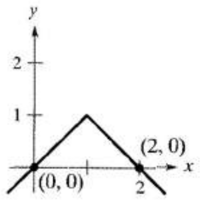
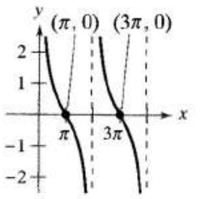
EXERCISES FOR SECTION 3.2

Think About It **In Exercises 1 and 2, state why Rolle's Theorem does not apply to the function even though there exist *a* and *b* such that $f(a) = f(b)$.**

1. $f(x) = 1 - |x - 1|$



2. $f(x) = \cot \frac{x}{2}$



In Exercises 3–16, determine whether Rolle's Theorem can be applied to *f* on the indicated interval. If Rolle's Theorem can be applied, find all values of *c* in the interval such that $f'(c) = 0$.

Function	*Interval*
3. $f(x) = x^2 - 2x$	$[0, 2]$
4. $f(x) = x^2 - 3x + 2$	$[1, 2]$
5. $f(x) = (x - 1)(x - 2)(x - 3)$	$[1, 3]$
6. $f(x) = (x - 3)(x + 1)^2$	$[-1, 3]$
7. $f(x) = x^{2/3} - 1$	$[-8, 8]$
8. $f(x) = 3 - \|x - 3\|$	$[0, 6]$
9. $f(x) = \frac{x^2 - 2x - 3}{x + 2}$	$[-1, 3]$
10. $f(x) = \frac{x^2 - 1}{x}$	$[-1, 1]$
11. $f(x) = \sin x$	$[0, 2\pi]$
12. $f(x) = \cos x$	$[0, 2\pi]$
13. $f(x) = \sin 2x$	$\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$
14. $f(x) = \frac{6x}{\pi} - 4 \sin^2 x$	$\left[0, \frac{\pi}{6}\right]$
15. $f(x) = \tan x$	$[0, \pi]$
16. $f(x) = \sec x$	$\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

In Exercises 17–20, use a graphing utility to graph the function on the indicated interval. Determine whether Rolle's Theorem can be applied to *f* on the interval and, if so, find all values of *c* in the interval such that $f'(c) = 0$.

Function	*Interval*
17. $f(x) = \|x\| - 1$	$[-1, 1]$
18. $f(x) = x - x^{1/3}$	$[0, 1]$
19. $f(x) = 4x - \tan \pi x$	$\left[-\frac{1}{4}, \frac{1}{4}\right]$
20. $f(x) = \frac{x}{2} - \sin \frac{\pi x}{6}$	$[-1, 0]$

21. ***Vertical Motion*** The height of a ball t seconds after it is thrown upward from a height of 32 feet and with an initial velocity of 48 feet per second is $f(t) = -16t^2 + 48t + 32$.

(a) Verify that $f(1) = f(2)$.

(b) According to Rolle's Theorem, what must be the velocity at some time in the interval $[1, 2]$?

22. ***Reorder Costs*** The ordering and transportation cost C of components used in a manufacturing process is approximated by

$$C(x) = 10\left(\frac{1}{x} + \frac{x}{x + 3}\right)$$

where C is measured in thousands of dollars and x is the order size in hundreds.

(a) Verify that $C(3) = C(6)$.

(b) According to Rolle's Theorem, the rate of change of cost must be 0 for some order size in the interval $[3, 6]$. Find this order size.

23. ***Think About It*** Let f be continuous on $[a, b]$ and differentiable on (a, b). If there exists c in (a, b) such that $f'(c) = 0$, does it follow that $f(a) = f(b)$? Explain.

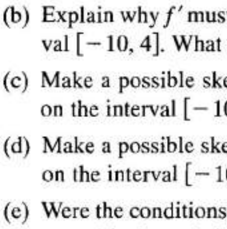
24. ***Think About It*** Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b). Also, suppose that $f(a) = f(b)$ and that c is a real number in the interval such that $f'(c) = 0$. Find an interval for the function g over which Rolle's Theorem can be applied, and find the corresponding critical number of g (k is a constant).

(a) $g(x) = f(x) + k$

(b) $g(x) = f(x - k)$

(c) $g(x) = f(kx)$

25. ***Graphical Reasoning*** The figure gives two parts of the graph of a continuous differentiable function f on $[-10, 4]$. The derivative f' is also continuous.



(a) Explain why f must have at least one zero in $[-10, 4]$.

(b) Explain why f' must also have at least one zero in the interval $[-10, 4]$. What are these zeros called?

(c) Make a possible sketch of the function with one zero of f' on the interval $[-10, 4]$.

(d) Make a possible sketch of the function with two zeros of f' on the interval $[-10, 4]$.

(e) Were the conditions of continuity of f and f' necessary to do parts (a) through (d)? Explain.

26. Consider the function $f(x) = 3\cos^2\left(\frac{\pi x}{2}\right)$.
   (a) Use a graphing utility to graph f and f'.
   (b) Is f a continuous function? Is f'?
   (c) Does Rolle's Theorem apply on the interval $[-1, 1]$? Does it apply on the interval $[1, 2]$? Explain.
   (d) Evaluate, if possible, $\lim_{x\to 3^-} f'(x)$ and $\lim_{x\to 3^+} f'(x)$.

In Exercises 27–34, apply the Mean Value Theorem to f on the indicated interval. In each case, find all values of c in the interval (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Function	*Interval*
27. $f(x) = x^2$	$[-2, 1]$
28. $f(x) = x(x^2 - x - 2)$	$[-1, 1]$
29. $f(x) = x^{2/3}$	$[0, 1]$
30. $f(x) = \frac{x+1}{x}$	$[\frac{1}{2}, 2]$
31. $f(x) = \sqrt{x-2}$	$[2, 6]$
32. $f(x) = x^3$	$[0, 1]$
33. $f(x) = \sin x$	$[0, \pi]$
34. $f(x) = 2\sin x + \sin 2x$	$[0, \pi]$

In Exercises 35–38, use a graphing utility to (a) graph the function f on the indicated interval, (b) find and sketch the secant line through points on the graph of f at the endpoints of the indicated interval, and (c) find and sketch any tangent lines to the graph of f that are parallel to the secant line.

Function	*Interval*
35. $f(x) = \frac{x}{x+1}$	$[-\frac{1}{2}, 2]$
36. $f(x) = x - 2\sin x$	$[-\pi, \pi]$
37. $f(x) = \sqrt{x}$	$[1, 9]$
38. $f(x) = -x^4 + 4x^3 + 8x^2 + 5$	$[0, 5]$

Writing **In Exercises 39 and 40, explain why the Mean Value Theorem does not apply to the function on the interval [0, 6].**

39. $f(x) = \frac{1}{x-3}$

40. $f(x) = |x - 3|$

Think About It **In Exercises 41 and 42, sketch the graph of an arbitrary function f that satisfies the given condition but does not satisfy the conditions of the Mean Value Theorem on the interval $[-5, 5]$.**

41. f is continuous on $[-5, 5]$.

42. f is not continuous on $[-5, 5]$.

43. ***Vertical Motion*** The height of an object t seconds after it is dropped from a height of 500 meters is $s(t) = -4.9t^2 + 500$.
   (a) Find the average velocity of the object during the first 3 seconds.
   (b) Use the Mean Value Theorem to verify that at some time during the first 3 seconds of fall the instantaneous velocity equals the average velocity. Find that time.

44. ***Sales*** A company introduces a new product for which the number of units sold S is

$$S(t) = 200\left(5 - \frac{9}{2+t}\right)$$

where t is the time in months.
   (a) Find the average value of $S(t)$ during the first year.
   (b) During what month does $S'(t)$ equal the average value during the first year.

45. ***Think About It*** A plane begins its takeoff at 2:00 P.M. on a 2500-mile flight. The plane arrives at its destination at 7:30 P.M. Explain why there were at least two times during the flight when the speed of the plane was 400 miles per hour.

46. ***Think About It*** When an object is removed from a furnace and placed in an environment with a constant temperature of 90°F, its core temperature is 1500°F. Five hours later the core temperature is 390°F. Explain why there must exist a time in the interval when the temperature is decreasing at a rate of 222°F per hour.

True or False? **In Exercises 47–50, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

47. The Mean Value Theorem can be applied to $f(x) = 1/x$ on the interval $[-1, 1]$.

48. If the graph of a function has three x-intercepts, then it must have at least two points at which its tangent line is horizontal.

49. If the graph of a polynomial function has three x-intercepts, then it must have at least two points at which its tangent line is horizontal.

50. If $f'(x) = 0$ for all x in the domain of f, then f is a constant function.

51. Prove that if $a > 0$ and n is any positive integer, then the polynomial function $p(x) = x^{2n+1} + ax + b$ cannot have two real roots.

52. Prove that if $f'(x) = 0$ for all x in an interval (a, b), then f is constant on $[a, b]$.

53. Let $p(x) = Ax^2 + Bx + C$. Prove that for any interval $[a, b]$, the value c guaranteed by the Mean Value Theorem is the midpoint of the interval.

54. Prove that if f is differentiable on $(-\infty, \infty)$ and $f'(x) < 1$ for all real numbers, then f has at most one fixed point. A **fixed point** of a function f is a real number c such that $f(c) = c$.

55. Use the result of Exercise 54 to show that $f(x) = \frac{1}{2}\cos x$ has at most one fixed point.

56. Prove that $|\cos x - \cos y| \le |x - y|$ for all x and y.