x^2$ and $y = -x^2 + 6x - 5$, and sketch the two lines that are tangent to both graphs. Find the equations of these lines.

56. Show that the graphs of the two equations $y = x$ and $y = 1/x$ have tangent lines that are perpendicular to each other at their point of intersection.

In Exercises 57 and 58, find an equation of the tangent line to the graph of the function f through the point (x_0, y_0) not on the graph. To find the point of tangency (x, y) on the graph of f, solve the equation

$$f'(x) = \frac{y_0 - y}{x_0 - x}.$$

57. $f(x) = \sqrt{x}$

$(x_0, y_0) = (-4, 0)$

58. $f(x) = \dfrac{2}{x}$

$(x_0, y_0) = (5, 0)$

59. ***Linear Approximation*** Consider the function $f(x) = x^{3/2}$ with the solution point $(4, 8)$.

(a) Use a graphing utility to obtain the graph of f. Use the zoom feature to obtain successive magnifications of the graph in the neighborhood of the point $(4, 8)$. After zooming in a few times, the graph should appear nearly linear. Use the trace feature to determine the coordinates of a point "near" $(4, 8)$. Find an equation of the secant line $S(x)$ through the two points.

(b) Find the equation of the line

$$T(x) = f'(4)(x - 4) + f(4)$$

tangent to the graph of f passing through the given point. Why are the linear functions S and T nearly the same?

(c) Use a graphing utility to graph f and T on the same set of coordinate axes. Note that T is a "good" approximation of f when x is "close to" 4. What happens to the accuracy of the approximation as you move farther away from the point of tangency?

(d) Demonstrate the conclusion in part (c) by completing the table.

Δx	-3	-2	-1	-0.5	-0.1	0
$f(x)$						
$T(x)$						

Δx	0.1	0.5	1	2	3
$f(x)$					
$T(x)$					

60. ***Linear Approximation*** Repeat Exercise 59 for the function $f(x) = x^3$ where $T(x)$ is the line tangent to the graph at the point $(1, 1)$. Explain why the accuracy of the linear approximation decreases more rapidly than in Exercise 59.

True or False? **In Exercises 61–64, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

61. If $f'(x) = g'(x)$, then $f(x) = g(x)$.

62. If $f(x) = g(x) + c$, then $f'(x) = g'(x)$.

63. If $y = \pi^2$, then $dy/dx = 2\pi$.

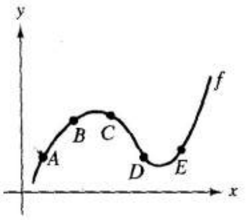
64. If $y = x/\pi$, then $dy/dx = 1/\pi$.

In Exercises 65–68, find the average rate of change of the function over the indicated interval. Compare this average rate of change with the instantaneous rates of change at the endpoints of the interval.

Function	*Interval*
65. $f(t) = 2t + 7$	$[1, 2]$
66. $f(t) = t^2 - 3$	$[2, 2.1]$

Function	*Interval*
67. $f(x) = \frac{-1}{x}$	$[1, 2]$
68. $f(x) = \sin x$	$\left[0, \frac{\pi}{6}\right]$

69. ***Think About It*** Use the graph of f to answer each question.



(a) Between which two consecutive points is the average rate of change of the function greatest?

(b) Is the average rate of change of the function between A and B greater than or less than the instantaneous rate of change at B?

(c) Sketch a tangent line to the graph between the points C and D such that the slope of the tangent line is the same as the average rate of change of the function between C and D.

(d) Give any sets of consecutive points for which the average rates of change of the function are approximately equal.

70. ***Think About It*** Sketch the graph of a function f such that $f' > 0$ for all x and the rate of change of the function is decreasing.

Vertical Motion **In Exercises 71 and 72, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.**

71. A silver dollar is dropped from the top of the World Trade Center, which is 1362 feet tall.

(a) Determine the position and velocity functions for the coin.

(b) Determine the average velocity on the interval $[1, 2]$.

(c) Find the instantaneous velocities when $t = 1$ and $t = 2$.

(d) Find the time required for the coin to reach ground level.

(e) Find the velocity of the dollar just before it hits the ground.

72. A ball is thrown straight down from the top of a 220-foot building with an initial velocity of -22 feet per second. What is its velocity after 3 seconds? What is its velocity after falling 108 feet?

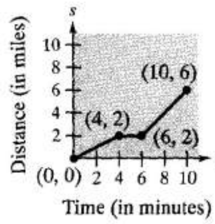
Vertical Motion **In Exercises 73 and 74, use the position function $s(t) = -4.9t^2 + v_0 t + s_0$ for free-falling objects.**

73. A projectile is shot upward from the surface of the earth with an initial velocity of 120 meters per second. What is its velocity after 5 seconds? After 10 seconds?

74. To estimate the height of a building, a stone is dropped from the top of the building into a pool of water at ground level. How high is the building if the splash is seen 6.8 seconds after the stone is dropped?

Think About It **In Exercises 75 and 76, the graph of a position function is given. It represents the distance in miles that a person drives during a 10-minute trip to work. Make a sketch of the corresponding velocity function.**

75.



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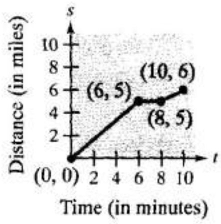
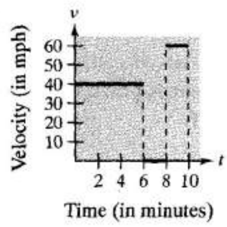


Think About It **In Exercises 77 and 78, the graph of a velocity function is given. It represents the velocity in miles per hour during a 10-minute drive to work. Make a sketch of the corresponding position function.**

77.



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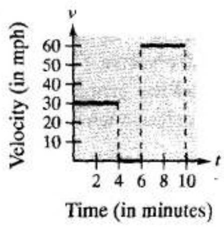

79. ***Modeling Data*** The stopping distance of an automobile traveling at a speed v (kilometers per hour) is the distance R (meters) the car travels during the reaction time of the driver plus the distance B (meters) the car travels after the brakes are applied. The table shows the results of an experiment.

v	20	40	60	80	100
R	3.3	6.7	10.0	13.3	16.7
B	2.3	8.9	20.2	35.9	56.7

(a) Use the regression capabilities of a graphing utility to find a linear model for reaction time.

(b) Use the regression capabilities of a graphing utility to find a quadratic model for braking time.

(c) Determine the polynomial giving the total stopping distance T.

(d) Use a graphing utility to graph the functions R, B, and T in the same viewing rectangle.

(e) Find the derivative of T and the rate of change of the total stopping distance for $v = 40$, $v = 80$, and $v = 100$.

(f) Use the results of this exercise to draw conclusions about the total stopping distance as speed increases.

80. *Velocity* Verify that the average velocity over the time interval $[t_0 - \Delta t, t_0 + \Delta t]$ is the same as the instantaneous velocity at $t = t_0$ for the position function.

$$s(t) = -\tfrac{1}{2}at^2 + c.$$

81. *Area* The area of a square with sides of length s is given by $A = s^2$. Find the rate of change of the area with respect to s when $s = 4$ meters.

82. *Volume* The volume of a cube with sides of length s is given by $V = s^3$. Find the rate of change of the volume with respect to s when $s = 4$ centimeters.

83. *Inventory Management* The annual inventory cost C for a certain manufacturer is

$$C = \frac{1{,}008{,}000}{Q} + 6.3Q$$

where Q is the order size when the inventory is replenished. Find the change in annual cost when Q is increased from 350 to 351, and compare this with the instantaneous rate of change when $Q = 350$.

84. *Fuel Cost* A car is driven 15,000 miles a year and gets x miles per gallon. Assume that the average fuel cost is \$1.25 per gallon. Find the annual cost of fuel C as a function of x and use this function to complete the table.

x	10	15	20	25	30	35	40
C							
$\frac{dC}{dx}$							

Who would benefit more from a 1-mile-per-gallon increase in fuel efficiency—the driver of a car that gets 15 miles per gallon or the driver of a car that gets 35 miles per gallon? Explain.

85. *Writing* The photo at the bottom of the page was taken at game 4 of the 1996 World Series between the New York Yankees and the Atlanta Braves. Do you think the runner was safe or out? Write a detailed explanation of your reasoning.

86. *Newton's Law of Cooling* This law states that the rate of change of the temperature of an object is proportional to the difference between the object's temperature T and the temperature T_a of the surrounding medium. Write an equation for this law.

87. Find an equation of the parabola $y = ax^2 + bx + c$ that passes through $(0, 1)$ and is tangent to the line $y = x - 1$ at $(1, 0)$.

88. Let (a, b) be an arbitrary point on the graph of $y = 1/x, x > 0$. Prove that the area of the triangle formed by the tangent line through (a, b) and the coordinate axes is 2.

89. Find the tangent line(s) to the curve $y = x^3 - 9x$ through the point $(1, -9)$.

90. Find the equation(s) of the tangent line(s) to the parabola $y = x^2$ through the given point.

(a) $(0, a)$ (b) $(a, 0)$

Are there any restrictions on the constant a?

91. Find a and b such that

$$f(x) = \begin{cases} ax^3, & x \le 2 \\ x^2 + b, & x > 2 \end{cases}$$

is differentiable everywhere.

92. Where are the functions $f_1(x) = |\sin x|$ and $f_2(x) = \sin |x|$ differentiable?

93. Prove that $\frac{d}{dx}[\cos x] = -\sin x$.

FOR FURTHER INFORMATION For a geometric interpretation of the derivatives of trigonometric functions, see the article "Sines and Cosines of the Times" by Victor J. Katz in the April 1995 issue of *Math Horizons*.