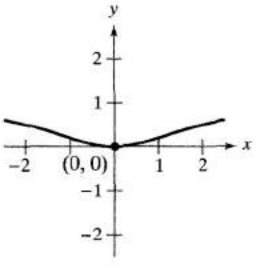
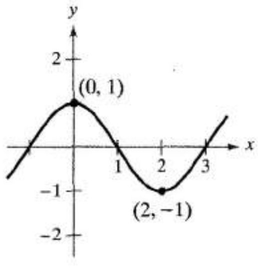
EXERCISES FOR SECTION 3.1

In Exercises 1–6, find the value of the derivative (if it exists) at each indicated extremum.

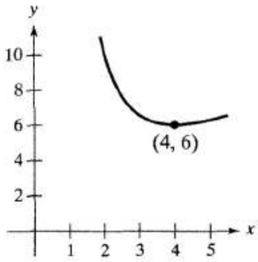
1. $f(x) = \dfrac{x^2}{x^2 + 4}$



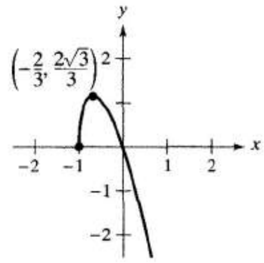
2. $f(x) = \cos \dfrac{\pi x}{2}$



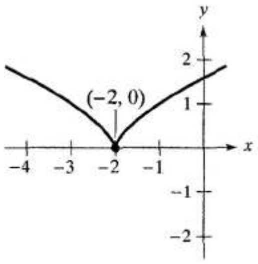
3. $f(x) = x + \dfrac{32}{x^2}$



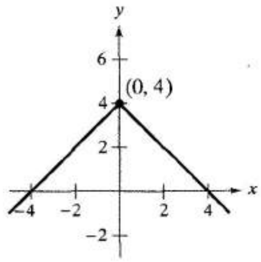
4. $f(x) = -3x\sqrt{x + 1}$



5. $f(x) = (x + 2)^{2/3}$



6. $f(x) = 4 - |x|$



In Exercises 7–12, find any critical numbers of the function.

7. $f(x) = x^2(x - 3)$

8. $g(x) = x^2(x^2 - 4)$

9. $g(t) = t\sqrt{4 - t}$

10. $f(x) = \dfrac{4x}{x^2 + 1}$

11. $h(x) = \sin^2 x + \cos x$
$0 \le x < 2\pi$

12. $f(\theta) = 2 \sec \theta + \tan \theta$
$0 \le \theta < 2\pi$

In Exercises 13–26, determine the absolute extrema of the function and the x-value in the closed interval where it occurs.

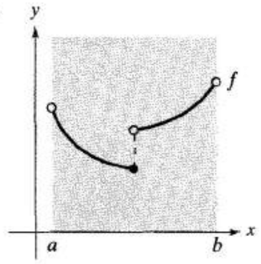
Function	*Interval*
13. $f(x) = 2(3 - x)$	$[-1, 2]$
14. $f(x) = \dfrac{2x + 5}{3}$	$[0, 5]$
15. $f(x) = -x^2 + 3x$	$[0, 3]$
16. $f(x) = x^2 + 2x - 4$	$[-1, 1]$
17. $f(x) = x^3 - 3x^2$	$[-1, 3]$
18. $f(x) = x^3 - 12x$	$[0, 4]$
19. $f(x) = 3x^{2/3} - 2x$	$[-1, 1]$
20. $g(x) = \sqrt[3]{x}$	$[-1, 1]$
21. $h(t) = 4 - \lvert t - 4 \rvert$	$[1, 6]$
22. $g(t) = \dfrac{t^2}{t^2 + 3}$	$[-1, 1]$
23. $h(s) = \dfrac{1}{s - 2}$	$[0, 1]$
24. $h(t) = \dfrac{t}{t - 2}$	$[3, 5]$
25. $f(x) = \cos \pi x$	$\left[0, \dfrac{1}{6}\right]$
26. $g(x) = \csc x$	$\left[\dfrac{\pi}{6}, \dfrac{\pi}{3}\right]$

27. ***Writing*** Explain why the function $f(x) = \tan x$ has a maximum on $[0, \pi/4]$ but not on $[0, \pi]$.

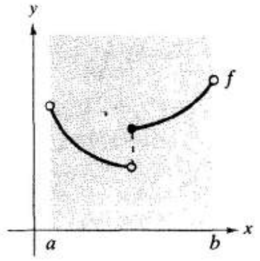
28. ***Writing*** Write a short paragraph explaining why a continuous function on an open interval may not have a maximum or minimum. Illustrate your explanation with a sketch of the graph of a function.

In Exercises 29–32, determine from the graph whether f has a minimum in the open interval (a, b).

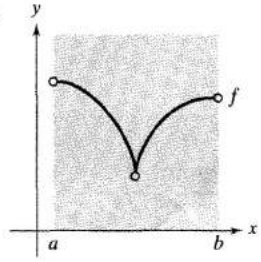
29. (a)



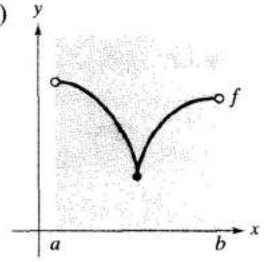
(b)



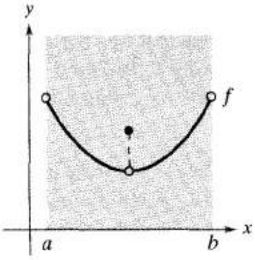
30. (a)



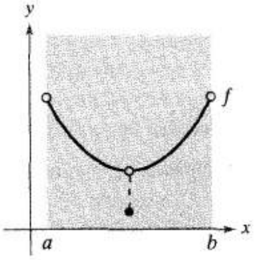
(b)



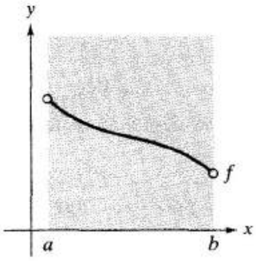
31. (a)



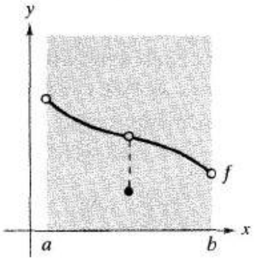
(b)



32. (a)



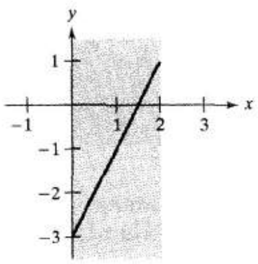
(b)

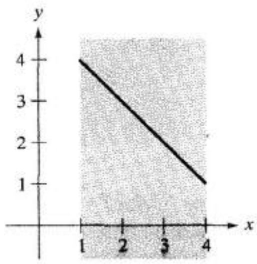


In Exercises 33–36, locate the absolute extrema of the function (if any exists) over the indicated intervals.

33. $f(x) = 2x - 3$
   (a) $[0, 2]$
   (b) $[0, 2)$
   (c) $(0, 2]$
   (d) $(0, 2)$

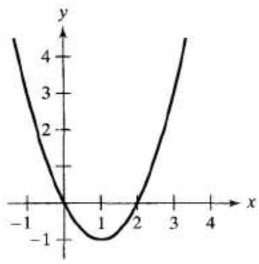
34. $f(x) = 5 - x$
   (a) $[1, 4]$
   (b) $[1, 4)$
   (c) $(1, 4]$
   (d) $(1, 4)$

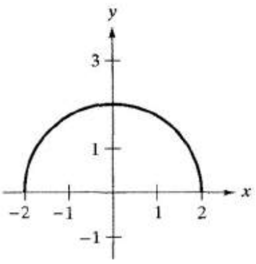




35. $f(x) = x^2 - 2x$
   (a) $[-1, 2]$
   (b) $(1, 3]$
   (c) $(0, 2)$
   (d) $[1, 4)$

36. $f(x) = \sqrt{4 - x^2}$
   (a) $[-2, 2]$
   (b) $[-2, 0)$
   (c) $(-2, 2)$
   (d) $[1, 2)$





In Exercises 37–40, use a graphing utility to graph the function. Determine the absolute extrema of the function and the *x*-value in the closed interval where it occurs.

Function	*Interval*
37. $f(x) = \begin{cases} 2x + 2, & 0 \le x \le 1 \\ 4x^2, & 1 < x \le 3 \end{cases}$	$[0, 3]$
38. $f(x) = \begin{cases} 2 - x^2, & 1 \le x < 3 \\ 2 - 3x, & 3 \le x \le 5 \end{cases}$	$[1, 5]$
39. $f(x) = \dfrac{3}{x - 1}$	$(1, 4]$
40. $f(x) = \dfrac{2}{2 - x}$	$[0, 2)$

In Exercises 41 and 42, (a) use a symbolic differentiation utility to graph the function and approximate any absolute extrema on the indicated interval. (b) Use the utility to find any critical numbers, and use them to find any absolute extrema not located at the endpoints. Compare the results with those in part (a).

Function	*Interval*
41. $f(x) = 3.2x^5 + 5x^3 - 3.5x$	$[0, 1]$
42. $f(x) = \frac{4}{3}x\sqrt{3 - x}$	$[0, 3]$

In Exercises 43 and 44, use a symbolic differentiation utility to find the maximum value of $|f''(x)|$ on the indicated interval. (This value is used in the error estimate for the Trapezoidal Rule, as discussed in Section 4.6.)

Function	*Interval*
43. $f(x) = \sqrt{1 + x^3}$	$[0, 2]$
44. $f(x) = \dfrac{1}{x^2 + 1}$	$\left[\frac{1}{2}, 3\right]$

In Exercises 45 and 46, use a symbolic differentiation utility to find the maximum value of $|f^{(4)}(x)|$ on the indicated interval. (This value is used in the error estimate for Simpson's Rule, as discussed in Section 4.6.)

Function	*Interval*
45. $f(x) = (x + 1)^{2/3}$	$[0, 2]$
46. $f(x) = \dfrac{1}{x^2 + 1}$	$[-1, 1]$

47. ***Power*** The formula for the power output P of a battery is $P = VI - RI^2$ where V is the electromotive force in volts, R is the resistance, and I is the current. Find the current (measured in amperes) that corresponds to a maximum value of P in a battery for which $V = 12$ volts and $R = 0.5$ ohm. Assume that a 15-ampere fuse bounds the output in the interval $0 \le I \le 15$. Could the power output be increased by replacing the 15-ampere fuse with a 20-ampere fuse? Explain.

48. ***Inventory Cost*** A retailer has determined that the cost C of ordering and storing x units of a certain product is

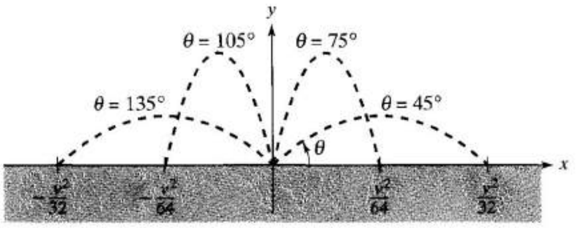
$$C = 2x + \frac{300{,}000}{x}, \qquad 1 \le x \le 300.$$

The delivery truck can bring at most 300 units per order. Find the order size that will minimize cost. Could the cost be decreased if the truck were replaced with one that could bring at most 400 units? Explain.

49. ***Lawn Sprinkler*** A lawn sprinkler is constructed in such a way that $d\theta/dt$ is constant, where θ ranges between 45° and 135° (see figure). The distance the water travels horizontally is

$$x = \frac{v^2 \sin 2\theta}{32}, \qquad \frac{\pi}{4} \le \theta \le \frac{3\pi}{4}$$

where v is the speed of the water. Find dx/dt and explain why this lawn sprinkler does not water evenly. What part of the lawn receives the most water?



Water sprinkler: $45° \le \theta \le 135°$

FOR FURTHER INFORMATION For more information on the "calculus of lawn sprinklers," see the article "Design of an Oscillating Sprinkler" by Bart Braden in the January 1985 issue of *Mathematics Magazine.*

50. ***Modeling Data*** The defense outlays as percents of the gross domestic product for the years 1976 through 1995 are as follows. *(Source: U.S. Office of Management and Budget)*

1976: (5.3%); 1977: (5.1%); 1978: (4.8%); 1979: (4.8%);
1980: (5.1%); 1981: (5.3%); 1982: (5.9%); 1983: (6.3%);
1984: (6.2%); 1985: (6.4%); 1986: (6.5%); 1987: (6.3%);
1988: (6.0%); 1989: (5.9%); 1990: (5.5%); 1991: (4.8%);
1992: (5.0%); 1993: (4.7%); 1994: (4.2%); 1995: (3.9%);

(a) Use the regression capabilities of a graphing utility to find a model of the form $y = at^4 + bt^3 + ct^2 + dt + e$ for the data. (Let t represent the time in years, with $t = 0$ corresponding to 1980.)

(b) Use a graphing utility to plot the data and graph the model.

(c) Locate the absolute extrema of the model on the interval $[-4, 15]$.

51. ***Honeycomb*** The surface area of a cell in a honeycomb is

$$S = 6hs + \frac{3s^2}{2}\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right)$$

where h and s are positive constants and θ is the angle at which the upper faces meet the altitude of the cell. Find the angle θ $(\pi/6 \le \theta \le \pi/2)$ that minimizes the surface area S.

FOR FURTHER INFORMATION For more information on the geometric structure of a honeycomb cell, see the article "The Design of Honeycombs" by Anthony L. Peressini in UMAP Module 502, published by COMAP, Inc., Suite 210, 57 Bedford Street, Lexington, MA.

52. ***Highway Design*** In order to build a highway it is necessary to fill a section of a valley where the grades (slopes) of the sides are 6% and 9% (see figure). The top of the filled region will have the shape of a parabolic arc that is tangent to the two slopes at the points A and B. The horizontal distance between the points A and B is 1000 feet.

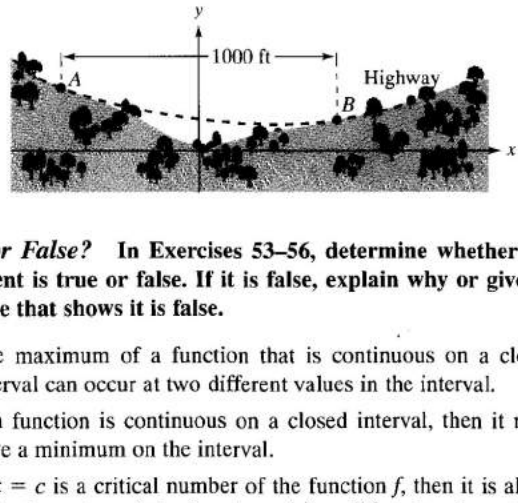
(a) Find a quadratic function $y = ax^2 + bx + c$, $-500 \le x \le 500$, that describes the top of the filled region.

(b) Complete the table giving the depths d of the fill at the specified values of x.

x	−500	−400	−300	−200	−100
d					

x	0	100	200	300	400	500
d						

(c) What will be the lowest point on the completed highway? Will it be directly over the point where the two hillsides come together?



True or False? **In Exercises 53–56, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

53. The maximum of a function that is continuous on a closed interval can occur at two different values in the interval.

54. If a function is continuous on a closed interval, then it must have a minimum on the interval.

55. If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x) + k$, where k is a constant.

56. If $x = c$ is a critical number of the function f, then it is also a critical number of the function $g(x) = f(x - k)$, where k is a constant.

57. Find all critical numbers of the greatest integer function $f(x) = [\![x]\!]$.