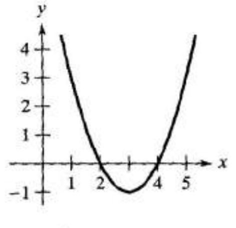
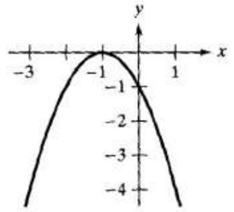
EXERCISES FOR SECTION 3.3

In Exercises 1–6, identify the open intervals on which the function is increasing or decreasing.

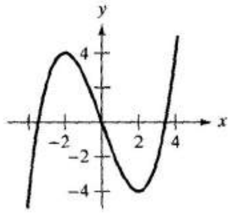
1. $f(x) = x^2 - 6x + 8$



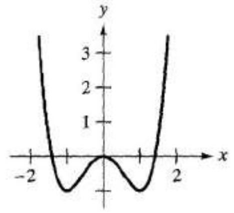
2. $y = -(x + 1)^2$



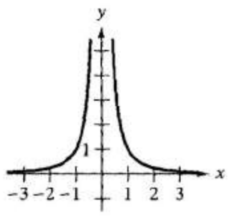
3. $y = \frac{x^3}{4} - 3x$



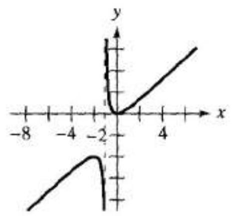
4. $f(x) = x^4 - 2x^2$



5. $f(x) = \frac{1}{x^2}$



6. $y = \frac{x^2}{x + 1}$



In Exercises 7–20, find the critical numbers of f (if any), find the open intervals on which the algebraic function is increasing or decreasing, and locate all relative extrema.

7. $f(x) = -2x^2 + 4x + 3$
8. $f(x) = x^2 + 8x + 10$
9. $f(x) = x^2 - 6x$
10. $f(x) = (x - 1)^2(x + 2)$
11. $f(x) = 2x^3 + 3x^2 - 12x$
12. $f(x) = (x - 3)^3$
13. $f(x) = \frac{x^5 - 5x}{5}$
14. $f(x) = x^4 - 32x + 4$
15. $f(x) = x^{1/3} + 1$
16. $f(x) = x^{2/3}(x - 5)$
17. $f(x) = 5 - |x - 5|$
18. $f(x) = |x + 3| - 1$
19. $f(x) = \frac{x^2}{x^2 - 9}$
20. $f(x) = \frac{x + 3}{x^2}$

In Exercises 21–28, find the open intervals on which the function is increasing or decreasing, and locate all relative extrema. Use a graphing utility to confirm your results.

21. $f(x) = x^3 - 6x^2 + 15$
22. $f(x) = x^4 - 2x^3$
23. $f(x) = (x - 1)^{2/3}$
24. $f(x) = (x - 1)^{1/3}$
25. $f(x) = x + \frac{1}{x}$
26. $f(x) = \frac{x}{x + 1}$
27. $f(x) = \frac{x^2 - 2x + 1}{x + 1}$
28. $f(x) = \frac{x^2 - 3x - 4}{x - 2}$

In Exercises 29–32, consider the trigonometric function on the interval $(0, 2\pi)$. Find the open intervals on which the function is increasing or decreasing, and locate all relative extrema. Use a graphing utility to confirm your results.

29. $f(x) = \frac{x}{2} + \cos x$
30. $f(x) = \sin x \cos x$
31. $f(x) = \sin^2 x + \sin x$
32. $f(x) = \frac{\cos x}{1 + \sin^2 x}$

In Exercises 33–36, (a) use a symbolic differentiation utility to differentiate the function, (b) sketch the graphs of f and f' on the same set of coordinate axes over the indicated interval, (c) find the critical numbers of f in the open interval, and (d) find the interval(s) on which f' is positive and the interval(s) on which it is negative. Compare the behavior of f and the sign of f'.

Function	*Interval*
33. $f(x) = 2x\sqrt{9 - x^2}$	$[-3, 3]$
34. $f(x) = 10(5 - \sqrt{x^2 - 3x + 16})$	$[0, 5]$
35. $f(t) = t^2 \sin t$	$[0, 2\pi]$
36. $f(x) = \frac{x}{2} + \cos \frac{x}{2}$	$[0, 4\pi]$

Think About It **In Exercises 37–42, assume that f is differentiable for all x. The sign of f' is as follows.**

$f'(x) > 0$ on $(-\infty, -4)$

$f'(x) < 0$ on $(-4, 6)$

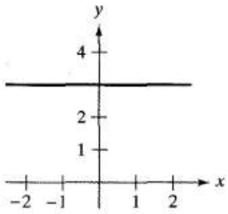
$f'(x) > 0$ on $(6, \infty)$

Supply the appropriate inequality for the indicated value of c.

Function	*Sign of $g'(c)$*
37. $g(x) = f(x) + 5$	$g'(0)$ ▒ 0
38. $g(x) = 3f(x) - 3$	$g'(-5)$ ▒ 0
39. $g(x) = -f(x)$	$g'(-6)$ ▒ 0
40. $g(x) = -f(x)$	$g'(0)$ ▒ 0
41. $g(x) = f(x - 10)$	$g'(0)$ ▒ 0
42. $g(x) = f(x - 10)$	$g'(8)$ ▒ 0

Think About It **In Exercises 43–48, a function f is shown in the figure. Sketch a graph of the derivative of f.**

43.



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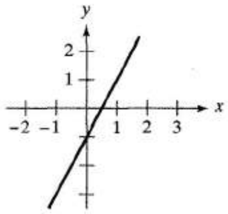

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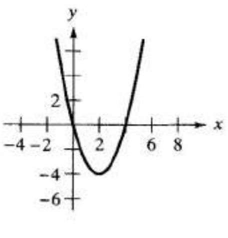

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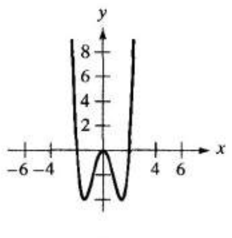

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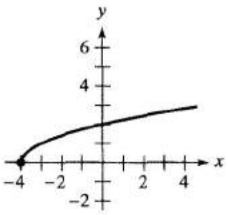

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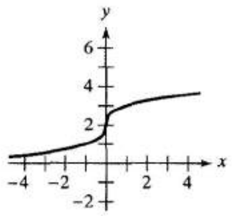


49. ***Think About It*** Sketch the graph of an arbitrary function f such that

$$f'(x)\begin{cases} > 0, & x < 4 \\ \text{undefined}, & x = 4 \\ < 0, & x > 4 \end{cases}$$

50. ***Think About It*** A differentiable function f has one critical number at $x = 5$. Identify the relative extrema of f at the critical number if $f'(4) = -2.5$ and $f'(6) = 3$.

51. ***Think About It*** The function f is differentiable on the interval $[-1, 1]$. The table gives the values of f' for selected values of x. Sketch a graph of f, approximate the critical numbers, and identify the relative extrema.

x	-1	-0.75	-0.50	-0.25
$f'(x)$	-10	-3.2	-0.5	0.8

x	0	0.25	0.50	0.75	1
$f'(x)$	5.6	3.6	-0.2	-6.7	-20.1

52. ***Rolling a Ball Bearing*** A ball bearing is placed on an inclined plane and begins to roll. The angle of elevation of the plane is θ. The distance (in meters) the ball bearing rolls in t seconds is $s(t) = 4.9(\sin \theta)t^2$.

(a) Determine the speed of the ball bearing.

(b) What value of θ will produce the maximum speed at a particular time?

53. ***Numerical, Graphical, and Analytic Analysis*** Consider the functions $f(x) = x$ and $g(x) = \sin x$ on the interval $(0, \pi)$.

(a) Complete the table and make a conjecture about which is the greater function on the interval $(0, \pi)$.

x	0.5	1	1.5	2	2.5	3
$f(x)$						
$g(x)$						

(b) Use a graphing utility to graph the functions and use the graphs to make a conjecture about which is the greater function on the interval $(0, \pi)$.

(c) Prove that $f(x) > g(x)$ on the interval $(0, \pi)$. [*Hint:* Show that $h'(x) > 0$ where $h = f - g$.]

54. ***Numerical, Graphical, and Analytic Analysis*** The concentration C of a chemical in the bloodstream t hours after injection into muscle tissue is

$$C = \frac{3t}{27 + t^3}, \quad t \geq 0.$$

(a) Complete the table and use the table to approximate the time when the concentration is greatest.

t	0	0.5	1	1.5	2	2.5	3
$C(t)$							

(b) Use a graphing utility to graph the concentration function and use the graph to approximate the time when the concentration is greatest.

(c) Use calculus to determine analytically the time when the concentration is greatest.

55. ***Trachea Contraction*** Coughing forces the trachea (windpipe) to contract, which affects the velocity v of the air passing through the trachea. Suppose the velocity of the air during coughing is

$$v = k(R - r)r^2, \quad 0 \leq r < R$$

where k is constant, R is the normal radius of the trachea, and r is the radius during coughing. What radius will produce the maximum air velocity?

56. ***Profit*** The profit P (in dollars) made by a fast-food restaurant selling x hamburgers is

$$P = 2.44x - \frac{x^2}{20{,}000} - 5000, \quad 0 \leq x \leq 35{,}000.$$

Find the open intervals on which P is increasing or decreasing.

57. ***Power*** The electric power P in watts in a direct-current circuit with two resistors R_1 and R_2 connected in series is

$$P = \frac{vR_1R_2}{(R_1 + R_2)^2}$$

where v is the voltage. If v and R_1 are held constant, what resistance R_2 produces maximum power?

58. *Electrical Resistance* The resistance R of a certain type of resistor is

$$R = \sqrt{0.001T^4 - 4T + 100}$$

where R is measured in ohms and the temperature T is measured in degrees Celsius.

(a) Use a symbolic differentiation utility to find dR/dT and the critical number of the function. Determine the minimum resistance for this type of resistor.

(b) Use a graphing utility to graph the function R and use the graph to approximate the minimum resistance for this type of resistor.

59. *Modeling Data* The number of bankruptcies (in thousands) for the years 1981 through 1994 are as follows.

1981: 360.3; 1982: 367.9; 1983: 374.7; 1984: 344.3;
1985: 364.5; 1986: 477.9; 1987: 561.3; 1988: 594.6
1989: 643.0; 1990: 725.5; 1991: 880.4; 1992: 972.5
1993: 918.7; 1994: 845.3

(Source: Administrative Office of the U.S. Courts)

(a) Use the regression capabilities of a graphing utility to find a model of the form

$$B = at^4 + bt^3 + ct^2 + dt + e$$

for the data. (Let $t = 1$ represent 1981.)

(b) Use a graphing utility to plot the data and graph the model.

(c) Analytically find the maximum of the model and compare the result with the actual data.

60. Use a graphing utility to graph $f(x) = 2 \sin 3x + 4 \cos 3x$. Find the maximum value of f. How could you use calculus to estimate the maximum?

Creating Polynomial Functions **In Exercises 61–64, find a polynomial function**

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

that has only the specified extrema. (a) Determine the minimum degree of the function and give the criteria you used in determining the degree. (b) Using the fact that the coordinates of the extrema are solution points of the function, and that the x-coordinates are critical numbers, determine a system of linear equations whose solution yields the coefficients of the required function. (c) Use a graphing utility to solve the system of equations and determine the function. (d) Use a graphing utility to confirm your result graphically.

61. Relative minimum: $(0, 0)$; Relative maximum: $(2, 2)$

62. Relative minimum: $(0, 0)$; Relative maximum: $(4, 1000)$

63. Relative minima: $(0, 0)$, $(4, 0)$
Relative maximum: $(2, 4)$

64. Relative minimum: $(1, 2)$
Relative maxima: $(-1, 4)$, $(3, 4)$

True or False? **In Exercises 65–68, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

65. The sum of two increasing functions is increasing.

66. The product of two increasing functions is increasing.

67. Every nth-degree polynomial has $(n - 1)$ critical numbers.

68. An nth-degree polynomial has at most $(n - 1)$ critical numbers.

69. Prove the second case of Theorem 3.5.

70. Prove the second case of Theorem 3.6.

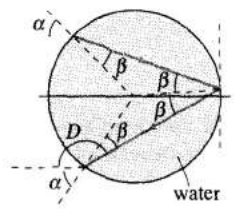
71. Let $x > 0$ and $n > 1$ be real numbers. Prove that $(1 + x)^n > 1 + nx$.

SECTION PROJECT

Rainbows Rainbows are formed when light strikes raindrops and is reflected and refracted, as shown in the figure. (This figure shows a cross section of a spherical raindrop.) The Law of Refraction states that $(\sin \alpha)/(\sin \beta) = k$, where $k \approx 1.33$ (for water). The angle of deflection is given by $D = \pi + 2\alpha - 4\beta$.

(a) Sketch the graph of D for $0 \le \alpha \le \pi/2$. Use a graphing utility with

$$D = \pi + 2\alpha - 4 \sin^{-1}\left(\frac{1}{k} \sin \alpha\right).$$



(b) Prove that the minimum angle of deflection occurs when

$$\cos \alpha = \sqrt{\frac{k^2 - 1}{3}}.$$

For water, what is the minimum angle of deflection, D_{min}? (The angle $\pi - D_{min}$ is called the *rainbow angle.*) What value of α produces this minimum angle? (A ray of sunlight that strikes a raindrop at this angle, α, is called a *rainbow ray.*)

FOR FURTHER INFORMATION For more information about the mathematics of rainbows, see the article "Somewhere Within the Rainbow" by Steven Janke in *UMAP Journal*, Volume 13, Number 2, 1992.