Before you begin the exercise set, be sure you realize that one of the most important steps in integration is *rewriting the integrand* in a form that fits the basic integration rules. To further illustrate this point, here are some additional examples.

Original Integral	*Rewrite*	*Integrate*	*Simplify*
$\int \frac{2}{\sqrt{x}}\,dx$	$2\int x^{-1/2}\,dx$	$2\left(\frac{x^{1/2}}{1/2}\right) + C$	$4x^{1/2} + C$
$\int (t^2+1)^2\,dt$	$\int (t^4 + 2t^2 + 1)\,dt$	$\frac{t^5}{5} + 2\left(\frac{t^3}{3}\right) + t + C$	$\frac{1}{5}t^5 + \frac{2}{3}t^3 + t + C$
$\int \frac{x^3+3}{x^2}\,dx$	$\int (x + 3x^{-2})\,dx$	$\frac{x^2}{2} + 3\left(\frac{x^{-1}}{-1}\right) + C$	$\frac{1}{2}x^2 - \frac{3}{x} + C$
$\int \sqrt[3]{x}(x-4)\,dx$	$\int (x^{4/3} - 4x^{1/3})\,dx$	$\frac{x^{7/3}}{7/3} - 4\left(\frac{x^{4/3}}{4/3}\right) + C$	$\frac{3}{7}x^{4/3}(x-7) + C$

EXERCISES FOR SECTION 4.1

In Exercises 1–4, verify the statement by showing that the derivative of the right side is equal to the integrand of the left side.

1. $\int \left(-\frac{9}{x^4}\right) dx = \frac{3}{x^3} + C$

2. $\int \left(4x^3 - \frac{1}{x^2}\right) dx = x^4 + \frac{1}{x} + C$

3. $\int (x-2)(x+2)\,dx = \frac{1}{3}x^3 - 4x + C$

4. $\int \frac{x^2-1}{x^{3/2}}\,dx = \frac{2(x^2+3)}{3\sqrt{x}} + C$

In Exercises 5–10, complete the table using the examples at the top of this page as a model.

Original Integral	*Rewrite*	*Integrate*	*Simplify*
5. $\int \sqrt[3]{x}\,dx$			
6. $\int \frac{1}{x^2}\,dx$			
7. $\int \frac{1}{x\sqrt{x}}\,dx$			
8. $\int x(x^2+3)\,dx$			
9. $\int \frac{1}{2x^3}\,dx$			
10. $\int \frac{1}{(2x)^3}\,dx$			

In Exercises 11–14, find the general solution of the differential equation and check the result by differentiation.

11. $\frac{dy}{dt} = 3t^2$

12. $\frac{dr}{d\theta} = \pi$

13. $\frac{dy}{dx} = x^{3/2}$

14. $\frac{dy}{dx} = 3x^{-4}$

In Exercises 15–30, evaluate the indefinite integral and check the result by differentiation.

15. $\int (x^3 + 2)\,dx$

16. $\int (x^2 - 2x + 3)\,dx$

17. $\int (x^{3/2} + 2x + 1)\,dx$

18. $\int \left(\sqrt{x} + \frac{1}{2\sqrt{x}}\right) dx$

19. $\int \sqrt[3]{x^2}\,dx$

20. $\int \left(\sqrt[4]{x^3} + 1\right) dx$

21. $\int \frac{1}{x^3}\,dx$

22. $\int \frac{1}{x^4}\,dx$

23. $\int \frac{x^2+x+1}{\sqrt{x}}\,dx$

24. $\int \frac{x^2+1}{x^2}\,dx$

25. $\int (x+1)(3x-2)\,dx$

26. $\int (2t^2 - 1)^2\,dt$

27. $\int y^2\sqrt{y}\,dy$

28. $\int (1 + 3t)t^2\,dt$

29. $\int dx$

30. $\int 3\,dt$

In Exercises 31–38, evaluate the trigonometric integral and check the result by differentiation.

31. $\int (2\sin x + 3\cos x)\,dx$

32. $\int (t^2 - \sin t)\,dt$

33. $\int (1 - \csc t \cot t)\,dt$

34. $\int (\theta^2 + \sec^2\theta)\,d\theta$

35. $\int (\sec^2\theta - \sin\theta)\,d\theta$

36. $\int \sec y(\tan y - \sec y)\,dy$

37. $\int (\tan^2 y + 1)\,dy$

38. $\int \frac{\sin x}{1 - \sin^2 x}\,dx$

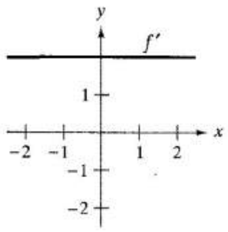
In Exercises 39 and 40, sketch the graphs of the function $g(x) = f(x) + C$ for $C = -2$, $C = 0$, and $C = 3$ on the same set of coordinate axes.

39. $f(x) = \cos x$

40. $f(x) = \sqrt{x}$

In Exercises 41–44, the graph of the derivative of a function is given. Sketch the graphs of *two* functions that have the given derivative. (There is more than one correct answer.)

41.



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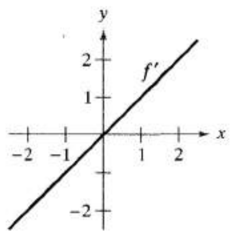

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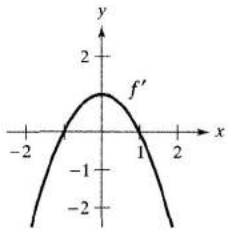

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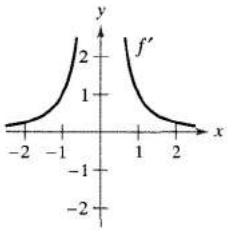
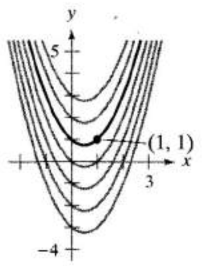
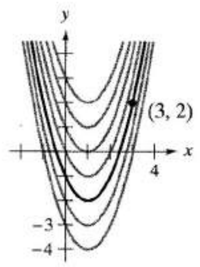


In Exercises 45–48, find the equation for y, given the derivative and the indicated point on the curve.

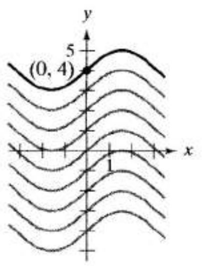
45. $\frac{dy}{dx} = 2x - 1$



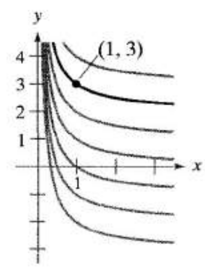
46. $\frac{dy}{dx} = 2(x - 1)$



47. $\frac{dy}{dx} = \cos x$

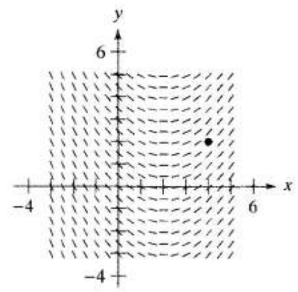


48. $\frac{dy}{dx} = -\frac{1}{x^2}, \quad x > 0$

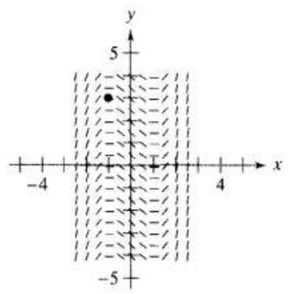


Direction Fields **In Exercises 49 and 50, a differential equation, a point, and a direction field are given. A *direction field* consists of line segments with slopes given by the differential equation. These line segments give a visual perspective of the directions of the solutions of the differential equation. (a) Sketch two approximate solutions of the differential equation on the direction field, one of which passes through the indicated point. (b) Use integration to find the particular solution of the differ- ential equation and use a graphing utility to graph the solution. Compare the result with the sketches in part (a).**

49. $\frac{dy}{dx} = \frac{1}{2}x - 1, \ (4, 2)$



50. $\frac{dy}{dx} = x^2 - 1, \ (-1, 3)$



In Exercises 51–54, solve the differential equation.

51. $f''(x) = 2, \ f'(2) = 5, \ f(2) = 10$

52. $f''(x) = x^2, \ f'(0) = 6, \ f(0) = 3$

53. $f''(x) = x^{-3/2}, \ f'(4) = 2, \ f(0) = 0$

54. $f''(x) = \sin x, \ f'(0) = 1, \ f(0) = 6$

55. ***Tree Growth*** An evergreen nursery usually sells a certain shrub after 6 years of growth and shaping. The growth rate during those 6 years is approximated by

$$\frac{dh}{dt} = 1.5t + 5$$

where t is the time in years and h is the height in centimeters. The seedlings are 12 centimeters tall when planted ($t = 0$).

(a) Find the height after t years.

(b) How tall are the shrubs when they are sold?

56. ***Population Growth*** The rate of growth dP/dt of a population of bacteria is proportional to the square root of t, where P is the population size and t is the time in days ($0 \le t \le 10$). That is,

$$\frac{dP}{dt} = k\sqrt{t}.$$

The initial size of the population is 500. After 1 day, the population has grown to 600. Estimate the population after 7 days.

Vertical Motion **In Exercises 57–60, use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)**

57. A ball is thrown vertically upward from the ground with an initial velocity of 60 feet per second. How high will the ball go?

58. Show that the height above the ground of an object thrown upward from a point s_0 feet above the ground with an initial velocity of v_0 feet per second is given by the function

$$f(t) = -16t^2 + v_0t + s_0.$$

59. With what initial velocity must an object be thrown upward (from ground level) to reach the top of the Washington Monument (approximately 550 feet)?

60. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant it is 64 feet above the ground.

(a) How many seconds after its release will the bag strike the ground?

(b) At what velocity will it hit the ground?

Vertical Motion **In Exercises 61–64, use $a(t) = -9.8$ meters per second per second as the acceleration due to gravity. (Neglect air resistance.)**

61. Show that the height above the ground of an object thrown upward from a point s_0 meters above the ground with an initial velocity of v_0 meters per second is given by the function

$$f(t) = -4.9t^2 + v_0t + s_0.$$

62. ***Grand Canyon*** The Grand Canyon is 1600 meters deep at its deepest point. A rock is dropped from the rim above this point. Express the height of the rock as a function of the time t in seconds. How long will it take the rock to hit the canyon floor?

63. A baseball is thrown upward from ground level with a velocity of 10 meters per second. Determine its maximum height.

64. With what initial velocity must an object be thrown upward (from ground level) to reach a maximum height of 200 meters?

65. ***Lunar Gravity*** On the moon, the acceleration due to gravity is -1.6 meters per second per second. A stone is dropped from a cliff on the moon and hits the surface of the moon 20 seconds later. How far did it fall? What was its velocity at impact?

66. ***Escape Velocity*** The minimum velocity required for an object to escape earth's gravitational pull is obtained from the solution of the equation

$$\int v\,dv = -GM\int \frac{1}{y^2}\,dy$$

where v is the velocity of the object projected from the earth, y is the distance from the center of the earth, G is the gravitational constant, and M is the mass of the earth.

Show that v and y are related by the equation

$$v^2 = v_0^2 + 2GM\left(\frac{1}{y} - \frac{1}{R}\right)$$

where v_0 is the initial velocity of the object and R is the radius of the earth.

Rectilinear Motion **In Exercises 67–70, consider a particle moving along the x-axis where $x(t)$ is the position of the particle at time t, $x'(t)$ is its velocity, and $x''(t)$ is its acceleration.**

67. $x(t) = t^3 - 6t^2 + 9t - 2, \quad 0 \le t \le 5$

(a) Find the velocity and acceleration of the particle.

(b) Find the open t-intervals on which the particle is moving to the right.

(c) Find the velocity of the particle when the acceleration is 0.

68. Repeat Exercise 67 for the position function

$$x(t) = (t - 1)(t - 3)^2, \quad 0 \le t \le 5.$$

69. A particle moves along the x-axis at a velocity of $v(t) = 1/\sqrt{t}$, $t > 0$. At time $t = 1$, its position is $x = 4$. Find the acceleration and position functions for the particle.

70. A particle, initially at rest, moves along the x-axis such that its acceleration at time $t > 0$ is given by $a(t) = \cos t$. At the time $t = 0$, its position is $x = 3$.

(a) Find the velocity and position functions for the particle.

(b) Find the values of t for which the particle is at rest.

71. ***Acceleration*** The maker of a certain automobile advertises that it takes 13 seconds to accelerate from 25 kilometers per hour to 80 kilometers per hour. Assuming constant acceleration, compute the following.

(a) The acceleration in meters per second per second

(b) The distance the car travels during the 13 seconds

72. ***Deceleration*** A car traveling at 45 miles per hour is brought to a stop, at constant deceleration, 132 feet from where the brakes are applied.

(a) How far has the car moved when its speed has been reduced to 30 miles per hour?

(b) How far has the car moved when its speed has been reduced to 15 miles per hour?

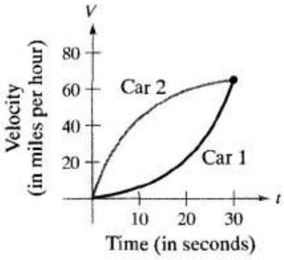
(c) Draw the real number line from 0 to 132, and plot the points found in parts (a) and (b). What can you conclude?

73. ***Acceleration*** At the instant the traffic light turns green, a car that has been waiting at an intersection starts with a constant acceleration of 6 feet per second per second. At the same instant, a truck traveling with a constant velocity of 30 feet per second passes the car.

(a) How far beyond its starting point will the car pass the truck?

(b) How fast will the car be traveling when it passes the truck?

74. ***Think About It*** Two cars starting from rest accelerate to 65 miles per hour in 30 seconds. The velocity of each car is shown in the figure. Are the cars side by side at the end of the 30-second time interval? Explain.



75. ***Acceleration*** Assume that a fully loaded plane starting from rest has a constant acceleration while moving down a runway. The plane requires 0.7 mile of runway and a speed of 160 miles per hour in order to lift off. What is the plane's acceleration?

76. ***Airplane Separation*** Two airplanes are in a straight-line landing pattern and, according to FAA regulations, must keep at least a 3-mile separation. Airplane A is 10 miles from touchdown and is gradually slowing its speed from 150 miles per hour to a landing speed of 100 miles per hour. Airplane B is 17 miles from touchdown and is gradually slowing its speed from 250 miles per hour to a landing speed of 115 miles per hour.

(a) Assuming the deceleration of each airplane is constant, find the position functions s_1 and s_2 for airplane A and airplane B. Let $t = 0$ represent the times when the airplanes are 10 and 17 miles from the airport.

(b) Use a graphing utility to graph the position functions.

(c) Find a formula for the magnitude of the distance d between the two airplanes as a function of t. Use a graphing utility to graph d. Is $d < 3$ for some time prior to the landing of airplane A? If so, find that time.

(d) If the airplanes do not keep the required separation, determine how much the 250 miles per hour speed for airplane B should be reduced in order to meet FAA requirements.

Marginal Cost **In Exercises 77 and 78, find the cost function and average cost for the given marginal cost and fixed cost (cost when $x = 0$).**

	Marginal Cost	*Fixed Cost*
77.	$\frac{dC}{dx} = 2x - 12$	\$50
78.	$\frac{dC}{dx} = \frac{\sqrt[4]{x}}{10} + 10$	\$2300

In Exercises 79 and 80, find the revenue and demand functions for the given marginal revenue.

79. $\frac{dR}{dx} = 100 - 5x$

80. $\frac{dR}{dx} = 100 - 6x - 2x^2$

True or False? **In Exercises 81–84, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.**

81. Each antiderivative of an nth-degree polynomial function is an $(n + 1)$st-degree polynomial function.

82. If $p(x)$ is a polynomial function, then p has exactly one antiderivative whose graph contains the origin.

83. If $F(x)$ and $G(x)$ are antiderivatives of $f(x)$, then $F(x) = G(x) + C$.

84. If $f'(x) = g(x)$, then $\int g(x)\,dx = f(x) + C$.

85. ***Think About It*** Use the graph of f' in the figure to answer the following, given that $f(0) = -4$.

(a) Approximate the slope of f at $x = 4$. Explain.

(b) Is it possible that $f(2) = -1$? Explain.

(c) Is $f(5) - f(4) > 0$? Explain.

(d) Approximate the value of x where f is maximum. Explain.

(e) Approximate any intervals in which the graph of f is concave upward and any intervals in which it is concave downward. Approximate the x-coordinates of any points of inflection.

(f) Approximate the x-coordinate of the minimum of $f''(x)$.

(g) Sketch an approximate graph of f.

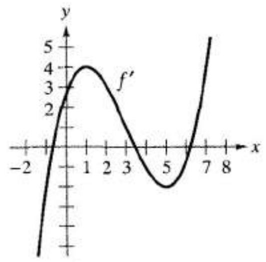


Figure for 85

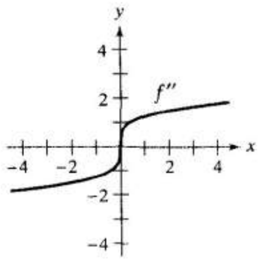


Figure for 86

86. ***Think About It*** The graphs of f and f' each pass through the origin. Use the graph of f'' shown in the figure to sketch the graphs of f and f'.

87. ***Acceleration*** Galileo Galilei (1564–1642) stated the following proposition concerning falling objects: The time in which any space is traversed by a uniformly accelerating body is equal to the time in which that same space would be traversed by the same body moving at a uniform speed whose value is the mean of the highest speed of the accelerating body and the speed just before acceleration began. Use the techniques of this section to verify this proposition.

88. Let $s(x)$ and $c(x)$ be two functions satisfying $s'(x) = c(x)$ and $c'(x) = -s(x)$ for all x. If $s(0) = 0$ and $c(0) = 1$, prove that $[s(x)]^2 + [c(x)]^2 = 1$.