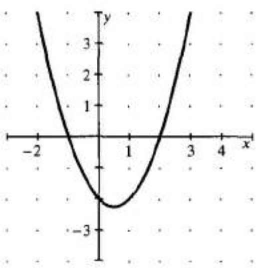
EXERCISES FOR SECTION 3.4

In Exercises 1–6, find the open intervals on which the graph is concave upward and those on which it is concave downward.

1. $y = x^2 - x - 2$



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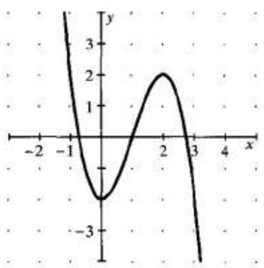
2. $y = -x^3 + 3x^2 - 2$



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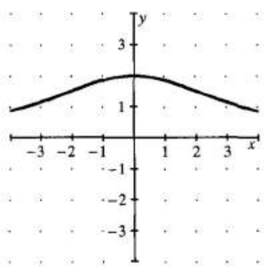
3. $f(x) = \dfrac{24}{x^2 + 12}$



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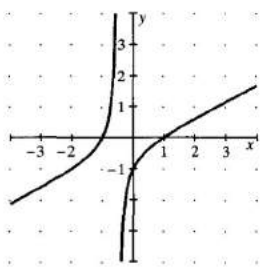
4. $f(x) = \dfrac{x^2 - 1}{2x + 1}$



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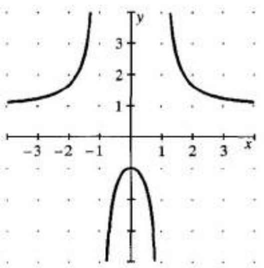
5. $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$



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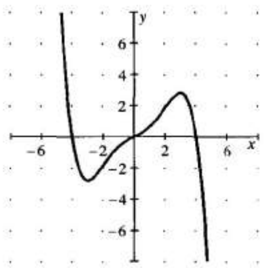
6. $y = \dfrac{-3x^5 + 40x^3 + 135x}{270}$



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In Exercises 7–20, find all relative extrema. Use the Second Derivative Test where applicable.

7. $f(x) = 6x - x^2$
8. $f(x) = x^2 + 3x - 8$
9. $f(x) = (x - 5)^2$
10. $f(x) = -(x - 5)^2$
11. $f(x) = x^3 - 3x^2 + 3$
12. $f(x) = 5 + 3x^2 - x^3$
13. $f(x) = x^4 - 4x^3 + 2$
14. $f(x) = x^3 - 9x^2 + 27x$
15. $f(x) = x^{2/3} - 3$
16. $f(x) = \sqrt{x^2 + 1}$
17. $f(x) = x + \dfrac{4}{x}$
18. $f(x) = \dfrac{x}{x - 1}$
19. $f(x) = \cos x - x$
    $0 \le x \le 4\pi$
20. $f(x) = 2 \sin x + \cos 2x$
    $0 \le x \le 2\pi$

In Exercises 21–36, find all relative extrema and points of inflection and use a graphing utility to graph the function.

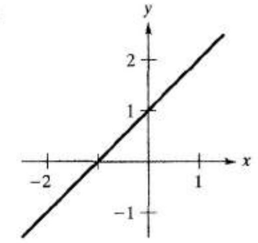
21. $f(x) = x^3 - 12x$
22. $f(x) = x^3 + 1$
23. $f(x) = x^3 - 6x^2 + 12x$
24. $f(x) = 2x^3 - 3x^2 - 12x$
25. $f(x) = \dfrac{1}{4}x^4 - 2x^2$
26. $f(x) = 2x^4 - 8x + 3$
27. $f(x) = x(x - 4)^3$
28. $f(x) = x^3(x - 4)$
29. $f(x) = x\sqrt{x + 3}$
30. $f(x) = x\sqrt{x + 1}$
31. $f(x) = \sin \dfrac{x}{2}$
    $0 \le x \le 4\pi$
32. $f(x) = 2 \csc \dfrac{3x}{2}$
    $0 < x < 2\pi$
33. $f(x) = \sec\left(x - \dfrac{\pi}{2}\right)$
    $0 < x < 4\pi$
34. $f(x) = \sin x + \cos x$
    $0 \le x \le 2\pi$
35. $f(x) = 2 \sin x + \sin 2x$
    $0 \le x \le 2\pi$
36. $f(x) = x - \sin x$
    $0 \le x \le 4\pi$

In Exercises 37–40, use a symbolic differentiation utility to analyze the function over the indicated interval. (a) Find the first- and second-order derivatives of the function. (b) Find any relative extrema and points of inflection. (c) Graph f, f', and f'' on the same set of coordinate axes and state the relationship between the behavior of f and the signs of f' and f''.

37. $f(x) = 0.2x^2(x - 3)^3$, $[-1, 4]$
38. $f(x) = x^2\sqrt{6 - x^2}$, $[-\sqrt{6}, \sqrt{6}]$
39. $f(x) = \sin x - \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x$, $[0, \pi]$
40. $f(x) = \sqrt{2x} \sin x$, $[0, 2\pi]$

Think About It **In Exercises 41–44, trace the graph of f. On the same set of coordinate axes, sketch the graphs of f' and f''.**

41.



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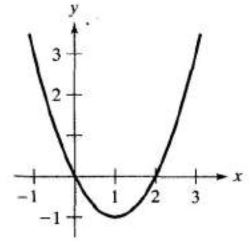

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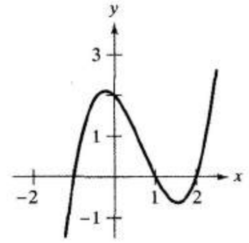

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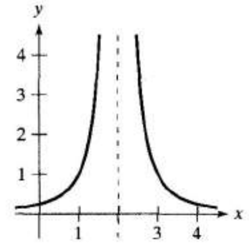


45. ***Think About It*** Consider a function f such that f' is increasing. Sketch graphs of f for (a) $f' < 0$ and (b) $f' > 0$.

46. ***Think About It*** Consider a function f such that f' is decreasing. Sketch graphs of f for (a) $f' < 0$ and (b) $f' > 0$.

Think About It **In Exercises 47–50, sketch the graph of a function f having the indicated characteristics.**

47. $f(2) = f(4) = 0$
$f(3)$ is defined.
$f'(x) < 0$ if $x < 3$
$f'(3)$ is undefined.
$f'(x) > 0$ if $x > 3$
$f''(x) < 0, x \neq 3$

48. $f(0) = f(2) = 0$
$f'(x) > 0$ if $x < 1$
$f'(1) = 0$
$f'(x) < 0$ if $x > 1$
$f''(x) < 0$

49. $f(2) = f(4) = 0$
$f'(x) > 0$ if $x < 3$
$f'(3)$ is undefined.
$f'(x) < 0$ if $x > 3$
$f''(x) > 0, x \neq 3$

50. $f(0) = f(2) = 0$
$f'(x) < 0$ if $x < 1$
$f'(1) = 0$
$f'(x) > 0$ if $x > 1$
$f''(x) > 0$

In Exercises 51 and 52, find a, b, c, and d such that the cubic $f(x) = ax^3 + bx^2 + cx + d$ satisfies the indicated conditions.

51. Relative maximum: $(3, 3)$
Relative minimum: $(5, 1)$
Inflection point: $(4, 2)$

52. Relative maximum: $(2, 4)$
Relative minimum: $(4, 2)$
Inflection point: $(3, 3)$

53. ***Think About It*** The figure shows the graph of the second derivative of a function f. Sketch a graph of f. (The answer is not unique.)

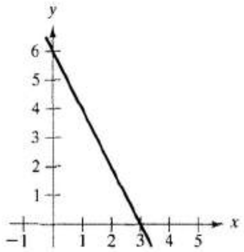


Figure for 53

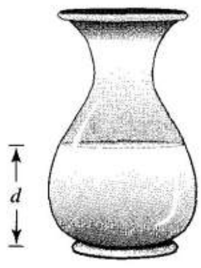


Figure for 54

54. ***Think About It*** Water is running into the vase shown in the figure at a constant rate.

(a) Sketch a graph of the depth d of water in the vase as a function of time.

(b) Does the function have any extrema? Explain.

(c) Give an interpretation of the inflection points of the graph of d.

55. ***Conjecture*** Consider the function $f(x) = (x - 2)^n$.

(a) Use a graphing utility to graph f for $n = 1, 2, 3$, and 4. Use the graphs to make a conjecture about the relationship between n and any inflection points of the graph of f.

(b) Verify your conjecture in part (a).

56. (a) Graph $f(x) = \sqrt[3]{x}$ and identify the inflection point.

(b) Does $f''(x)$ exist at the inflection point? Explain.

57. ***Think About It*** S represents weekly sales of a product. What can be said of S' and S'' for each of the following?

(a) The rate of change of sales is increasing.

(b) Sales are increasing at a slower rate.

(c) The rate of change of sales is constant.

(d) Sales are steady.

(e) Sales are declining, but at a slower rate.

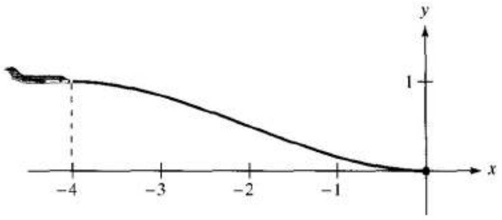
(f) Sales have bottomed out and have started to rise.

58. ***Think About It*** Sketch the graph of an arbitrary function that does *not* have a point of inflection at $(c, f(c))$ even though $f''(c) = 0$.

59. ***Aircraft Glide Path*** A small aircraft starts its descent from an altitude of 1 mile, 4 miles west of the runway (see figure).

(a) Find the cubic $f(x) = ax^3 + bx^2 + cx + d$ on the interval $[-4, 0]$ that describes a smooth glide path for the landing.

(b) If the glide path of the plane is described by the function in part (a), when would the plane be descending at the most rapid rate?



FOR FURTHER INFORMATION For more information on this type of modeling, see the article "How Not to Land at Lake Tahoe!" by Richard Barshinger in the May 1992 issue of the *The American Mathematical Monthly.*

60. ***Highway Design*** A section of highway connecting two hillsides with grades of 6% and 4% is to be built between two points that are separated by a horizontal distance of 2000 feet (see figure). At the point where the two hillsides come together, there is a 50-foot difference in elevation.

(a) Design a section of highway connecting the hillsides modeled by the function $f(x) = ax^3 + bx^2 + cx + d\ (-1000 \le x \le 1000)$. At the points A and B, the slope of the model must match the grade of the hillside.

(b) Use a graphing utility to graph the model.

(c) Use a graphing utility to graph the derivative of the model.

(d) Determine the grade at the steepest part of the transitional section of the highway.