LAB SERIES
Lab 2.1

EXERCISES FOR SECTION 2.3

In Exercises 1–6, find $f'(x)$ and $f'(c)$.

Function	*Value of c*
1. $f(x) = \frac{1}{3}(2x^3 - 4)$	$c = 0$
2. $f(x) = (x^2 - 2x + 1)(x^3 - 1)$	$c = 1$
3. $f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$	$c = 0$
4. $f(x) = \dfrac{x + 1}{x - 1}$	$c = 2$
5. $f(x) = x \cos x$	$c = \dfrac{\pi}{4}$
6. $f(x) = \dfrac{\sin x}{x}$	$c = \dfrac{\pi}{6}$

In Exercises 7–12, complete the table without using the Quotient Rule (see Example 6).

Function	*Rewrite*	*Differentiate*	*Simplify*
7. $y = \dfrac{x^2 + 2x}{x}$			
8. $y = \dfrac{4x^{3/2}}{x}$			
9. $y = \dfrac{7}{3x^3}$			
10. $y = \dfrac{4}{5x^2}$			
11. $y = \dfrac{3x^2 - 5}{7}$			
12. $y = \dfrac{x^2 - 4}{x + 2}$			

In Exercises 13–26, find the derivative of the algebraic function.

13. $f(x) = \dfrac{3x - 2}{2x - 3}$

14. $f(x) = \dfrac{x^3 + 3x + 2}{x^2 - 1}$

15. $f(x) = \dfrac{3 - 2x - x^2}{x^2 - 1}$

16. $f(x) = x^4\left(1 - \dfrac{2}{x + 1}\right)$

17. $f(x) = \dfrac{x + 1}{\sqrt{x}}$

18. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

19. $h(s) = (s^3 - 2)^2$

20. $h(x) = (x^2 - 1)^2$

21. $h(t) = \dfrac{t + 1}{t^2 + 2t + 2}$

22. $f(x) = \dfrac{x(x^2 - 1)}{x + 3}$

23. $f(x) = (3x^3 + 4x)(x - 5)(x + 1)$

24. $f(x) = (x^2 - x)(x^2 + 1)(x^2 + x + 1)$

25. $f(x) = \dfrac{x^2 + c^2}{x^2 - c^2}$, c is a constant

26. $f(x) = \dfrac{c^2 - x^2}{c^2 + x^2}$, c is a constant

In Exercises 27–42, find the derivative of the trigonometric function.

27. $f(t) = t^2 \sin t$

28. $f(\theta) = (\theta + 1) \cos \theta$

29. $f(t) = \dfrac{\cos t}{t}$

30. $f(x) = \dfrac{\sin x}{x}$

31. $f(x) = -x + \tan x$

32. $y = x + \cot x$

33. $g(t) = \sqrt{t} + 4 \sec t$

34. $h(s) = \dfrac{1}{s} - 10 \csc s$

35. $y = 5x \csc x$

36. $y = \dfrac{\sec x}{x}$

37. $y = -\csc x - \sin x$

38. $y = x \sin x + \cos x$

39. $y = x^2 \sin x + 2x \cos x$

40. $f(x) = \sin x \cos x$

41. $f(x) = x^2 \tan x$

42. $h(\theta) = 5 \sec \theta + \tan \theta$

In Exercises 43–46, use a symbolic differentiation utility to differentiate the function.

43. $g(x) = \left(\dfrac{x + 1}{x + 2}\right)(2x - 5)$

44. $f(x) = \left(\dfrac{x^2 - x - 3}{x^2 + 1}\right)(x^2 + x + 1)$

45. $g(\theta) = \dfrac{\theta}{1 - \sin \theta}$

46. $f(\theta) = \dfrac{\sin \theta}{1 - \cos \theta}$

In Exercises 47–50, evaluate the derivative of the function at the indicated point. Use a graphing utility to verify your result.

Function	*Point*
47. $y = \dfrac{1 + \csc x}{1 - \csc x}$	$\left(\dfrac{\pi}{6}, -3\right)$
48. $f(x) = \tan x \cot x$	$(1, 1)$
49. $h(t) = \dfrac{\sec t}{t}$	$\left(\pi, -\dfrac{1}{\pi}\right)$
50. $f(x) = \sin x(\sin x + \cos x)$	$\left(\dfrac{\pi}{4}, 1\right)$

In Exercises 51–56, (a) find an equation of the tangent line to the graph of f at the indicated point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the derivative feature of a graphing utility to confirm your results.

Function	*Point*
51. $f(x) = \dfrac{x}{x - 1}$	$(2, 2)$
52. $f(x) = (x - 1)(x^2 - 2)$	$(0, 2)$
53. $f(x) = (x^3 - 3x + 1)(x + 2)$	$(1, -3)$
54. $f(x) = \dfrac{(x - 1)}{(x + 1)}$	$\left(2, \dfrac{1}{3}\right)$

Function	*Point*
55. $f(x) = \tan x$	$\left(\frac{\pi}{4}, 1\right)$
56. $f(x) = \sec x$	$\left(\frac{\pi}{3}, 2\right)$

In Exercises 57 and 58, determine the point(s) at which the graph of the function has a horizontal tangent.

57. $f(x) = \dfrac{x^2}{x-1}$

58. $f(x) = \dfrac{x^2}{x^2+1}$

Think About It **In Exercises 59–62, find $f'(2)$ given the following.**

$g(2) = 3$ and $g'(2) = -2$

$h(2) = -1$ and $h'(2) = 4$

59. $f(x) = 2g(x) + h(x)$

60. $f(x) = 4 - h(x)$

61. $f(x) = \dfrac{g(x)}{h(x)}$

62. $f(x) = g(x)h(x)$

In Exercises 63 and 64, find the derivative of the function f for $n = 1, 2, 3,$ and 4. Use the result to write a general rule for $f'(x)$ in terms of n.

63. $f(x) = x^n \sin x$

64. $f(x) = \dfrac{\cos x}{x^n}$

65. ***Inventory Replenishment*** The ordering and transportation cost C of the components used in manufacturing a certain product is

$$C = 100\left(\frac{200}{x^2} + \frac{x}{x+30}\right), \quad 1 \le x$$

where C is measured in thousands of dollars and x is the order size in hundreds. Find the rate of change of C with respect to x when (a) $x = 10$, (b) $x = 15$, and (c) $x = 20$. What do these rates of change imply about increasing order size?

66. ***Boyle's Law*** This law states that if the temperature of a gas remains constant, its pressure is inversely proportional to its volume. Use the derivative to show that the rate of change of the pressure is inversely proportional to the square of the volume.

67. ***Population Growth*** A population of 500 bacteria is introduced into a culture and grows in number according to the equation

$$P(t) = 500\left(1 + \frac{4t}{50 + t^2}\right)$$

where t is measured in hours. Find the rate at which the population is growing when $t = 2$.

68. ***Rate of Change*** Determine whether there exist any values of x in the interval $[0, 2\pi)$ such that the rate of change of $f(x) = \sec x$ and the rate of change of $g(x) = \csc x$ are equal.

69. Prove the following differentiation rules.

(a) $\dfrac{d}{dx}[\sec x] = \sec x \tan x$

(b) $\dfrac{d}{dx}[\csc x] = -\csc x \cot x$

(c) $\dfrac{d}{dx}[\cot x] = -\csc^2 x$

70. ***Think About It*** Sketch a graph of a differentiable function f such that $f > 0$ and $f' < 0$ for all real numbers x.

71. ***Think About It*** Sketch a graph of a differentiable function f such that $f(2) = 0$, $f' < 0$ for $-\infty < x < 2$, and $f' > 0$ for $2 < x < \infty$.

72. ***Modeling Data*** The federal tax burden per capita is given in the table. *(Source: U.S. Internal Revenue Service)*

Year	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Tax	\$2738	\$2982	\$3090	\$3414	\$3598	\$3884	\$4026	\$4064	\$4153	\$4382	\$4728	\$4996

Let T be the per capita tax burden and let t be the time in years. Then $T = (2{,}231{,}291 + 110{,}636t)/(1000 - 14t)$ is a model for the data, with $t = 4$ corresponding to 1984.

(a) Find T' and use a graphing utility to graph the derivative.

(b) Interpret the graph in part (a), assuming that this model will be used to forecast the per capita tax burden through the year 2000.

(c) Use the model to predict the per capita tax burden in the year 2000.

In Exercises 73–78, find the second derivative of the function.

73. $f(x) = 4x^{3/2}$

74. $f(x) = \dfrac{x^2 + 2x - 1}{x}$

75. $f(x) = \dfrac{x}{x-1}$

76. $f(x) = x + \dfrac{32}{x^2}$

77. $f(x) = 3 \sin x$

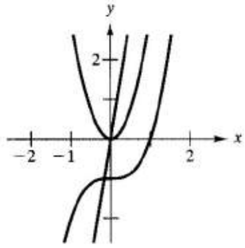
78. $f(x) = \sec x$

In Exercises 79–82, find the higher-order derivative.

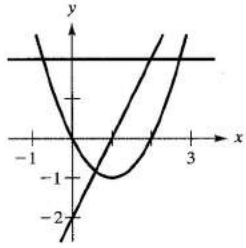
Given	*Find*
79. $f'(x) = x^2$	$f''(x)$
80. $f''(x) = 2 - \frac{2}{x}$	$f'''(x)$
81. $f'''(x) = 2\sqrt{x}$	$f^{(4)}(x)$
82. $f^{(4)}(x) = 2x + 1$	$f^{(6)}(x)$

Think About It **In Exercises 83 and 84, the graphs of f, f', and f'' are shown on the same set of coordinate axes. Which is which?**

83.



84.



85. *Finding a Pattern* Consider the function $f(x) = g(x)h(x)$.

(a) Use the product rule to generate rules for finding $f''(x)$, $f'''(x)$, and $f^{(4)}(x)$.

(b) Use the results in part (a) to write a general rule for $f^{(n)}$.

86. *Finding a Pattern* Develop a general rule for $f^{(n)}(x)$ if

(a) $f(x) = x^n$ and (b) $f(x) = \frac{1}{x}$.

87. *Acceleration* The velocity of an object in meters per second is

$$v(t) = 36 - t^2, \quad 0 \le t \le 6.$$

Find the velocity and acceleration of the object when $t = 3$. What can be said about the speed of the object when the velocity and acceleration have opposite signs?

88. *Acceleration* An automobile's velocity starting from rest is

$$v(t) = \frac{100t}{2t + 15}$$

where v is measured in feet per second. Find the acceleration at each of the following times.

(a) 5 seconds (b) 10 seconds (c) 20 seconds

89. *Stopping Distance* A car is traveling at a rate of 66 feet per second (45 miles per hour) when the brakes are applied. The position function for the car is

$$s(t) = -8.25t^2 + 66t$$

where s is measured in feet and t is measured in seconds. Use this function to complete the table, and find the average velocity during each time interval.

t	0	1	2	3	4
$s(t)$					
$v(t)$					
$a(t)$					

90. *Acceleration on the Moon* An astronaut standing on the moon throws a rock into the air. The height of the rock is

$$s = -\frac{27}{10}t^2 + 27t + 6$$

where s is measured in feet and t is measured in seconds.

(a) Find expressions for the velocity and acceleration of the rock.

(b) Find the time when the rock is at its highest point by finding the time when the velocity is zero. What is its height at this time?

(c) How does the acceleration of the rock compare with the acceleration due to gravity on the earth?

Linear and Quadratic Approximations **The linear and quadratic approximation of a function f at $x = a$ are**

$$P_1(x) = f'(a)(x - a) + f(a) \text{ and}$$

$$P_2(x) = \tfrac{1}{2}f''(a)(x - a)^2 + f'(a)(x - a) + f(a).$$

In Exercises 91 and 92, (a) find the specified linear and quadratic approximations of f, (b) use a graphing utility to graph f and the approximations, (c) determine whether P_1 or P_2 is the better approximation, and (d) state how the accuracy changes as you move farther from $x = a$.

91. $f(x) = \cos x$, $a = \frac{\pi}{3}$

92. $f(x) = \tan x$, $a = \frac{\pi}{4}$

True or False? **In Exercises 93–98, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

93. If $y = f(x)g(x)$, then $dy/dx = f'(x)g'(x)$.

94. If $y = (x + 1)(x + 2)(x + 3)(x + 4)$, then $d^5y/dx^5 = 0$.

95. If $f'(c)$ and $g'(c)$ are zero and $h(x) = f(x)g(x)$, then $h'(c) = 0$.

96. If $f(x)$ is an nth-degree polynomial, then $f^{(n+1)}(x) = 0$.

97. The second derivative represents the rate of change of the first derivative.

98. If the velocity of an object is constant, then its acceleration is zero.

99. Find the derivative of the function $f(x) = x|x|$. Does $f''(0)$ exist?

100. *Think About It* Let f and g be functions whose first and second derivatives exist on an interval I. Which of the following formulas is (are) true?

(a) $fg'' - f''g = (fg' - f'g)'$

(b) $fg'' + f''g = (fg)''$